## Highway Structures
### Design Handbook
#### Volume II

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Introduction

The Through Truss

Early in the development of highways there were few crossings that required grade separation structures. Roads were generally dirt, and they followed the earth's contour without regard to the extent of either vertical or horizontal visibility. At stream-crossings, approach roads were curved to permit bridge-building perpendicular to the stream. Often the road had to be raised at the end of the bridge so that the bridge could clear the local high water mark. Loads were few and light; roads and bridges were narrow.

For the most part, construction depended on mulepower. Most mechanical equipment was light and could handle only light members. In general, the available heavy steam powered equipment lacked mobility; and any that had mobility faced a problem of narrow roads. Except for rail, transportation equipment was also light; and construction sites, unless adjacent to railroads, were not readily accessible. Engineers therefore found it necessary to design bridges made up of relatively small, light members easy to transport and erect.

Nearly all early steel highway bridges were through trusses. These were economically satisfactory even for short spans (30 feet). Because roadways were narrow and loadings light, through and pony trusses presented virtually no problems related to heavy floor systems or to efficient use of materials.

A through truss gave maximum clearance over high water, with minimum increase in approach grades. Fabrication and material costs were low; and even in the short spans, dead loads were a relatively large part of the total load. Frequently, higher stresses were allowed for dead loads than for live loads.

Truss members were generally made up of

FIG. 1. EARLY HIGHWAY CONSTRUCTION, GEORGIA
angles and plates riveted together; or angles with lattice bracing were riveted to form open I-shaped or box-type members. Riveted angle and plate construction made up the floor beams—sometimes as little as 15 feet long. Over these beams, timber stringers were often used—floored with edge-laid timbers spanning the stringers transversely.

The steel in general use had a yield point of about 30,000 psi. High strength steels were available, but their use was restricted to longer spans—usually over 1000 feet. Stringers and floor beams on the longer spans were riveted. Stringers were spanned by steel buckle plate or timber. For the bridges utilizing buckle plate, some of which are still in use, brick or wood block provided the riding surface.

**Plate Girders**

Railroads gave the first big push to the use of girder-type bridges in place of the early iron bridges and timber trestles. To minimize interruptions of train service, the railroads naturally sought a structural system allowing the shortest possible construction time. They also needed, for short spans, a structure with high shear capacity. Steel plate girders provided the solution for many of their short-span structures.

Girders could be and were completely fabricated at shops having railroad sidings. It was easy to load the girders on flat cars with shop equipment and ship by rail directly to the bridge site. The mobile steam powered railroad cranes (for righting derailed trains) that were already in widespread use had capacity for erecting bridge girders.

Because of railroad requirements for flat grades, elevations of approaches and bridges at many stream crossings were greater than high water clearance. Therefore, both deck girders and through girders were used. Because of stability requirements for nailing and locomotive and train sway, deck girders were sometimes spaced more than track width and needed floor beams and stringers as on through girders and trusses. The floor system when used was made up—as in highway bridges—of riveted floor beams and stringers.

**Rolled Beam Bridges**

The combined requirements of railroads and highways, augmented by building needs, were recognized by the steel industry. In the 1920’s, to help meet these needs at the lowest practical cost, wide-flange rolled beams large enough for use in bridges were produced. These not only provided ready-made, economic members but were less susceptible to fatigue. At this time, fatigue was usually considered to be taken care of in design by the use of large impact factors.

About the time large rolled beams became available, gasoline power had replaced mule-power. To handle the requirements of the automobile and truck, highways were improved throughout the country. Paving became a commonplace instead of a rarity in highway construction. Bridge sites became more accessible; and mobile construction equipment, transportable over the highways, was developed.

Highway bridge engineers, spurred by the need for wider bridges to handle the new traffic, recognized the advantages of deck-type bridge systems. The availability of rolled beams, decked either with reinforced concrete or timber, provided the economic feasibility for the deck system as a complete superstructure for highway bridges. For the shorter spans, weights and lengths of individual beams were little more than those of some truss members previously used. Further, the rolled beams were little deeper and sometimes less deep than the floor system required for the through truss. This allowed bridge elevations to remain the same.

As transportation, construction techniques, and construction equipment advanced, longer and heavier members were designed for the deck structures. As width requirements continued to increase, through truss construction became relatively more expensive. By the late thirties, rolled beam deck-type highway bridges were in general use for spans up to about 70 feet and were sometimes used for spans up to 90 feet. Use of riveted plate girders increased deck spans to 120 to 150 feet. Simple spans were used almost exclusively; continuous construction was rarely considered. Longer spans remained as through trusses. Over 400 feet or so cantilever construction and occasionally arches were used.

**Continuous Construction**

Bridge engineers were aware of the efficiencies of continuous construction. Through this method, maximum moments could be concentrated in short distances near the supports, resulting in a sizeable reduction in the relatively flat and long midspan portion of the moment curve; moreover, there could be a desirable reduction in the number of expensive expansion
joints and bearing assemblies.

At this time classical methods of analysis had not been extensively used on continuous members of variable cross section. Not only was analysis a problem; there was also much concern about possible differential pier settlement and its effect on continuous construction. There was a general lack of highly trained foundation exploration teams; in some areas, soil mechanics was not considered a fully developed science.

To obtain part of the advantage of continuity, cantilevers with suspended spans came into use on plate girder spans. The upper span length of plate girders was extended well above 100 feet, and economies were improved in spans under 100 feet. However, while cantilevers reduced the number of bearing assemblies, they still required the same number of expensive expansion assemblies as did simple spans. Further, hanger and hanger assembly costs were added.

In the late 1930's, moment distribution began to be used as a tool in the analysis of indeterminate structures. This procedure simplified the problem considerably and designers began to investigate more thoroughly the potential of continuous construction. Initial studies were made primarily on bridges founded on rock or on virtually unyielding piles. Then, highway construction was brought almost to a standpoint by the Second World War.

After the war, highway construction surged. Under 70 feet, simple spans made of rolled beams were still dominant. From 70 to 90 feet, cantilever spans of rolled beams or riveted plate girders were common, although some continuous construction was being used. Spans over 90 feet and up to 150 feet were riveted plate girders—some simple spans, some cantilever, and some continuous.

In the late 1940's, college graduates entering the field of engineering were versed in the easier design solutions for indeterminate structures, and were eager to put these approaches to work. With this knowledge available increasing numbers of investigations were made on the use of continuous girder bridges.

Nearly all girder bridges were still using riveted construction. Possible extension of deck plate girder span length was being investigated, and some continuous spans exceeding 300 feet were built. With the addition of riveted cover plates, primarily in negative moment regions, continuous rolled beam construction for deck structures was found economical in spans up to 100 feet.

Welding

Post-war highway construction changed radically from prewar construction. Greater emphasis was put on speed of travel, both by individuals and by trucking firms eager to compete with the railroads. It was no longer feasible or desirable to make a road fit a bridge location by putting curves at each end of the bridge. Instead, bridges were designed and constructed to fit the requirement of a total system. Skews and horizontal curves in bridges became usual rather than rare, and bridges over highways became a necessity for speed and safety.

During the war, considerable experience was gained in the use of welding. To conserve materials and speed construction, welded ships were produced at a fast rate. The technology of welding was greatly increased and its application to bridges was realized. In Europe, where materials were still in short supply, it was important to make as efficient use of steel as possible. Welded construction, with savings of about 15 per cent in material, was common for most steel bridges—long or short span. This resulted in more functional structures with less material and, therefore, less dead load. Girder spans could be, and were, extended beyond the previous economic lengths.

Although most of the fabricating shops in this country were geared to riveted construction, designers investigated the possibilities of welded bridges. Extensive use was made of welded cover plates on rolled beam structures and of weldments for bearing assemblies. Since less equipment and less costly setups were required for welding, smaller fabricators found the means to handle larger bridge projects and rapidly gained experience in such work.

Composite Construction

During this period, composite construction—with the concrete deck slab an integral part of the superstructure beams—was finding favor with some designers. New types of connectors (to resist the horizontal shear between concrete deck and steel beam) were developed and marketed by the manufacturers. Through research and sales promotion, they added impetus to composite construction and furthered economies in bridge construction.

Studs, channels and spirals were the primary shear connectors used for composite construction. Besides tying the deck slab to the steel beams and girders, forming an integral unit with
a large capacity for overloads, composite construction allowed a reduction in the structural steel required for the primary beams. Most of this was reflected in smaller areas for top flanges of beams and girders, although bottom flanges were also somewhat reduced.

The greatest materials saving was apparent in plate girder spans, where top flanges could be reduced to a minimum consistent with erection requirements. Rolled beams were made asymmetrical by the addition of welded cover plates to the bottom flanges only. This not only increased economy in the spans where rolled beams were previously applicable, but also increased their usable length of span.

Composite construction led to a further change in the application of plate girders. For spans 150 feet and over, girders with floor beams and stringers had been considered the most economical construction. However, with the concrete deck supplying a large part of the top flange, economies were available in longer span multi-girder bridges without floor beams and stringers.

With the experience gained on welded cover plates, designers pushed toward all-welded plate girder construction in spans greater than 100 feet. Fabricating shops acquired automatic and semi-automatic welding equipment; and portable radiographic equipment became available for inspecting welds. In a further development of connections, high strength bolts (which make joints less influenced by fatigue) were introduced and began to supplant field riveting on some structures. Except for truss spans, riveted construction declined rapidly, as did the art of riveting itself.

As welding came to the fore, new steels were produced with the toughness needed for welded construction. These steels, with yield points ranging from 33,000 psi up to 100,000 psi, were made available and were quick to catch on. With the reduction in weight made possible by welding and by the higher strength steels, continuous girders began to be used in spans greater than 250 feet.

Labor rates increased rapidly in all industries during the 1950's. Fabricating shops were no exception, and fabricating costs for steel bridges started on a steady climb. Materials costs also increased, but not at a comparable rate. It therefore became more and more important to simplify fabrication wherever possible. There was a further increase in the economic span length of plate girders requiring more material than trusses. "In line" fabrication methods were exploited wherever practical in bridge design and construction.

By late 1950, except for the larger fabricating shops, few shops were capable of handling riveted construction efficiently. Where riveting had previously been specified, welding or high strength bolts were used. Towers for suspension bridges had been designed and constructed using shop welding, with high strength bolts for field connections; and the new 100,000 psi yield point steels were being used for welded construction of truss members.

Orthotropic Plate

As esthetics in bridge construction received greater consideration, the demand increased for girder type structures in the longer span ranges. In all long spans, dead load plays a dominant role; and methods were sought to reduce this loading wherever possible. Higher strength steels aided in this quest; an additional means of reducing dead loads was to utilize stiffened steel decks having a two-purpose function—both as the top flange of the girder and as the deck.

Appropriate design procedures were devised for the "orthotropic" plate, and girders with stiffened steel deck flanges are currently in place. Some of these structures have spans of more than 700 feet. Probably for all bridges in excess of 400 to 500 feet, trusses still offer maximum economies although they may not be considered as pleasing esthetically.

Advances

Bridge design and construction has now become very sophisticated. Methods of analysis have advanced so that multi-redundant indeterminate structures can be handled with comparative ease. The electronic computer has provided tremendous aid in this direction.

Nearly all state highway departments and bridge consultants have (or have access to) excellent foundation investigating teams. Soil mechanics have improved steadily, and it is now possible to predict future settlement of piers founded on elastic materials. With this knowledge, continuous structures can be designed taking into full account the stresses that may be induced by future pier settlement.

Research facilities have improved, particularly at the university level, and are in considerable demand for work in the highway field. Many re-
search projects directly related to bridges have led to improved design solutions and, to a large degree, many past unknowns have been clarified. This, in turn, has led to a better knowledge of the true action and capacity of bridge structures.

Construction and fabrication techniques and equipment have kept pace with design. Methods have been devised for continuous structures whereby all splices can be made at the bridge site prior to the erection of beams. Full length beams covering several spans are erected without the need for false bents. In fabrication, semi-automatic and automatic equipment and procedures are in use for welding, punching and drilling. All these have improved the economics of fabrication.

Materials have also improved. Higher strength steels are available for specific as well as for general use. Weldable steels are available in almost all strength ranges applicable to bridges.

All of these factors have been considered and are currently being used to economic advantage. In design, structures are being viewed more integrally, rather than as a combination of related but independent pieces. These structures have greater total capacity and at the same time make more efficient and economic use of materials.

Traffic and its related problems have increased at an almost alarming rate. By means of traffic counts and loadometer readings, much information has been gained on the actual loads and load frequencies that bridges are subjected to. However, while more knowledge has been gained about actual loads, control of these loads has not necessarily kept pace.

In highway design the accent is on speedy movement of traffic. This, plus the related needs of other methods of travel, has brought bridge engineers into closer contact with one another. Mutual assistance in the solution of common problems has made better use of available engineering talent. Various engineering groups outside of highway groups, such as ASCE, have also been active in the interchange of ideas and knowledge. And industry has helped by spreading the best available knowledge of particular materials or products.

However, in many respects, bridges have remained unchanged. The basic objective is the same — to cross a relatively wide expanse usually with a relatively narrow elevated path. The primary design problem is therefore longitudinal. Although trusses and other forms have been supplanted in many spans by beams and girders, the structures still perform the same function in a similar manner. Hence, the great strides in bridge engineering have been in better representations of the same systems.

The resulting structures are much more functional and at the same time, have gained in aesthetic values. Spans once considered economically and technically impractical are now designed as a matter of everyday engineering practice.

**Short Span Bridges**

**General**

In this discussion of economy, short span bridges will be considered those for which rolled beams (in areas where they are available) are generally applicable. For simple spans, this definition provides for a maximum of about 80 feet. For continuous construction, the maximum intermediate span is about 110 feet, and a maximum end span is about 90 feet.

Deck structures utilizing rolled beams are now in vogue because they are the most economical type of construction for these spans. However, in some areas where fabricating costs are relatively low, there is use of welded plate girders rather than rolled beams in both simple and continuous spans over 80 feet.

Composite construction, which received serious consideration in the 1950's, is now in general use and is applicable for spans over 40 feet. The minimum span for continuous composite construction is about 60 feet. For shorter spans (less than 40 to 60 feet), where composite action is not advantageous, fabrication should be kept at a minimum; therefore beams not requiring cover plates are generally selected.

**Beam Spacing**

In all short span deck bridges, it will be economical to use designs that do not require floor beams or excessive deck thicknesses but do require a minimum practical number of longitudinal beams. The validity of the latter requirement can be shown by considering the properties of available wide flange rolled beams. The section modulus of WF beams can be expressed as a constant times the product of the area and depth of the beam. Published data indicate that the
constant is equal to about 1/3 for nearly all WF beams used in bridges. Hence, four beams having the same depth and total area as five beams will have the same total capacity. Since the total area is constant each of the four beams must have a greater area than each of the five beams, the greater individual beam area will nearly always allow selection of beams with greater depth and consequently greater total capacity.

To achieve maximum economy, the typical 28’ roadway bridge with a total width of 33’ to 34’ should have four beams at a spacing of about 8’-6”. This provides a distance of 25’-6” out to out of beams, and requires deck cantilevers of 3’-9” to 4’-3” for the remaining roadway plus safety curbs and handrails. With this spacing and a design concrete strength of 4,000 psi (the minimum suggested for durability), a 7” deck will be necessary. Since this is the minimum deck depth allowed in many states, the deck will be used at its maximum efficiency. The 4’ ± cantilever balances the deck moments in the 8’-6” distance between beams, and also produces nearly equal longitudinal moments in the interior and exterior beams.

As a general guide for all spans and bridge widths, a spacing of 7’-0” to 8’-6” for rolled beams should be used wherever practical. However, at the upper limit of the rolled beam spans, it may be desirable to decrease the beam spacing and increase the number of beams. For the same typical 28’ roadway bridge, better results will be obtained with five beams at a spacing of about 6’-6”.

There are, of course, exceptions to these general guides to transverse spacing of wide flange beams. Occasionally, when it is not possible to obtain an efficient section for the beams when the suggested minimum number is used, economy may be found by adding an extra beam. Some economies may also be available in certain short span designs—such as the T-Beam, or those which use widely spaced WF beams and a two way deck slab—that deviate from the above general guides.

**Spans to 40’**

At the lower end of short span bridges non-composite beams of A36 steel are used almost exclusively. Some gain in efficiency of materials can be achieved even in these spans by the use of composite construction. However, the actual material savings in pounds is small, and the corresponding cost savings is more than offset by fabricating costs.

Beam depths required will be between 21” and 30”, so that there is ample latitude for selection of efficient sections. Since the total weight of the steel will be small, each fabrication item will be reflected as a relatively large percentage of the materials cost; simplicity in details is therefore especially desirable.

By AASHO specifications, diaphragms or cross frames are required at intervals not to exceed 25’. Since, in these short spans, the stiffness of the concrete deck is large compared to the stiffness of the diaphragms, the latter serve primarily as beam spacers and should be as light as practical. Lightweight channels (depth 1/2 that of the beams) will be ample, and should be field connected to the beams by bolting or welding to plates that have been either shop or field welded to the beam web. For spans over 25’ and up to 50’, one such diaphragm will be required at the center line of the span.

Over intermediate supports of continuous bridges, channel diaphragms similar to those at the span center line are satisfactory. At expansion joints, which occur at the ends of simple spans and at the ends of continuous units, the end of the concrete deck should be protected by increasing the deck thickness—this provides a concrete edge beam. Slotted holes should be called for in the ends of the steel beams, to allow placement of reinforcing steel required for the edge beam.

Continuity should always be considered for these spans. Savings will be obtained primarily through reduction in number of bearing assemblies and expansion joints. For example, a four span continuous bridge will require five bearing assemblies per beam, whereas four simple spans will require eight bearing assemblies. Cover plates should not be used, since the cost of fabrication will more than offset the material savings.

An end to intermediate span ratio of 1 to 1 1/2 will provide near-balanced continuous design—i.e. negative moments at all intermediate supports will be nearly the same and positive moments in each span will also be nearly the same. Since the negative moments will be greater than the positive moments, the design will be for the negative moments. Necessary splices should be shown as optionally welded or bolted.

A design for short spans that deviates from the general guides is one making use of widely spaced longitudinal beams, transverse beams
and a two-way deck. Beam spacing on constructed spans has varied from 12' to 14'. Transverse beams, forming the additional supports required for the 2-way slab, have been spaced from 15' to 20'. The advantages of this design include reduced requirements for the deck, more efficient use of longitudinal beams, and reduced number of bearing assemblies. The chief disadvantage is the additional fabrication required for transverse beam connections. Bearings should be either flat steel plates or elastomeric pads. The flat steel plates will be inexpensive since cutting of curved surfaces is not required. Both steel plates and elastomeric pads should bear on thin sole plates welded to the beam flange. On continuous spans, sole plates at intermediate supports should be wider than the beam flange. Round or slotted holes for anchor bolts can then be placed in the extension beyond the beam flange, so that the full beam section can be utilized for moment resistance.

**Spans 40' to 110'**

Rolled wide flange beams and welded plate girders have been used economically in spans from 40' to 110'. When the general beam spacing guides can be followed, wide flange beams will usually provide the greater economy. Spans in the upper limit of this range may require an excessive number of wide flange beams. When six or more beams (or beams heavier than 170 pounds) are necessary, welded plate girders will frequently be more economical.

Wide flange rolled beams can be used for simple spans up to about 90'—with 80' as about the economic limit. There are indications that ASTM A441 steel will provide some economies where deflections are not a controlling design criteria. In all cases, simple spans should be investigated as composite beams.

Partial length welded cover plates added to the bottom flange will improve the economies of composite construction. It is preferable to use a single unplated plate of constant width and thickness. The width should be less than that of the beam flange and the ends should be cut square, so that the attaching fillet welds can be continuous around the plate perimeter. This arrangement provides the best resistance to fatigue (by eliminating the danger of undercuts at corners), and also simplifies welding procedures.

Composite welded plate girders should be investigated for simple spans over 80'. Girders of ASTM A 36 and A 441 steels will cost about the same. Spacing should be much the same as for rolled beams except on wider structures, where it is better to increase the spacing of the welded girders if a change from the 8'-6" figure is necessary. The reason for this is the increase in efficiency that can be obtained with an increase in depth for the same total beam area.

A minimum top flange, consistent with erection requirements, will usually satisfy design stresses. Top flanges will rarely exceed 12" in width and ¾" in thickness and will often be only ½" thick. The 12" width provides ample space for placing shear connectors, such as studs, in groups of sufficient numbers to allow a longitudinal spacing that will not interfere with the deck reinforcing steel. As an aid to deck construction, the connectors can be spaced longitudinally in multiples of one half the deck steel spacing. This prevents interference of the connector with the deck steel and also provides a guide for deck construction.

The bottom flange generally should be made from three plates of two sizes: a center plate covering approximately the mid 60% of the span, and two end plates butt welded to the center plate. Plate sizes correspond to the requirements of the moment curve. Splice costs will usually offset any material savings available through increasing the number of changes in flange plate size beyond the two that are recommended.

Frequently, ¾" unstiffened or ½" stiffened web plates will satisfy the shear and buckling requirements for welded plate girders in spans under 100'. While the ¾" stiffened plate has less material, the final cost after fabrication will usually be less with the ½" web. If unstiffened web plates thicker than ¾" are required, it will be economical to use thinner webs with transverse stiffeners. For maximum savings, the stiffeners should be welded.

Non-composite construction is economical for continuous spans having end spans less than 50' and intermediate spans less than 65'. In these spans, it will be advantageous to use beams requiring cover plates in the regions of negative moment. Only one cover plate should be used on each flange; plates should be of constant size, with a width less than the beam flanges. Beams requiring positive moment cover plates usually will not offer economies. Other fabrication should preferably be restricted to connections for angle or channel diaphragms and to beam end preparation for field splices.
It is desirable to locate field splices at or near points of dead load contraflexure. Designing and detailing the splices for both welding and bolting will permit the contractor to select the method suitable for his equipment. In these spans, splices at each point of contraflexure may not be necessary. Here, too, a design option will allow the contractor and fabricator freedom to select the cheapest construction method.

Composite construction should be considered for continuous spans greater than 65'. Usually WF beams can be selected requiring partial length cover plates in both positive and negative moment regions. In the positive moment region, a cover plate on the bottom flange alone will be effectively balanced by the concrete deck serving as top flange cover plate.

Studies to date indicate that costs are about the same for composite construction used only in the positive moment regions as for composite construction over the total continuous span. Where composite construction is used in the negative moment region, top flange cover plates will be smaller than those on the bottom flange. Longitudinal reinforcing steel in the concrete deck provides the additional resistance needed for live load moments. Although the cost of reinforcing steel per pound is less than that of structural steel, this usually will be offset by the cost of the additional connectors.

In areas of relatively low cost fabrication, welded plate girders may be less expensive than wide flange beams for continuous spans in excess of about 80'. The girder in the positive moment regions, should have the same characteristics as those of a simple span composite welded girder. Symmetrical girders are necessary in non-composite negative moment regions. Where composite construction is utilized in negative moment regions, the top flange plate will be smaller than the bottom flange plate. In either case, flange plates required at the point of maximum moment (over the support) should be continued for a short distance of about 1/4 the span into end spans, and 3/8 the span into intermediate spans. A reduction in flange plate size should be made at these points. No other reduction should be made in the negative moment region.

Maintaining composite construction over the total span is advantageous in satisfying fatigue requirements at the dead load points of contraflexure. At these points in the partially composite girder, fatigue may govern plate sizes; this will not ordinarily be true in the totally composite structure.

An empirical factor of S/5.5 is authorized in the AASHO specifications to determine the number of wheel loads for which each beam is designed. This factor is based on studies confirming the ability of the concrete deck to distribute live loads transversely. No credit is given in this factor for additional distribution effects resulting from the use of heavy diaphragms. Although the empirical factor is authorized, the use of a more sophisticated procedure such as grid analysis is not prohibited.

When designs are made on the basis of the empirical factor, diaphragms should be of minimum size. Three angles, one placed horizontally near the bottom flange and two placed diagonally, will usually be adequate and will result in a minimum amount of material. The angles can be connected by welding or bolting to vertical plates welded to the beam webs. At the ends of simple spans and at the ends of continuous units, diaphragms must also serve to support the end of the concrete deck—the recommended support is a channel of minimum weight consistent with the edge beam requirements. Connection to the beam web should be similar to that for the cross frames.

In some areas elastomeric pads have recently replaced the more expensive fabricated bearing assemblies. While these pads have been used primarily in concrete construction, they can also be economical in steel construction. For spans under 80', a single-thickness pad will allow the necessary movement for expansion and contraction; laminated pads will be required for movement in excess of that anticipated for an 80' span.

It is common in simple span construction for each span to have one fixed end and one expansion end. On many of these structures, armored expansion joints are used over each pier. In addition to being expensive, they are rarely leak-proof. Extruded or preformed plastics give promise of providing a better and less expensive joint requiring less maintenance. These are applicable not only for simple span joints but also for joints at the ends of continuous units.

Precambering of rolled beams is expensive and unnecessary for the typical bridge, since some inaccuracies will always exist for which provision must be made. This is usually done by means of a variable coping depth over the beams. The same method can be used in obtain-
ing the correct deck thickness and final deck elevation for the uncambered beam. The minor additional coping concrete required will cost less than the precambering.

The subject of deflections is of considerable importance, especially where wide flange beams of higher strength steel are used. Wide flange beams of A441 steel are economical in the 80’ span range, provided the design is governed by stresses and not by deflections. The AASHO specification requires that live load deflection be limited to 1/800 of the spans; this will often be difficult to meet with wide flange beams of steels other than A36, unless composite construction is utilized. When both A441 steel and composite construction are used in the same structure, the economies are additive, producing an attractive cost picture.

The composite wide flange rolled beam example in Chapter 4 illustrates some of the economic considerations previously discussed. For this two span continuous structure, the requirements for negative moments are satisfied almost precisely by a 36 WF 135 pounds, with 10” x 1½” top and bottom cover plates in the negative moment region. The beam is the minimum weight in its section depth, and requires cover plates of maximum thickness allowed by AASHO. This results in a negative moment section having maximum economies.

Unfortunately, the positive moment section is not as economical as the negative moment section, because—if constant depth is maintained—the minimum weight section satisfying the requirements of negative moment must necessarily be used for the full length of the structure. Had the negative moment section required the next larger section (36 WF 150) the 135 pound beam could still have been used in the positive moment region. A larger cover plate would have been necessary than the 10” x ½” one shown in the example and the economies of the positive moment section would have increased.

The box girder design example shown in Chapter 7 illustrates another type of construction and its related economies. The design of this example, as noted in the discussion and calculations, was made for transverse distribution of the live load by the composite concrete deck only. The addition of suitable transverse cross frames would have improved this distribution.

The main advantage of the box girder over the plate girder lies in its torsional rigidity, which improves the transverse live load distribution and reduces the total load for which the structure must be designed. This, in turn, reduces the amount of steel necessary to resist the forces and loads to which the structure is subjected.

Quantities and costs for simple span and continuous span box girders have been compared with quantities and costs for the latest wide flange beam design suggested by the Bureau of Public Roads for the same span. The assigned unit prices reflected those to be anticipated in two different areas of the country. Judging from these comparisons, the simple span box girder appears to be about 10% cheaper than the wide flange beam design, and the continuous box girder about 10% cheaper than the simple span box girder. The continuity savings will increase for units of more than two spans.

Medium Span Bridges

General

This discussion will consider medium span bridges to be those with spans from 110’ to 350’. Except where absolute minimum clearances are required, welded deck plate girders are generally applicable. For spans above 150’, absolute minimum clearances can usually be provided by utilizing through trusses. Since these are special applications, they will not be discussed here.

All medium span bridges containing two or more spans should be designed and constructed as either continuous or cantilever bridges. Continuous construction is preferred, since it makes more efficient use of materials and provides a greater reduction in number of expansion joints. Continuous construction also obviates the expensive fabrication required at cantilever joints.

Composite construction has been used to economic advantage for all spans in medium span bridges. This is true both for construction similar to rolled beam design (multiple girders) and for construction with girders, floor beams and stringers.

Where floor beams and stringers are not used on two-lane structures, girder spacing should be about the same as for rolled beam bridges. For structures with more than two lanes, girder spacing should increase with increasing span length. Girders for floor beam and stringer construction should be spaced inside the curb faces, so that
cantilever deck moments will closely equal deck moments between and over the stringers.

In general, medium span girders should use webs with transverse stiffeners. Longitudinal web stiffeners should also be used for girders in the upper part of the medium span range.

**Spans 110' to 175'**

Composite construction is very suitable for these spans. In two-lane bridges, of either composite or non-composite construction, maximum economies will be obtained with four girders. For bridges with more than two lanes, the best solution will be provided by five or more girders at 7' to 9' spacing.

Studies made for girders in this span range indicate that ASTM A36 and A441 steels can be used at about the same cost. Where A441 steel is utilized, webs should be made as thin as practical using both transverse and longitudinal stiffeners. Girders utilizing A36 steel may show some economies with longitudinal stiffeners; transverse stiffeners are necessary in any case.

It is generally more economical to build plate girders in the medium span range with straight soffits. In such girders, A441 steel should be utilized in the negative moment region while A36 steel should be used for positive moments. Haunched girders, while providing greater efficiency in use of materials, are more expensive in fabrication. The increased cost of haunched fabrication will usually not offset the material savings.

In the regions of negative moment, no more than three different flange plate sizes should be shown, requiring two butt welded splices. In the regions of positive moment, three plate sizes requiring four butt splices may also be desirable. Because of cost variations, showing flange plate splices as optional will give the fabricator the opportunity to select the cheapest method for his shop.

Bracing should consist of cross frames and a lower lateral system. Cross frames generally require less material and avoid fabrication and erection problems if they are composed of two horizontal angles (one at each flange level) and two diagonal angles forming a cross between girders. Cross frames spaced as near 25' as practical should go between all girders.

The lower lateral system should also be made up of angles, resembling the cross frames in geometric form. The lower horizontal member of the cross frame also serves as a part of the lateral system. Lateral bracing should connect pairs of girders and should be used in alternate bays only.

Bearing assemblies for these spans are usually rockers or rollers. Prior to 1935, these were almost always castings. However, welding has made it feasible to fabricate these assemblies from plates at much lower cost. Some laminated elastomeric pads have also been used as bearings for these spans. They appear to be even less expensive than weldments.

Twin box girders (Chapter 7) are expected to show maximum economy in spans from 110' to 175'. The economies indicated in short spans should improve with increased span lengths. Transverse live load distribution characteristics, which are considerably better than those of the usual plate girder, will remain about the same as for the shorter spans. More efficient use of the wide bottom flange will be an added advantage of the longer span box girder. Although diaphragms or cross frames are not shown in the example, their use is recommended. When used, they should be considered in the design and account taken of their additional aid in transverse live load distribution.

In addition to relatively low cost, box girders provide a structure with low maintenance. Depth to span ratios are minimal; this, together with the clean straight lines, makes for a functional structure with high esthetic appeal.

**Spans 175' to 350'**

As in spans under 175', continuous construction should be used wherever possible. Plate girders utilizing floor beam and stringers may show economies for non-composite construction. For composite construction, the multi-girder system (similar to that for rolled beams) has been applied to spans up to 350'. In these spans, the girders should be spaced about 14' on centers. This tends to limit the multi-girder system to roadway widths in excess of about 40'. For two lane structures, two composite girders should be used with floor beams and stringers. Spacing of girders should be as given in previous discussion.

With span lengths above 175', use of the higher strength steels becomes more economical. They have a particular advantage in the regions of negative moment where A36 steel flange plates may have excessive thickness. By present specifications, webs must be of the same material as flanges. Therefore, where A441 or A514 steels are used in the flanges they must also be used in the webs.
Girders utilizing the higher strength steels should have stiffeners, bracing and connection details of A36 steel. Higher strength steels for these items will reduce neither thicknesses nor widths.

Estimates made for each structure will be necessary to determine the relative economies of haunched girders vs. girders with straight soffits. An advantage of straight soffits for these spans lies in obtaining web depths that may not require horizontal splices. The additional cost of these splices plus the additional cost of haunching may offset the material savings to be gained through use of a haunch.

Lateral bracing, cross frames and bearing assemblies should be the same as those suggested for spans between 110' and 175'. Obviously, larger bearing assemblies are necessary since reactions are greater. However, welded bearing assemblies will still provide the least costly solution. Elastomeric pads, laminated or otherwise, are generally not considered applicable for these spans.

In selecting flange plates, it will usually be advantageous to use the maximum practical width. This is particularly true for the higher strength steels, where the lower yield point of thicker steel requires a corresponding reduction in allowable design stresses. Maximum width of plate will of course result in minimum thickness, maximum yield point and maximum allowable stress.

Box girder bridges, utilizing a single box subdivided into cells about 12' wide, have indicated economies for spans greater than 175'. A variation of this, using two relatively narrow box girders, has also shown promise. This type of structure is very similar to regular plate girder construction, in that floor beams and stringers are used to span the space between the boxes. Usually the two box girders are spaced so that they are near the roadway edges but require deck slab cantilevers, thus making efficient use of the deck thickness required to span the stringers.

**Spans Greater Than 350'**

Each bridge in this span range should be considered a special study and thoroughly examined before deciding which of the various possible solutions is most applicable to the specific structure. Orthotropic plate girder bridges, box girders, trusses, and arches have all been used to advantages. The final choice depends on specific bridge site i.e., vertical clearance, foundations, number of traffic lanes, etc. Continuous or cantilever trusses probably offer the most economical solution for spans in excess of 500' but less than suspension bridge range.

Orthotropic plate girder bridges are considered more aesthetically pleasing than trusses, and have been used for spans up to about 750'. However, the economy of this construction for spans above 600' is doubtful. The girders supporting the steel deck may be either of the conventional type or they may be box sections. In any event, continuous or cantilever construction is a must.

For all of these structures welding has proven to be advantageous. On trusses it has been primarily in shop fabrication for the truss members. Field connections for the truss members are generally made with high strength bolts. On the orthotropic plate girder bridge, welding is almost a necessity — fabrication of the stiffened deck and attachment of this deck to the girder webs could hardly be accomplished by other means. Field splices of the girder sections are usually made with high strength bolts.

**Other Considerations**

**Continuity**

The primary reason for using continuous construction is to obtain the economies it offers over simple and cantilever spans. Although not considered a design feature, the continuous structure also has greater overload capacity. This is true because maximum positive and maximum negative moments occur with the loads at different locations. Thus, yielding at a given location under an overload redistribute the moments and does not mean that failure is imminent.

The maximum dead load moment in continuous spans occurs at the supports. For interior spans, this moment is about 2/3 that of the maximum dead load moment of a simple span that has the same length. The moment decreases rapidly and equals zero at about 1/4 the span length away from the support. At the center of an interior span the dead load moment is about 1/3 that of a corresponding simple span.

The envelope of maximum live load moments for continuous spans will show near equal moments at the span ends and at the center. For an interior span, these moments are about 60% of the maximum live load moments for the equal length simple span. The combined live and dead load continuous beam moments are greater.
at the supports than at the center of the span. Since both the live and dead load support moments reduce rapidly, the maximum beam or girder section required is not only less than that of a simple span, but also occurs over a shorter distance.

Two span continuous structures are not as efficient as those containing more than two spans. This is true because each span in a two span structure has one pin rod end and therefore does not realize the full advantage of continuity. Conversely, the intermediate spans of a three or more span unit has a degree of fixity at each end.

The most advantageous ratio of interior to end spans in continuous construction varies with the ratio of dead to live loads. As the dead to live load ratio increases, the most efficient ratio of interior to end span approaches 1.22. For small dead to live load ratios, as will occur in spans less than 60', interior spans should have a length about ½ greater than end spans. For spans from 60' to about 110', the ratio should decrease to about 1.3. For spans greater than 100', a ratio of 1.3 to 1.25 will provide balanced designs. However, these ratios are more important for wide flange beam spans. Since in plate girder construction the resistance to moment at any section can be changed readily by increasing or decreasing flange plates, minor deviations from the suggested ratios will have negligible effect on overall economy.

Curved Girders

In the past, bridges constructed on horizontal curves made use of straight beams or girders placed on chords between the supporting piers. Curved steel girders were rarely considered since there was little readily available design information. Recent publications, including Chapter 12, Volume I of this Handbook, have begun to fill this void. Curved girders have been designed and are now in service.

The curved girder has greatly improved the aesthetics of bridges on horizontal curves. In addition it allows continuous construction in much the same manner as for straight girders on tangent bridges. Form work and the placing of deck reinforcing steel are also simplified.

The primary difference between the curved girder bridge and the straight girder bridge of the same span length arises in the outermost and innermost girders. Because of the torsional stresses resulting from curvature, additional loads are induced in both these girders. These loads increase moments in the outer girder and decrease moments in the inner girder. Depending on the radius of curvature, the change in moments will be from 10 to 30%. Although the increased load on the outside girder is about equal to the decreased load on the inside girder, additional steel will be required in the curved girder bridge. The amount of this increase will usually be on the order of about 5% to 10%.

Girder spacing for these bridges should be the same as that for tangent bridges having the same span. Composite construction and continuity should be used wherever practical. Bracing details will vary somewhat. Recommendations have been made that cross frames or diaphragms be placed closer together than the spacing normally used on tangent bridges. This is intended to offset the effect of radial loads horizontal to the flanges.

Cross frames and diaphragms act as supports in resisting the radial loads. Therefore, the closer the transverse bracing is spaced the less the effect of the radial components. However, even when cross frames are spaced at more than 20', welded girder bottom flange plates of the greatest practical width consistent with the thickness will usually prevent these radial stresses from exceeding 5% of the allowable design stress. In the top flange the radial components need be considered only for dead load stresses of the girder, provided the concrete deck is so detailed and constructed that it can serve as the top lateral system.

Although fabrication and erection costs for curved girders will be greater than those for straight girders, the advantages previously mentioned may offset these cost increases.

Steels

Most steels used in bridges have ASTM Specifications that establish minimum properties regardless of the supplier. The steels primarily used in bridge construction are ASTM A36, A572 Grade 50, A588 and A514. These steels are all weldable steels and have minimum yield points of 36,000 psi (A36), 50,000 psi (A572 G50), 50,000 psi (A588) and 90,000 psi to 100,000 psi (A514). A588 and A514 grades can be used in unpainted bridges where the enhanced atmospheric corrosion resistance of these steels is desired to help minimize maintenance costs.
Each of these steels has definite applications in bridge construction; some of these applications have been mentioned earlier in this chapter. However, it is difficult to know which steel will furnish greatest economies without an evaluation for the specific structure. Since total fabrication costs for most plate girder spans will be about the same regardless of the steels used, mill prices of the specific steels in the sizes desired will be a helpful guide in determining their economy for a specific structure. In general, as span lengths increase steels with greater yield strengths should be used.

All Steel Superstructures

Chapter 9 illustrates two examples of designs for all steel construction. They are for the most part designed in accordance with the AASHO specifications. Each of these examples makes use of construction similar to that used for decks on orthotropic plate girders. In Design 1, the deck forms the total superstructure; in Design 2, welded longitudinal beams and transverse wide flange beams support the stiffened steel deck.

The weight of these structures, for a 50’ span, varies from about 35 to 45 pounds per square foot. This steel weight is considerably more than that of a wide flange beam span of the same length. However, these weights include the steel deck; with fabrication set up for mass production, costs for this type of structure could be satisfactory.

The two biggest advantages of all steel superstructures are:

1. The rapidity with which they can be placed and opened to traffic.
2. They require neither form work nor false work, allowing complete freedom during construction for traffic to move underneath.

In addition, these superstructures can easily and readily be widened when future traffic volumes make this desirable. The use of corrosion resistant steels in such structures should obviate maintenance.

Future Bridges

One of the most promising economies for the immediate future is in the use of steels of different strengths in the same beam cross section. In present designs, when high strength steel is used in the webs. The thickness of these webs is governed by buckling and therefore the strength properties can not be fully utilized. Since the unit price of steels generally increases with strength, economies will obviously result from the use of high strength steel flanges in combination with lower strength steel webs.

Girders of this type are commonly referred to as "hybrid" girders. Present data indicate that steels with yield points of 36,000 psi can be used as webs with girder flanges of 50,000 psi yield point steel. Girders with flanges of 100,000 psi yield point steel will probably require webs with steels having a 50,000 psi yield point.

Considerable work is being done to prepare suggested design guides for the hybrid girder. These recommendations should be forthcoming in the very near future. Substantiation will be provided so that designers can interpret the information to suit their particular needs.

It is well known that composite girders have toughness and safety in excess of that of non-composite girders. This has not yet been recognized in bridge design specifications, partly because there has been insufficient published information on composite construction as it relates to bridges.

Design procedures allowing different factors of safety for dead and live loads are also to be anticipated. In many respects this is a return to the past, as noted in the introduction to this chapter. However, the application of these factors of safety will differ from the past. The newer procedures will give designers more information on the actual capacity of beams for overloads, yield loads and failure loads.
Note:

Designs presented in this chapter are in accordance with the Tenth Edition of the Standard Specifications for Highway Bridges of the American Association of State Highway and Transportation Officials (AASHTO) dated 1969. They should be reviewed for adequacy in conforming to the latest edition of the AASHTO Specifications and subsequent interims, especially with regard to fatigue provisions and welded stud shear connectors.
Composite: Wide-Flange Beam

Introduction

Composite and noncomposite wide-flange beam bridges are in common use. Noncomposite beams generally are economical for simply supported spans shorter than about 40 ft and for continuous spans shorter than about 60 ft. Composite beams generally are economical for longer spans.

In this chapter, five design examples are given for wide-flange beams:

I. A 40-ft simple-span noncomposite beam.
II. A 40-ft simple-span, composite beam.
III. A two-span, continuous beam (70 ft—70 ft), composite for positive moment only.
IV. A two-span, continuous beam (70 ft—70 ft), composite for positive moment and negative moment.
V. A four-span, continuous beam (70 ft—90 ft—90 ft—70 ft), composite for positive moment only.

The design example for a 40-ft noncomposite, simple-span bridge is given to permit comparison with a 40-ft composite, simple-span bridge. All design examples are for an interior stringer.

All designs are in accordance with the Tenth Edition of the Standard Specifications for Highway Bridges of the American Association of State Highway Officials, dated 1969. Other specifications or other editions of the AASHO Specifications may give procedures that vary from those used.

The design examples are for ASTM A36 steel. Other structural steels such as ASTM A441, A572, and A588 have also proven economical for such structures. Design procedures for these high-strength low-alloy steels are similar to those for A36 steel except that the allowable stresses are generally higher.

Nomenclature for structural shapes adopted by the American Iron and Steel Institute in 1969 is used in this chapter; for example, the designation W33×130 replaces the former 33WF130.

General Design Considerations

In composite construction, shear connectors are provided between steel stringers and a concrete slab to make them act as a unit. Three elements, therefore, must be considered in design: (1) the reinforced concrete slab, (2) steel stringers, and (3) shear connectors.
Longitudinally, the reinforced concrete slab, located on the compression side of the steel stringer, acts as an effective cover plate. Transversely, the slab is designed to span between stringers as in noncomposite construction. Combined stresses due to composite action and due to slab action, which act at right angles to each other, are not critical and need not be investigated.

The steel stringers may be rolled beams, rolled beams with cover plates, or welded girders. Unsymmetrical sections, such as rolled beams with cover plates on the bottom flange or welded girders with bottom flange plates heavier than top flange plates, generally are economical for composite construction. Design of Composite, Welded Plate Girders is discussed in Chapter II.4.

Shear connectors provide mechanical connection between the slab and the steel stringers. (While there is a natural bond between the concrete slab and steel stringers, this bond is considered unreliable for providing the horizontal shearing resistance essential to composite action.) The connectors must be able to transfer horizontal shear between concrete slab and steel stringers so that the entire structure deforms as a unit for its full life.

Composite construction offers the following advantages over conventional steel-stringer-and-slab construction:

1. Greater economy.
2. Shallower construction.
3. Less deflection.
5. Better lateral bracing of the top flange.

Composite bridges may be built with or without temporary shoring. The design examples in this chapter are prepared for unshored construction, since most highway bridges are built without shores.

In unshored construction, the steel stringers must support their own weight plus the weight of the concrete slab. The composite section supports the weight of any additional dead load placed after the slab has hardened, plus all live load and impact. In shored construction, the steel stringers are temporarily supported during placing and hardening of the concrete slab and the composite section supports all loads after removal of the supports.

Sustained loads, such as dead loads, on concrete cause it to creep. In flexural members, creep reduces the intensity of the compressive stresses in the concrete. Thus under sustained loads, the concrete deck is less effective than for temporary loads.

The effect of creep is accounted for in composite construction by increasing the modular ratio \( E_n \) by a factor of 3. Stresses due to long-time dead loads on the composite section are computed with section properties based on the increased modular ratio \( 3n \).

Concrete is assumed ineffective in resisting tension. Thus, the slab is not considered part of the composite section in the negative-moment region of continuous, composite construction. Continuous designs are based upon one of the following assumptions:

1. Positive moments due to loads applied after the concrete slab has been placed and hardened are resisted by the composite section. Negative moments are resisted by the rolled beam plus cover plates. Shear connectors need be provided only in the positive-moment regions.

2. Positive moments due to loads applied after the concrete slab has been placed and hardened are resisted by the composite section. Negative moments also are resisted by a composite section that consists of the rolled beam and cover plates plus the longitudinal reinforcing bars in the deck. Shear connectors must be provided over the full length of the stringer.

A composite stringer bridge is designed as a series of T-beams. Each consists of one steel stringer and a portion of the concrete slab. The concrete is transformed into an equivalent area of steel by dividing the area of the slab by the modular ratio \( n \) (or \( 3n \) when creep is considered). The properties of the transformed section, and stresses at the top and bottom of the steel stringer and top of the concrete slab, are computed.
The assumed effective width of the slab as a T-beam flange must not exceed the following:

One-fourth the span of the stringer.
The spacing, center to center, of stringers.
Twelve times the least thickness of the slab.

For stringers having a flange on one side only, the effective flange width must not exceed one-twelfth the span of the stringer, six times the thickness of the slab, one-half the distance, center to center, to the adjacent stringer.

**LATERAL DISTRIBUTION OF DEAD LOAD**

Each interior stringer carries the weight of that portion of concrete slab extending a distance of one-half the stringer spacing on either side of the stringer. An outer stringer carries the weight of that portion of slab extending from the outer edge to a point midway between the outer stringer and the adjacent interior stringer.

The dead load of curbs, parapets, railings, and wearing surface, if placed after the slab has cured, may be considered equally distributed to all stringers.

If the overhang of the slab beyond the outer stringer is maintained at one-half the stringer spacing, total dead load on all stringers will be nearly equal.

**LATERAL DISTRIBUTION OF LIVE LOAD**

Live-load bending moments for an interior stringer are determined by applying to the stringer a fraction of the wheel loads, as prescribed in AASHO specifications. For a bridge consisting of a concrete slab on steel stringers and designed for two or more traffic lanes, this fraction is:

\[
\text{Live-load distribution factor} = \frac{S}{5.5} \text{ wheels}
\]

where \( S \) = average stringer spacing, ft, but not more than 14 ft.

Live-load bending moments for an outer stringer are determined by applying to the stringer the reactions due to wheel loads on the concrete slab, which is assumed to act as a simply supported beam between stringers. The fraction of wheel loads used, however, should not be less than:

\[
\text{Live-load distribution factor} = \frac{S}{5.5} \text{ wheels}
\]

when \( S \) = 6 ft or less.

\[
\text{Live-load distribution factor} = \frac{S}{4 + 0.25S} \text{ wheels}
\]

when \( S \) is more than 6 ft but less than 14 ft.

The live load applied to an outer stringer as determined by these formulas generally exceeds that obtained by assuming the slab to act as a simple span between stringers.

The live load applied to an outer stringer of a bridge designed for two or more lanes of traffic will be slightly less than that for an interior stringer. If stringers are positioned under the roadway to give equal dead loads to interior and outer stringers, often only the interior stringer need be designed and the same beam section may be used for the outer stringer.

In the calculation of stringer reactions and end shears, the live load of the wheel adjacent to the support should be distributed by assuming the concrete slab to act as a simple beam between stringers. For loads in other positions on the span, the same live-load distribution factors are used as for moment. (Many designs are made assuming the live-load distribution factor for moment applies for all shears and reactions. The resulting errors in reactions and shears are small.)
ALTERNATE INTERSTATE LOADING
Interstate loading, often called military loading, must be considered in the design of bridges on the interstate system. This loading governs moment in simple spans shorter than 38 ft and governs end shear in simple spans shorter than 30 ft.

Interstate loading does not govern any of the design examples in this chapter.

MAXIMUM MOMENTS IN CONTINUOUS STRINGERS
Curves of maximum moments in continuous spans may be computed or determined from tables based upon uniform moment of inertia of stringer. (See References 3 and 4 at end of chapter.) For the usual composite stringer with partial-length cover plates, these tables will yield a design well within reasonable accuracy. Moment curves for dead load and for live load based upon variable moment of inertia, when compared with moment curves based upon uniform moment of inertia, will show some variations. These, however, tend to be compensating; so the variations in total-moment curves are small. This is explained as follows:

To determine moments due to weight of concrete slab, stringers and framing details, the variable moment of inertia of the steel section alone is used. Since the moment of inertia is greater over the supports than in the positive-moment regions, negative moments will exceed and positive moments will be less than those based on uniform moment of inertia.

To determine moments due to live load plus impact, the moment of inertia of the composite section with an increased modular ratio of 3a is used in the positive-moment area and the moment of inertia of the steel section alone is used in the negative-moment area. Both positive and negative moments differ very little from those based upon uniform moment of inertia.

To determine moments due to live load plus impact, the moment of inertia of the composite section with a modular ratio of n is used in the positive-moment area and the moment of inertia of the steel section alone is used in the negative-moment area. The moment of inertia over the supports is less than in the positive-moment region. Hence, negative moments will be less than and positive moments will exceed those based on uniform moment of inertia.

The difference in negative moments for dead load tends to balance the difference in negative moments for live load and impact. The difference in positive moments for dead load tends to balance the difference in positive moments for live load and impact. Thus, the difference in total moments for uniform and variable moments of inertia depends on the relative magnitudes of dead-load and live-load moments. Stresses based on uniform moment of inertia generally will be within 3 or 4% of stresses based on variable moment of inertia. This is well within the limits of accuracy of the predicted loads on the stringer.

Curves of maximum moments based on uniform and variable moments of inertia are shown for Designs III and V, with stresses in the maximum-positive-moment section and the maximum-negative-moment section calculated under each assumption. The maximum increase in stress for variable moment of inertia was 0.3% and the maximum decrease was 4.3%.

COVER-PLATE CUTOFFS
The theoretical cutoff point of a cover plate is located where the stress in the rolled beam without the cover plate equals the allowable stress, exclusive of fatigue considerations. If the compression flange is unsupported, reduced allowable stresses, governed by lateral buckling requirements, should be used. From the theoretical cutoff point, the cover plate should be extended a terminal distance, as defined later. Stresses in the rolled beam at the actual end of the cover plate should not exceed allowable fatigue stresses.

The exact computation of theoretical cover-plate cutoff locations is very tedious when a portion of the load is carried by the steel section alone. For simply supported
spans, an approximate calculation of the theoretical length of the cover plate can be made on the assumption that the ratios of the section modulus without a cover plate to the section modulus with a cover plate are the same for the composite section without creep, for the composite section with creep, and for the steel section alone. The following approximate formula based on a parabolic moment diagram may be used:

\[ L_{cp} = (L - 2a) \sqrt{1 - \frac{Z'_{bs}}{Z_{et}}} + 2a \]

where \( L_{cp} \) = theoretical length of cover plate

\( L \) = span, ft

\( a \) = distance of maximum-moment section from midspan. For HS20 truck loading, \( a = 0 \) for spans less than 23.8 ft, 3.50 ft for spans between 23.8 and 33.8 ft, 2.33 ft for spans over 33.8 ft

\( Z'_{bs} \) = section modulus of rolled beam without a cover plate (steel section alone)

\( Z_{et} \) = section modulus of rolled beam with a cover plate (steel section alone)

After the approximate theoretical cover-plate cutoff location has been found, stresses at that point in the section without the cover plate are determined. If the controlling stress (usually steel stress in the bottom flange) exceeds the allowable stress, the theoretical cover-plate cutoff point is moved slightly and stresses are recalculated. This procedure is repeated until the controlling stress equals the allowable stress.

For continuous spans when reinforcing bars are not considered part of the resisting section for negative moments, theoretical cover-plate cutoff locations in the non-composite negative-moment region can be determined by plotting the resisting moment of the steel section without a cover plate on the curve of maximum total moments and scaling the length of cover plate required. Theoretical cover-plate cutoff locations in the positive-moment regions are most easily determined by assuming locations, scaling moments at these locations from the curves of maximum moments, and calculating stresses. Assumed cutoff locations are adjusted until the controlling stress equals the allowable stress. When reinforcing bars are considered part of the resisting section for negative moments, theoretical cover-plate cutoff locations in the negative-moment regions are determined by the same procedure as described for positive moments.

All cover plates must extend beyond their theoretical ends by the terminal distance, or to a point where the stress in the beam flange is equal to the allowable fatigue stress in base metal adjacent to fillet welds, the greater length governing. The terminal distance is two times the nominal cover-plate width for cover plates not welded across their ends, and 11/4 times the nominal cover-plate width for cover plates welded across their ends. The weld connecting the cover plate to the flange beyond the theoretical cutoff must be continuous and of sufficient size to develop a total force not less than the computed force in the cover plate at its theoretical end.

**FATIGUE**

For rolled beams with cover plates, allowable fatigue stresses at the ends of the cover plates (rather than allowable stresses exclusive of fatigue considerations at the theoretical cutoffs of cover plates) usually control cover-plate cutoff locations. AASHO specifications give the number of cycles of maximum stress to be assumed and corresponding formulas for allowable fatigue stress.

For freeways, expressways, and major highways and streets, the allowable fatigue stress in a rolled beam at the fillet-welded end of a cover plate is based on 500,000 cycles of stress and is given by

\[ F_r = \frac{12,000k_1}{1 - R} \]
where $F_r =$ allowable fatigue stress, psi

$k_i =$ factor that depends on type of steel $= 1.00$ for A36 steel

$R =$ ratio of minimum to maximum stress (tensile stress is positive; compressive stress, negative)

The allowable shear in a fillet weld for A36 steel is

$$F_s = \frac{10,800}{1 - 0.55R}$$

For minor highways and streets, the allowable fatigue stress in a rolled beam at the fillet-welded end of a cover plate is based on 100,000 cycles of repeated stress and is given by

$$F_r = \frac{18,000k_i}{1 - R}$$

The allowable fatigue stresses at cover-plate cutoffs in continuous spans near inflection points are low, particularly when the design is based on 500,000 cycles of stress. Consequently, the designer may economically specify a light rolled beam and extend top and bottom cover plates continuously from the negative-moment region to the positive-moment region, varying the thickness of the cover plates by butt welding plates of different thicknesses. Or the designer may specify a heavier rolled beam with thinner cover plates that can be cut off several feet each side of the inflection point.

Allowable fatigue stresses for base metal, for weld metal or base metal adjacent to butt welds, and for base metal adjacent to fasteners at field splices are given in AASHO specifications. Fatigue of base metal (not adjacent to a butt weld) controls design only if reversal of stress occurs. Fatigue of weld metal or base metal adjacent to a butt weld controls design only if the ratio of minimum to maximum stress is less than 0.226 in a design for 500,000 cycles of stress, or if reversal occurs in a design for 100,000 cycles of stress.

If butt-welded splices conform to the following conditions, they may be designed in accordance with the allowable fatigue stress for base metal:

1. The parts joined are of equal thickness.
2. The parts joined are of equal width, or tapered as illustrated on p. II/4.2.
3. Weld soundness, established by radiographic inspection, meets specified requirements.
4. Welds are made smooth and flush by grinding in the direction of applied stress.

Allowable fatigue stresses for A36 base metal at 500,000 cycles are given by AASHO specifications as follows:

Tension: $F_r = \frac{20,500}{1 - 0.55R}$

Compression: $F_r = \frac{0.55F_y}{1 - \left(\frac{0.55F_y}{13,300} - 1\right)R}$

where $F_y =$ minimum yield strength of the material, psi.

Butt-welded splices that do not meet the above conditions must be designed for allowable fatigue stresses for weld metal or base metal adjacent to butt welds. For 500,000 cycles, these stresses are given by

Tension: $F_r = \frac{17,200}{1 - 0.62R}$
Compression: \( F_r = \frac{0.55F_v}{1 - \left(\frac{0.55F_v}{10.6}\right)R} \)

**LATERAL BUCKLING**

The bottom flanges of the stringers are laterally unsupported between diaphragms. Consequently, lateral buckling must be considered in determining the allowable compressive stresses in the negative-moment regions of continuous spans. The allowable compressive stress for A36 steel is \(20,000 - 7.5(L/b)^2\) for \(L/b \leq 36\), where \(L\) = length, in., of unsupported flange between diaphragms or other points of support, or the distance, in., from interior support to point of dead-load contraflexure, the shorter length governing, and \(b\) = flange width, in.

The allowable compressive stress at interior supports of continuous spans may be increased 20% over that permitted by the above formula, but may in no case exceed the allowable unit stress for a compression flange laterally supported its full length. The allowable stress at the theoretical cutoff points of cover plates in negative-moment regions is determined by the above formula, with no percentage increase.

**WEB SHEARING STRESS**

Web shearing stress may be determined on the basis that the web of the steel stringer carries the total external shear. This assumption neglects shear taken by the steel flanges and concrete slab. The shear is assumed to be uniformly distributed over the gross area of the web. Web shearing stress seldom is critical.

**DESIGN OF SHEAR CONNECTORS**

Many mechanical devices have been used to provide the necessary resistance to horizontal shear at the junction of concrete slabs and steel stringers. Steel studs automatically end-welded to the stringers and channels fillet welded to the stringers are the most common types used. Allowable loads for stud and channel shear connectors are given in the AASHO specifications. Welded studs are used in all the following design examples.

To insure serviceability and durability of shear connectors, AASHO specifications recommend design criteria based on fatigue under service loading. The number of connectors determined by these criteria are then checked to insure that they can develop the ultimate strength of the section.

The shear connectors required to resist fatigue are determined by an elastic analysis with the following formula:

\[ S_r = \frac{V_rQ}{T} \]

where \(S_r\) = range of horizontal shear, pounds per linear inch, at junction of slab and stringer at point under consideration

\(V_r\) = range of shear, pounds, due to live load plus impact. At any section, the range of shear should be taken as the difference between minimum and maximum shear envelopes (excluding dead loads)

\(Q\) = statical moment, in.\(^3\), of transformed compressive concrete area, with modular ratio \(n\), about neutral axis of composite section; or statical moment of area of concrete reinforcement for negative moment

\(I\) = moment of inertia, in.\(^4\), of transformed composite stringer, with modular ratio \(n\) in positive-moment regions; moment of inertia provided by steel beam and area of concrete reinforcement in negative-moment regions.
The allowable design range of load, pounds, on an individual shear connector is given for welded studs by

\[ Z_r = \alpha d^2 \]

where \( \alpha \) = constant that depends on number of design cycles
\( d \) = diameter of stud, in.

The required pitch of shear connectors is determined by dividing the resistance of all connectors at a stringer cross section by \( S_r \), the horizontal range of shear per linear inch. The maximum pitch should not exceed 24 in.

If slab reinforcement steel is not used in computing composite-section properties in negative-moment regions of the span, AASHO specifications require additional shear connectors to be placed at inflection points. The number of additional connectors equals

\[ N_r = \frac{A_r f_r}{Z_r} \]

where \( A_r \) = area of reinforcement steel over interior support, sq in.
\( f_r \) = range of stress, psi, due to live load plus impact in slab reinforcement over support; 10,000 psi may be used in lieu of more accurate computations

These additional connectors must be placed within a distance of the inflection point equal to one-third the effective slab width.

The number of shear connectors established by fatigue considerations must be investigated for ultimate strength. This is relatively easy to do and assures the capability of developing the full plastic-stress distribution in the beam.

The procedure can be derived from an examination of the state of stress in a composite beam at ultimate moment.

For most practical cross sections the neutral axis is located within the concrete slab. The plastic-stress distribution is as shown in (a) below. Occasionally, the neutral axis falls below the top of the stringer, with a resultant stress distribution as in (b).

**STRESSES IN STRINGER**

In case (a), the maximum compressive force in the concrete equals the plastic force in the entire steel area, \( F_y A \), where \( F_y \) is the steel yield stress, psi, and \( A \), the area of steel section, sq in. In case (b) the maximum compressive force in the concrete equals \( 0.85 f'_c b t \), where \( f'_c \) is the 28-day strength of the concrete, psi, \( b \) the effective width of slab, in., and \( t \) the slab thickness, in. The smaller of the two values, \( F_y A \) and \( 0.85 f'_c b t \), is the maximum possible compressive force that can occur in the concrete slab.

When a composite beam, such as shown on the next page, is loaded to ultimate moment, equilibrium with respect to the concrete slab must be satisfied over the length \( L_x \), between the point of plastic moment \( M_p \) and the point of zero moment.
COMPOSITE BEAM LOADED TO ULTIMATE

Thus, the sum of the ultimate strengths of the shear connectors over the length $L_e$ are required to balance the compressive slab force $C$. Tests have shown that the ultimate strength $Q_u$, pounds, of stud shear connector is proportional to the square root of the concrete strength:

$$Q_u = 930d^2\sqrt{f_c'}$$

where $d$ is the stud diameter, in.

The required shear resistance to develop the compressive force $C$ in the slab is furnished by $N$ shear connectors between the point of maximum moment and the ends of span or inflection points, as determined by the following equation from AASHO specifications:

$$N = \frac{C}{\phi Q_u}$$

where $\phi$ is a reduction factor equal to 0.85.

Since by concepts of ultimate-strength design, plastic moment is theoretically developed at every transition in section such as at ends of cover plates along the beam, $N$ shear connectors should be provided between each transition location and the ends of span or inflection points. In this case, the $C$ value used in computing $N$ should be based on the weaker section at the transition when $F_y A_e < 0.85 f'_c bt$.

DEAD-LOAD DEFLECTIONS

The final elevations of the bridge deck under dead load should be in accordance with the finished elevations established in the plans. It is therefore necessary to establish the dead-load deflections of the beam to set the forms for the concrete slab and the screed guides for finishing the concrete slab at their proper elevations.

If esthetics are important for straight roadways, stringers may be cambered so that they will be straight under full dead load. For roadways on vertical curves, stringers may be cambered an amount equal to the dead-load deflection plus or minus sufficient additional camber so that the stringers will be a constant distance below the roadway surface under full dead load. For rolled-beam spans shorter than about 50 ft, however, dead-load deflections are small and cambering of the beam will be of little value.

Rolled beams may have a slight mill camber when received from the rolling mill.
This mill camber should be turned upward when no other cambering is required.

For unshored construction, a composite stringer is subjected to both composite and noncomposite dead loads. Hence, dead-load deflections must be computed in two steps. Because the weight of the concrete slab, stringer and framing details act on the steel section alone, deflections due to these loads are calculated with the moment of inertia of the steel section alone. Because the weight of curbs, parapets, railing and wearing surface act on the composite section, deflections due to these loads are calculated with the moment of inertia of the composite section, with an increased modular ratio of 3n to allow for creep.

If exact deflections are required, calculations should include the variable moment of inertia of the stringer and composite section. The conjugate beam method or other methods may be used for this purpose. (See References 6, 7, and 8 at end of chapter.) In most cases, however, sufficient accuracy is obtained if deflection computations are based on the moment of inertia of the maximum-positive-moment section and variations in moment of inertia along the span are neglected. The following formula may be used:

\[
\Delta = \frac{72wL^4}{E_s I} ab[1 + ab - 4(C_n - C_L)(1 + a) - 12C_L]
\]

**Uniformly Loaded Stringers**

where \( \Delta \) = deflection, in., at distance \( aL \) from left support

\( b = 1 - a \)

\( w = \) dead load, kips per ft

\( L = \) span, ft

\( E_s = \) modulus of elasticity of steel, ksi

\( I = \) moment of inertia, in.\(^4\), of steel section or of composite section with modular ratio 3n, computed at point of maximum positive moment

\( C_n = M_n / wL^2 \)

\( C_L = M_L / wL^2 \)

\( M_n = \) bending moment at right support, kip-ft

\( M_L = \) bending moment at left support, kip-ft

For simply supported stringers, \( M_n = M_L = 0 \), reducing the above formula to:

\[
\Delta = \frac{72wL^4}{E_s I} ab(1 + ab)
\]

At midspan of a simple span:

\[
\Delta = \frac{45wL^4}{2E_s I}
\]
LIVE-LOAD DEFLECTIONS

AASHO specifications set minimum depth-to-span ratios and limit maximum deflections under live load plus impact.

For simply supported, composite stringers, the ratio of over-all depth of stringer (concrete slab plus steel stringer) to span should not be less than 1/25. The ratio of depth of steel stringer alone to span should not be less than 1/30. For continuous stringers, the span may be taken as the distance between dead-load inflection points.

Maximum deflection due to live load plus impact must not exceed 1/800 of the span. For bridges in urban areas used in part by pedestrians, the ratio preferably is 1/1000.

Live-load deflections are computed with the moment of inertia of the composite section with a modular ratio \( n \). Deflections seldom control the design of composite stringers, because of the large moment of inertia of the composite section.

If more exact deflections are required, calculations should include the variable moment of inertia of the composite section and live load should be placed for maximum positive moment in the span.

The following formulas give the approximate maximum live-load deflection at midspan of a continuous stringer:

For HS truck loading:

\[
\Delta = \frac{324}{E_s I_c} \left[ P_r (L^3 - 555L + 4,780) - \frac{1}{3} (M_R + M_L) L^2 \right]
\]

For lane loading:

\[
\Delta = \frac{45L^2}{2E_s I_c} \left[ L (w_l L + 1.6P_L) - 4.8(M_R + M_L) \right]
\]

where \( \Delta \) = deflection at midspan, in.

\( P_r \) = weight of one front truck wheel, kips, multiplied by the live-load distribution factor, plus impact.

\( I_c \) = moment of inertia, in.\(^4\), of the composite section with modular ratio \( n \), computed at point of maximum positive moment

\( w_l \) = one-half weight of uniform lane load, kips per ft, multiplied by the distribution factor, plus impact

\( P_L \) = one-half weight of concentrated lane load for moment, kips, multiplied by the distribution factor, plus impact.

\( M_R \) = bending moment at right support, kip-ft

\( M_L \) = bending moment at left support, kip-ft

For simple beams, \( M_R = M_L = 0 \).

The following formulas give the live-load deflection at the 0.4 point in the end span of a continuous stringer, the approximate point of maximum deflection:

For HS truck loading:

\[
\Delta_{0.4} = \frac{300}{E_s I_c} \left[ P_r (L^3 + 3.89L^2 - 680L + 5,910) - 0.32ML^2 \right]
\]

For lane loading:

\[
\Delta_{0.4} = \frac{43L^2}{2E_s I_c} \left[ L (w_l L + 1.5P_L) - 4.5M \right]
\]

where \( \Delta_{0.4} \) = deflection at distance of 0.4 \( L \) from simple support, in.

\( M \) = bending moment at continuous support, kip-ft
Truck loading will cause maximum positive moments and maximum live-load deflections in rolled-beam spans and in most welded girder spans.

**DESIGN EXAMPLES**
Five design examples as listed at the beginning of this chapter are presented to illustrate the design of wide-flange beams.

The following applies to all designs:

**Roadway Section:** See sketch in Design I.

**Specifications:** 1969 AASHO Standard Specifications for Highway Bridges.

**Loading:** HS20-44

**Structural Steel:** A36
- Allowable bending stress = 20,000 psi
- Allowable shearing stress = 12,000 psi

**Concrete:**
- $f'_c = 4,000$ psi
- $f_c = 1,600$ psi
- $n = 8$

**Loading Conditions:**
Case 1—Weight of stringer and slab ($DL_1$) supported by the steel stringer alone.

Case 2—Superimposed dead load ($DL_2$) (curbs and railings) supported by the composite section with the modular ratio $n = 8$.

Case 3—Superimposed dead load ($DL_3$) (curbs and railings) supported by the composite section with the increased modular ratio $3n = 3 \times 8 = 24$.

Case 4—Live load plus impact ($LL+I$) supported by the composite section with the modular ratio $n = 8$.

**Loading Combinations:**
- Combination A = Case 1 + 3 + 4.
- Combination B = Case 2 + 4.
- Combination C = Case 1 + 2 + 4.

**Fatigue Considerations:** 500,000 cycles of maximum stress for all designs except Alternate A of Design III. 100,000 cycles of maximum stress for Alternate A of Design III.

**Design I—40-Ft Simple-Span Noncomposite Beam**

**LOADS, SHEARS AND MOMENTS**
The dead load consists of the weights of the concrete slab, steel beam, framing details, concrete curbs and steel railings. The weight of curbs and railings may be assumed to be distributed equally to the four stringers by the deck slab and the diaphragms between stringers. Live load plus impact on a stringer are determined in accordance with the AASHO wheel-load distribution for HS20-44 truck loading and impact factors. Since the span is greater than 38 ft, military loading does not govern. HS truck loading is used for design.
TYPICAL SECTION

Dead Loads

Slab = 7/12 x 8.33 x 0.150 = 0.730
Steel beam, details, haunches, diaphragms = 0.140
Load per stringer = 0.870 k/ft
Curbs = 2.67 x 0.75 x 0.15 x 2 = 0.600
Railings = 0.060
Load on 4 stringers = 0.660 k/ft
Curb and railing load per stringer = 0.660 / 4 = 0.165 k/ft
Total dead load per stringer = 0.870 + 0.165 = 1.035 k/ft

Live Load

Live load distribution = S = 8.33
5.5
= 1.51 wheels = 0.755 axle

Impact = 50
40 + 125
= 0.303; maximum = 30%

Maximum bending moments occur near midspan. Dead-load moment is calculated for uniform dead load. Live-load moment may be obtained from the AASHO Table of Maximum Moments, Shears and Reactions—Simple Spans, One Lane for HS20-44 loading. (These tables do not cover military loading.) End shears are calculated similarly.

Maximum Moment

DL: \[ M = \frac{wL^2}{8} = \frac{1.035 \times (40)^2}{8} = 207 \]

LL: \[ M = 0.755 \times 449.8 = 340 \]

I: \[ M = 0.30 \times 340 \]

= 102

649 kip-ft

Maximum Shear (End Reaction)

DL: \[ V = 1.035 \times 20 = 20.7 \]

LL: \[ V = 0.755 \times 55.2 = 41.7 \]

I: \[ V = 0.30 \times 41.7 = 12.5 \]

74.9 kips

*AASHO Specifications, Section 1.3.1, states that for end shears and end reactions, no lateral or longitudinal distribution should be assumed for the wheel or axle adjacent to the end at which the stress is being calculated. Wheel loads in other positions are distributed for shear in the same manner as for moment. For simplicity in the above case, all wheel loads are distributed for shear the same as for moment. The resulting shear value is very little different from that which would follow from strict adherence to the Specifications.
SELECTION OF SECTION
For a short, simply supported noncomposite beam, it is usually economical to use a wide-flange beam without cover plates. Thus, the midspan moment controls the design, and it is unnecessary to determine an envelope of maximum moments for the span.

The required section modulus is calculated for an allowable bending stress of 20 ksi. A W33×130 beam is selected, and flexural and shearing stresses are checked. Shear in the webs of wide-flange beams is rarely critical.

Section Required
Required \( Z = \frac{649 \times 12}{20} = 389.4 \text{ in.}^3 \)

Use W33×130 \((Z = 404.8 \text{ in.}^3, I = 6,699 \text{ in.}^4)\). Check of bending stress shows

\[ f_b = \frac{649 \times 12}{404.8} = 19.24 \text{ ksi} < 20 \]

Check of shearing stress shows

\[ f_s = \frac{74.9}{33.10 \times 0.580} = 3.9 \text{ ksi} < 12 \]

DEFLECTIONS
For beam spans exceeding about 50 ft, stringers usually are cambered for dead-load deflection. A single camber ordinate at midspan is sufficient for cambering rolled beams to a parabolic curve in a fabrication shop. For all spans, the live-load deflection is limited to 1/800 of the span by AASHO specifications.

Although mill camber is adequate for the stringers in this example, the dead-load deflection is calculated for illustrative purposes. The formulas used are those for deflection under uniform load and constant moment of inertia.

![Deflections Due to Dead Load](image)

Deflections Due to Dead Load

\( DL: w = 1.035 \text{ kips per ft} \)

The dead-load deflection at midspan is

\[ \Delta = \frac{45wL^4}{2EI} \]

where \( \Delta = \text{midspan deflection, in.} \)
\( w = \text{dead load, kips per ft} \)
\( L = \text{span, ft} \)
\( E = \text{modulus of elasticity of steel} = 29(10)^6 \text{ ksi} \)
\( I = \text{moment of inertia at midspan, in.}^4 \)
\[ \Delta = \frac{45 \times 1.035 \times (40)^4}{2 \times 29(10)^3 \times 6,699} = 0.307 \text{ in.} \]

Note: Turn mill camber upward. No other camber is required.

**Deflection Due to Live Load + Impact**

Maximum live-load deflections occur under two lanes of the HS20-44 truck loading, assumed to be distributed equally to the four stringers. The formula used is that for approximate maximum live-load deflection discussed under General Design Considerations. For simple beams, \( M_R = M_L = 0 \).

\[ \Delta = \frac{324}{E_t I} P_r (L^3 - 555L + 4,780) \]

where \( \Delta \) = midspan deflection, in.

\( P_r \) = concentrated load, kips, on four stringers = weight of front truck wheels \times distribution factor, plus impact, kips

\( I_s \) = moment of inertia at midspan, in.\(^4\)

\( L \) = span, ft

\( E_t \) = modulus of elasticity of steel = 29(10)\(^3\) ksi

With two lanes of live load (four wheels abreast) plus 30% impact carried by four stringers,

\[ P_r = 4 \times 4 \times 1.30 = 20.8 \text{ kips} \]

\[ I_s = 4 \times 6,699 = 26,796 \text{ in.}^4 \]

\[ \Delta = \frac{324 \times 20.8[(40)^3 - 555(40) + 4,780]}{29(10)^3 \times 26,796} = 0.402 \text{ in.} \]

The ratio of live-load deflection to span is

\[ \frac{0.402}{40 \times 12} = \frac{1}{1,194} < \frac{1}{800} \]

**Design II—40-Ft Simple-Span Composite Beam**

Normally, the most economical section for a composite, simply supported, rolled beam is obtained when a cover plate is welded to the bottom flange over part of its length. Curves of maximum moments and maximum shears are required to determine the location of cover-plate cutoffs and shear-connector spacing.

**LOADS, SHEARS AND MOMENTS**

The dead load to be carried by the steel section alone, \( DL_1 \), consists of the weight of the concrete slab plus the weight of the beam, cover plate and framing details. Weight of concrete curbs and steel railings makes up the additional dead load, \( DL_2 \), to be carried by the composite section. This weight is assumed to be distributed equally to the four stringers by the deck slab and diaphragms between stringers. For live load plus impact, the AASHO wheel load distribution for HS20-44 truck loading and impact factors are used.
Dead Load Carried by Steel

Slab = \( \frac{7}{12} \times 8.33 \times 0.150 = 0.730 \)

Steel beam, details, haunches, diaphragms = 0.108

\( DL_1 \) per stringer = 0.838 k/ft

Dead Load Carried by Composite Section

Curbs and railings, \( DL_0 = 0.660 \) k/ft

\( DL_2 \) per stringer = 0.660/4 = 0.165 k/ft

Live Load

Live-load distribution = \( \frac{S}{5.5} = \frac{8.33}{5.5} = 1.51 \) wheels = 0.755 axle

\[ \text{Impact} = \frac{50}{40 + 125} = 0.303; \text{ maximum 30\%} \]

The curve of maximum moments for a system of two or three moving concentrated loads is closely approximated by two half-parabolas joined near midspan by a straight line of length 2\( a \), where 2\( a \) = distance of maximum-moment section from midspan (see Maximum Moment Curves). For HS truck loading on spans greater than 33.8 ft, an accurate value of 2\( a \) is 4.67 ft.

In this example, the midspan moment is 441 kip-ft, extending over the 2\( a \) distance. Moment ordinates then decrease parabolically from the maximum.
The curves of maximum and minimum live-load shears are calculated by moving a standard HS20-44 truck across the span to produce maximum and minimum shear at each tenth point.

MAXIMUM-SHEAR CURVES

PROPERTIES OF COMPOSITE SECTION

Design of composite stringers is essentially a trial-and-error procedure involving selection of a steel section and investigation of steel and concrete stresses. By adjusting the steel section, the designer approaches an optimum stress condition.

Tables are available to facilitate selection of a trial steel section for certain conditions of slab thickness, stringer spacing, haunch depth and concrete strength. (See References 1 and 2 at end of chapter.) The tables list section properties for steel sections alone and composite sections. If tables are unavailable or out of the range of a given design problem, very little additional work is involved in a purely arbitrary selection of the initial trial section. The trial-and-error procedure converges rapidly to an acceptable design.

Maximum moment at midspan controls the design of the 40-foot stringer in this example. A W27×84 with an 8 x ½-in. bottom cover plate is investigated as a possible section. Section properties are computed for the steel beam and cover plate alone, the composite section with n = 8, and the composite section with 3n = 24.
Effective Flange Width

\( \frac{3}{4} \text{ span} = \frac{3}{4} \times 40 \times 12 = 120 \text{ in.} \)

Stringer spacing, \( c \) to \( c = 8.33 \times 12 = 100 \text{ in.} \)

12 x slab thickness = 12 x 7 = 84 in. (governs)

**Steel Section for Maximum Positive Moment**

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I_s</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>W27 x 84</td>
<td>24.71</td>
<td>13.60</td>
<td>-54.4</td>
<td>740</td>
<td>2,825</td>
<td>740</td>
</tr>
<tr>
<td>Bot. Cover Plate 8 x ( \frac{1}{2} )</td>
<td>4.00</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

\( d_s = \frac{-54.4}{28.71} = -1.89 \text{ in.} \)

\[ L = 28.71 \text{ in.}^2, \quad d_{\text{Top of steel}} = 13.35 + 1.89 = 15.24 \text{ in.} \]

\[ d_{\text{Bot of steel}} = 13.85 - 1.89 = 11.96 \text{ in.} \]

\[ Z_{\text{Top of steel}} = \frac{3462}{15.24} = 227 \text{ in.}^3 \]

\[ Z_{\text{Bot of steel}} = \frac{3462}{11.96} = 289 \text{ in.}^3 \]

**Composite Section, 3n = 24, for Maximum Positive Moment**

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I_s</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>28.71</td>
<td>17.85</td>
<td>-54.4</td>
<td>7,806</td>
<td>100</td>
<td>3,565</td>
</tr>
<tr>
<td>Conc. 84 x 7.24</td>
<td>24.50</td>
<td>-</td>
<td>+437.3</td>
<td>7,806</td>
<td>100</td>
<td>7,906</td>
</tr>
</tbody>
</table>

\( d_{41} = \frac{382.9}{53.21} = 7.20 \text{ in.} \)

\[ d_{41} = 53.21 \text{ in.}^2, \quad +382.9 \text{ in.}^3 \]

\[ I_{N_4} = 11,471 \text{ in.}^4 \]

\[ I_{N_4} = 7.20 \times 382.9 = 2,757 \text{ in.}^4 \]

\[ I_{N_4} = 8,714 \text{ in.}^4 \]
\[ d_{\text{Top of steel}} = 13.35 - 7.20 = 6.15 \text{ in.} \quad d_{\text{Bot. of steel}} = 13.85 + 7.20 = 21.05 \text{ in.} \]
\[ Z_{\text{Top of steel}} = \frac{8.714}{6.15} = 1.418 \text{ in.}^3 \quad Z_{\text{Bot. of steel}} = \frac{8.714}{21.05} = 0.414 \text{ in.}^3 \]

**Composite Section, n = 8, for Maximum Positive Moment**

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>( A^d )</th>
<th>( A^d )</th>
<th>( I_o )</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>28.71</td>
<td>17.85</td>
<td>-54.4</td>
<td>23,419</td>
<td>300</td>
<td>3,565</td>
</tr>
<tr>
<td>Conc. 84 x 3/8</td>
<td>73.50</td>
<td></td>
<td>+1,312.0</td>
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</tbody>
</table>

\[ d_s = \frac{1,257.6}{102.21} = 12.30 \text{ in.} \quad +1,257.6 \text{ in.}^3 \]
\[ 12.30 \times 1,257.6 = -15,468 \quad I_{NA} = \frac{11,816}{26.15} = 452 \text{ in.}^4 \]

\[ d_{\text{Top of conc.}} = 1.05 + 8.00 = 9.05 \text{ in.} \]
\[ Z_{\text{Top of conc.}} = \frac{11,816}{9.05} = 1,306 \text{ in.}^3 \]

**STRESSES IN COMPOSITE SECTION**

The stresses at top and bottom of steel and top of concrete are computed for \( DL_1 \), \( DL_2 \) and \( LL+I \). With unshored construction, steel stresses are governed by Loading Combination A and concrete stresses by Loading Combination B. When rolled sections are used in composite design, the critical stress normally occurs in the bottom flange.

**Midspan Bending Moments**

<table>
<thead>
<tr>
<th></th>
<th>( DL_1 )</th>
<th>( DL_2 )</th>
<th>( LL+I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M ), kip-ft</td>
<td>168</td>
<td>33</td>
<td>441</td>
</tr>
</tbody>
</table>

**Steel Stresses—Combination A**

<table>
<thead>
<tr>
<th></th>
<th>Bottom of Steel (Tension)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( DL_1 ): ( f_b = \frac{168 \times 12}{227} ) = 7.32</td>
<td>( f_b = \frac{168 \times 12}{289} ) = 5.98</td>
</tr>
<tr>
<td>( DL_2 ): ( f_b = \frac{33 \times 12}{418} ) = 0.78</td>
<td>( f_b = \frac{33 \times 12}{414} ) = 0.96</td>
</tr>
<tr>
<td>( LL+I ): ( f_b = \frac{441 \times 12}{11,253} ) = 0.38</td>
<td>( f_b = \frac{441 \times 12}{452} ) = 11.71</td>
</tr>
</tbody>
</table>

**Concrete Stresses—Combination B**

**Top of Concrete**

<table>
<thead>
<tr>
<th></th>
<th>Bottom of Steel (Tension)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( DL_1 ): ( f_c = \frac{33 \times 12}{1,306 \times 8} ) = 0.038</td>
<td>( f_b = \frac{441 \times 12}{1,306 \times 8} ) = 0.507</td>
</tr>
<tr>
<td>( LL+I ): ( f_c = \frac{441 \times 12}{1,306 \times 8} ) = 0.507</td>
<td>( f_b = \frac{441 \times 12}{1,306 \times 8} ) = 0.545</td>
</tr>
</tbody>
</table>
Maximum Shear Stress
Shear stress in the web of the beam is checked, although it is rarely critical in wide-flange beams that meet flexural requirements.

End Shears

<table>
<thead>
<tr>
<th></th>
<th>(DL_1)</th>
<th>(DL_2)</th>
<th>(LL+I)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V), kips</td>
<td>16.8</td>
<td>3.3</td>
<td>54.2</td>
<td>74.3</td>
</tr>
</tbody>
</table>

\[
f_s = \frac{74.3}{26.69 \times 0.463} = 6.0\text{ ksi} < 12
\]

LOCATION OF COVER-PLATE CUTOFFS
For locating cover-plate cutoffs, the section properties of the composite section without the bottom cover plate are computed. Properties of the steel section alone are obtained directly from Chapter 1/4, *Hot Rolled Shapes and Plates* or the AISC Steel Construction Manual.

Composite Section, \(3n = 24\), Near Supports

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad(^2)</th>
<th>I(_o)</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>W27\times 84</td>
<td>24.71</td>
<td>49.21</td>
<td>437.3</td>
<td>7,806</td>
<td>2,825</td>
<td>2,825</td>
</tr>
<tr>
<td>Conc. 84\times 7/24</td>
<td>24.50</td>
<td>17.85</td>
<td>+437.3</td>
<td>10,731</td>
<td>7,906</td>
<td></td>
</tr>
</tbody>
</table>

\[
d_{st} = \frac{437.3}{49.21} = 8.99\text{ in.}
\]

\[
d_{\text{Top of steel}} = 13.35 - 8.99 = 4.46\text{ in.}
\]

\[
d_{\text{Bot of steel}} = 13.35 + 8.99 = 22.24\text{ in.}
\]

\[
Z_{\text{Top of steel}} = \frac{6,843}{4.46} = 1,534\text{ in.}^3
\]

\[
Z_{\text{Bot of steel}} = \frac{6,843}{22.24} = 308\text{ in.}^3
\]

Composite Section, \(n = 8\), Near Supports

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad(^2)</th>
<th>I(_o)</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>W27\times 84</td>
<td>24.71</td>
<td>73.50</td>
<td>17.85</td>
<td>+1,312</td>
<td>2,825</td>
<td>2,825</td>
</tr>
<tr>
<td>Conc. 84\times \frac{7}{8}</td>
<td>73.50</td>
<td>17.85</td>
<td>23,419</td>
<td>23,719</td>
<td>23,719</td>
<td></td>
</tr>
</tbody>
</table>

\[
d_s = \frac{1,312}{98.21} = 13.36\text{ in.}
\]

\[
d_{\text{Top of steel}} = 13.35 - 13.36 = 0
\]

\[
d_{\text{Bot of steel}} = 13.35 + 13.36 = 26.71
\]

\[
Z_{\text{Top of steel}} = \frac{9,016}{0} = \infty
\]

\[
Z_{\text{Bot of steel}} = \frac{9,016}{26.71} = 338\text{ in.}^3
\]

Change in Section
The approximate cover-plate cutoff location is determined, for a parabolic moment diagram, by the formula for cover-plate length given in General Design Considerations. Stresses are checked at the theoretical cutoff points 8.5 ft from the bearings, with moment values scaled from the Maximum Moment Curves. The stresses are compared with the allowable static tensile and compressive stresses and found satisfactory.
Approximate length $L_{cp}$, ft, for the 8 x 1/2-in. cover plate:

$$L_{cp} = (L - 2a) \sqrt{\frac{1-Z_{ps}}{Z_{bs}} + 2a}$$

where $L$ = beam span = 40 ft

$a$ = distance of maximum-moment section from midspan = 2.33 ft

$Z'_{bs}$ = section modulus of W27 x 84 = 212 in.$^3$

$Z_{bs}$ = section modulus of beam with cover plate = 289 in.$^3$

$$L_{cp} = \left[40 - (2 \times 2.33)\right] \sqrt{\frac{1 - \frac{212}{289} + (2 \times 2.33)}{22.89, \text{ say 23 ft}}}$$

Approximate cutoff point for the cover plate from center of bearing:

$$\frac{L}{2} - \frac{L_{cp}}{2} = 20.0 - 11.5 = 8.5 \text{ ft}$$

A location 8.5 ft from the bearing is investigated as the theoretical cutoff point of the cover plate. Stresses are checked there, in the beam without cover plate.

### Bending Moments 8.5 ft from Support

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$LL + I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$, kip-ft</td>
<td>112</td>
<td>22</td>
<td>320</td>
</tr>
</tbody>
</table>

### Steel Stresses—Combination A

**Top of Steel (Compression)**

$$DL_1: f_b = \frac{112 \times 12}{212} = 6.34$$

$$DL_2: f_b = \frac{22 \times 12}{1534} = 0.17$$

$$LL + I: f_b = \frac{320 \times 12}{651} = 0 \text{ ksi}$$

**Bottom of Steel (Tension)**

$$f_b = \frac{112 \times 12}{212} = 6.34$$

$$f_b = \frac{22 \times 12}{308} = 0.86$$

$$f_b = \frac{320 \times 12}{338} = 11.36 \text{ ksi}$$

A theoretical cutoff 8.5 ft from the support is satisfactory; but due to fatigue considerations, the plate must be extended. The terminal distance is computed as 1.5 times the cover-plate width: 1.5 x 8 = 12 in.

Try cutoffs at 7 ft 6 in. from the bearings.

### Fatigue Check at Cover-Plate End

Fatigue stresses are checked in the beam adjacent to the fillet weld across the end of the cover plate. Dead- and live-load moments are scaled from the moment curve at this point, and stresses are calculated for the extreme fiber of the rolled section. Stresses alternate from full dead-load stress to dead- plus live-load stress, with no reversal. The allowable fatigue stress is given by the AASHO specification formula for expressway structures at 500,000 cycles of loading.

### Bending Moments 7.5 ft from Bearings

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$LL + I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$, kip-ft</td>
<td>103</td>
<td>21</td>
<td>297</td>
</tr>
</tbody>
</table>
Steel Stresses—Combination A

Bottom of Steel (Tension)

\[ DL_1: f_b = \frac{103 \times 12}{212} = 5.83 \]

\[ DL_2: f_b = \frac{21 \times 12}{308} = 0.82 \]

\[ LL + I: f_b = \frac{297 \times 12}{338} = 10.54 \quad \frac{17.19}{ksi} \]

Ratio of minimum to maximum stress in beam flange at cover-plate weld:

\[ R = \frac{5.83 + 0.82}{17.19} = 0.387 \]

The allowable fatigue stress is

\[ F_r = \frac{12}{1 - R} = \frac{12}{1 - 0.387} = 19.58 \text{ ksi} > 17.19 \]

Weld at Cover-Plate End

The fillet weld connecting the cover plate to the beam flange must develop the total force in the cover plate at its theoretical cutoff. This force is approximately equal to the extreme fiber stress in the cover plate at that point times the cross sectional area of the cover plate. A \( \frac{3}{8} \)-in. fillet weld 32 in. long would be adequate based on the AASHO specification of 12.4 ksi allowable shear stress. In this case, however, minimum requirements rather than strength considerations govern weld size. The \( \frac{3}{8} \)-in. thickness of the beam flange requires at least a \( \frac{1}{4} \)-in. fillet weld.

The allowable fatigue stress usually exceeds 12.4 ksi, except near inflection points of continuous beams. Fatigue should be checked where the designer believes it could govern.

Stresses 8.5 ft from End Support—Combination A

Cover Plate

\[ DL_1: f_b = \frac{112 \times 12}{289} = 4.65 \]

\[ DL_2: f_b = \frac{22 \times 12}{414} = 0.64 \]

\[ LL + I: f_b = \frac{320 \times 12}{452} = 8.50 \quad \frac{13.79}{ksi} \]

Ratio of minimum to maximum stress in cover plates:

\[ R = \frac{5.29}{13.79} = 0.384 \]

The allowable weld fatigue stress in shear is

\[ F_r = \frac{10.8}{1 - 0.55 (0.384)} = 13.7 \text{ ksi} > 12.4 \]

Allowable load on weld = 12.4 \( \times \) 0.707 = 8.76 kips per in.

Force in cover plate = 8 \( \times \) \( \frac{3}{4} \) \( \times \) 13.79 = 55.16 kips

Weld size required = \( \frac{55.16}{8.76 \times 32} \) = 0.195 in., say \( \frac{3}{8} \)-in.

Use \( \frac{1}{4} \)-in. fillet weld, required for flange thickness.
DESIGN OF SHEAR CONNECTORS

For shear connectors, %1-in.-dia. studs, 4-in. high, are used. The studs must satisfy the requirement: \( H/d \geq 4.0 \). The AASHO specifications formula for ultimate strength of welded studs is used to determine the allowable load per stud. Strength requirements are satisfied with 27 shear connectors placed between the point of maximum moment at midspan and the end of the span. In addition, at least 24 shear connectors should be placed between the cover-plate cutoffs and nearest bearings.

Concrete: \( f'_c = 4,000 \text{ psi; } n = 8 \)

Studs: %1-in.-dia, 4-in. high, \( H/d = 4.0/0.875 = 4.6 > 4.0 \)

The ultimate strength of a shear connector equals

\[
Q_u = 0.93d^2\sqrt{f'c} = 0.93(0.875)^2\sqrt{4,000} = 45.0 \text{ kips per stud}
\]

With \( \alpha \) given as 10.6 for 500,000 cycles of load in AASHO specifications, the load range per shear connector is

\[
Z_r = \alpha d^2 = 10.6(0.875)^2 = 8.11 \text{ kips per stud}
\]

**Shear Connectors—Strength Requirements**

At midspan, the maximum compressive stress in the concrete is

\[
H_1 = A_rF_y = 28.71 \times 36.0 = 1,033.6 \text{ kips (governs)}
\]

\[
H_2 = 0.85f'c/bt = 0.85 \times 4.0 \times 84.0 \times 7.0 = 1,999.2 \text{ kips}
\]

The number of studs required between midspan and each support is

\[
N = \frac{H_1}{\phi Q_u} = \frac{1,033.6}{0.85 \times 45.0} = 27
\]

where \( \phi = \text{reduction factor} = 0.85 \)

At cover-plate cutoffs:

\[
H_1 = A_rF_y = 24.71 \times 36.0 = 889.6 \text{ kips (governs)}
\]

\[
H_2 = 1,999.2 \text{ kips}
\]

The number of studs required between the cover-plate cutoffs and nearest bearing is

\[
N = \frac{H_1}{\phi Q_u} = \frac{889.6}{0.85 \times 45.0} = 23.3; \text{ use 24 studs}
\]

**Shear Connector Spacing for Service Behavior (Fatigue)**

Shear-connector-spacing requirements for service behavior under repeated loads are checked. Here, a variable pitch will result, in accordance with the variation in range of shear along the span. The allowable range of load per stud is 8.11 kips. Spacing is calculated at the end of the span first.

At supports, shear range for the live load \( v_r = 54.2 - 0 = 54.2 \) kips. For \( n = 8 \), the horizontal shear per linear inch is

\[
S_r = \frac{V_rQ}{I} = \frac{54.2(73.5 \times 4.5)}{9,016} = 1.99 \text{ kips per in.}
\]

Spacing required (3 studs) \( \frac{3Z_r}{S_r} = \frac{3(8.11)}{1.99} = 12.23 \text{ in.} \)

Computation for shear-connector spacing under service behavior is repeated at each tenth point along the span, and a curve of required spacing is plotted. The curve provides an envelope within which the spacing diagram for fabrication is drawn. A single step in the spacing, from 12 to 15 in., is made at the quarter point of the span.
This spacing furnishes 54 studs from midspan to each bearing and 24 studs from the cover-plate cutoffs to nearest bearing. The number of studs supplied exceed the number required for strength and fatigue.

![Graph showing shear-connector spacing](image)

**SHEAR-CONNECTOR SPACING**

**BEARING STIFFENERS**

For rolled beams, end bearing stiffeners are not required if the shearing stress in the web does not exceed 75% of the allowable shearing stress for girder webs.

<table>
<thead>
<tr>
<th>Reactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DL_1$</td>
</tr>
<tr>
<td>$R$, kips</td>
</tr>
</tbody>
</table>

$$f_r = \frac{74.3}{26.69 \times 0.463} = 6.01 \text{ ksi}$$

The allowable shear without stiffeners is

$$f_r = 12 \times 0.75 = 9.0 \text{ ksi} > 6.01$$

No end stiffeners are required.

**DEFLECTIONS**

Dead-load-deflection computations are carried out in two parts. $DL_1$ deflections are based on the moment of inertia of the steel section alone; $DL_2$ deflections are based on the moment of inertia of the composite section with $3n = 24$. Calculations show that the total dead-load deflections are small. Ordinary mill camber turned upward is sufficient.

**Deflections Due to Dead Load**

$DL_1$: $w = 0.838$ kips per ft

$DL_2$: $w = 0.165$ kips per ft

The dead-load deflection at midspan is

$$\Delta = \frac{45wL^4}{2EI}$$
where $\Delta =$ midspan deflections, in.

$w =$ dead load, kips per ft

$L =$ span, ft

$E_s =$ modulus of elasticity of steel $= 29(10)^3$ ksi

$I =$ moment of inertia at midspan, in.$^4$

**Deflections Under DL$_1$**

$I_s =$ 3,462 in.$^4$

$\Delta = \frac{45 \times 0.838(40)^4}{2 \times 29(10)^3 \times 3,462} = 0.481$ in.

**Deflections Under DL$_2$**

$I_{s4} =$ 8,714 in.$^4$

$\Delta = \frac{45 \times 0.165(40)^4}{2 \times 29(10)^3 \times 8,714} = 0.039$ in.

**Note:** Turn mill camber upward. No other camber is required.

**Deflection Due to Live Load + Impact**

The deflection from live load plus impact is computed for two lanes of HS20-44 truck loading distributed equally to the four stringers. Computations show that the maximum live-load deflection is considerably below the allowable limit of $1/800$ of the span.

$\Delta = \frac{324}{E_s I_s} P_T (L^4 - 555L + 4,780)$

where $\Delta =$ midspan deflection, in.

$P_T =$ concentrated load, kips, on four stringers $= $ weight of front truck wheels $\times$ distribution factor, plus impact, kips

$I_s =$ moment of inertia of composite section at midspan, in.$^4$

$L =$ span, ft

$E_s = 29(10)^3$ ksi

Assume that two lanes of live load (four wheels abreast) are carried by four stringers. Then,

$P_T = 4 \times 4 \times 1.30 = 20.8$ kips

$I_s = 4 \times 11,816 = 47,264$ in.$^4$

$\Delta = \frac{324 \times 20.8(40)^4 - 555(40) + 4,780}{29(10)^3 \times 47,264} = 0.229$ in.

The ratio of live-load deflection to span is

$$\frac{0.229}{40 \times 12} = \frac{1}{2,096} \approx \frac{1}{800}$$

**FINAL DESIGN**

Section and elevation views of the composite-beam design are shown on the next page. See also the detail drawing at the end of this chapter.
Comparison Of Design I And Design II

Unit prices that may be used in a cost comparison of the composite and noncomposite beams for the 40-ft span differ from one part of the country to another. While the price per pound for the composite beam with a cover plate will be higher than the unit price for a noncomposite beam that requires minimum fabrication, the composite structure usually will cost less. Assume, for example, erected unit prices of $0.15 per pound for the noncomposite beam, $0.17 per pound for the composite beam, and $0.50 for each stud shear connector. Then, one noncomposite beam for the 40-ft span costs $800, and one composite beam for this span costs $699 (see table).

### Cost Comparison of 40-Ft Composite and Noncomposite Stringers

<table>
<thead>
<tr>
<th></th>
<th>Structural Steel (A 36)</th>
<th>7/8-In.-Dia Stud Connectors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pounds</td>
<td>Unit Price</td>
</tr>
<tr>
<td>Noncomposite stringer</td>
<td>5,330</td>
<td>$0.15</td>
</tr>
<tr>
<td>Composite stringer</td>
<td>3,784</td>
<td>$0.17</td>
</tr>
</tbody>
</table>
While live-load deflections are well within allowable limits for noncomposite and composite beams, the live-load deflection of the composite beam is less than 60% of that for the noncomposite beam. As span increases, live-load deflection may become an important consideration in the design of noncomposite beams. In composite design, however, live-load deflection seldom is critical.

Design III—Two-Span Continuous Beam (70-70 Ft), Composite For Positive Movement Only

This type of structure generally is applicable for spans of about 60 to 80 ft. For shorter continuous spans, noncomposite rolled beams usually will be more economical.

The design procedure for a two-span, continuous, composite, rolled-beam stringer in the positive-moment portion of the span is similar to the procedure for simple spans. Negative moments are assumed to be carried solely by the steel stringer in this example.

Economic design of a two-span, continuous composite beam requires a varying steel section. In negative-moment areas, cover plates are added to both flanges of the rolled beam, and in positive-moment areas, to the bottom flange only.

LOADS, SHEARS AND MOMENTS
An initial analysis of the two-span continuous stringer is made for moments and shears based on constant moment of inertia.

$DL_1$ is calculated as the weight of the 7-in.-thick concrete slab and an assumed weight of 0.170 kips per ft for the stringer and framing details. Weight of curbs and railings comprises $DL_2$. Live load is the standard HS20-44 truck loading. Impact is computed for a 70-ft span.

Dead Load Carried by Steel

Slab = $7/12 \times 8.33 \times 0.150 = 0.730$

Steel beam, details, haunches, diaphragms = 0.170

$DL_1$ per stringer = $0.900$ k/ft

Dead Load Carried by Composite Section

Curbs and railings, $DL_2 = 0.660$ k/ft

$DL_2$ per stringer = $0.660/4 = 0.165$ k/ft

Live Load

Live-load distribution = $S \over 5.5 = 8.33 \over 5.5 = 1.51$ wheels = 0.755 axle

Impact = $50 \over 70 + 125 = 0.256$
Curves of maximum moments and maximum shears are calculated from tables of influence-line coefficients (see References 3 and 4 at end of chapter).

*Resisting Moment of W36×135 Alone
**Resisting Moment of W36×135 Plus 10" x ½" Cover Plates Top and Bottom
***Resisting Moment of W36×135 Plus 10" x 1" Cover Plates Top and Bottom

MAXIMUM-MOMENT CURVES—CONSTANT I
MAXIMUM-SHEAR CURVES—CONSTANT I

DESIGN OF STRINGER SECTION
As a first step in the design procedure, the control sections at the points of maximum negative moment and maximum positive moment are selected. A W36 \times 135 beam with 10 \times 1-in. cover plates top and bottom is investigated for maximum negative moment. Calculations show that the section satisfies stress limitations.

MAXIMUM-NEGATIVE MOMENT
From the maximum moment curves, the total moment at the interior support is \(-1,253\) kip-ft. This moment must be taken by the W36 \times 135 with top and bottom cover plates. For the W36 \times 135, the moment of inertia \(I_n = 7,796\) in.\(^4\) and the half-depth \(c = 17.78\) in. The allowable tensile stress \(f\) in the plates is 20 ksi.

\[
\text{Required area of cover plates} = \frac{12M}{fc} = \frac{12(1,253)}{20(18)} = \frac{7,796}{(18)^2} = 17.71 \text{ in.}^2
\]
Try 2 cover plates 10 x 1 in., area = 20 in.²

**Steel Section at Interior Support**

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>Iₜ</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>W36 x 135</td>
<td>39.70</td>
<td>18.28</td>
<td>365.6</td>
<td>6,683</td>
<td>7,796</td>
<td>7,796</td>
</tr>
<tr>
<td>2 Plates 10 x 1</td>
<td>20.00</td>
<td>18.28</td>
<td>365.6</td>
<td>6,683</td>
<td>7,796</td>
<td>7,796</td>
</tr>
</tbody>
</table>

59.70 in.²

\[ I_{N,i} = \frac{14,479}{18.78} = 771 \text{ in.}^4 \]

\[ Z = \frac{14,479}{18.78} = 771 \text{ in.}^3 \]

Maximum stress \( f_i = \frac{1,253 \times 12}{771} = 19.50 \text{ ksi} < 20 \)

Resisting moment for 20 ksi allowable stress \( M_u = \frac{20 \times 771}{12} = 1,285 \text{ kip-ft} \)

**Allowable Compressive Stress near Support**

Length \( L \) for lateral buckling is 17.5 ft, the distance from interior bearing to dead-load inflection point and \( b = \text{flange width} = 12 \text{ in.} \)

\[ F_b = 20,000 - 7.5\left(\frac{L}{b}\right)^2 = 20,000 - 7.5\left(\frac{17.5 \times 12}{12}\right)^2 = 17.70 \text{ ksi} \]

Because of continuity, AASHO specifications permit a 20% increase in allowable stresses up to 20 ksi at the interior support.

\[ F_b = 17.70 \times 1.20 = 21.2 \text{ ksi}. \text{ Use 20 ksi.} \]

**POSITIVE-MOMENT SECTION**

**MAXIMUM-POSITIVE MOMENT**

A steel section consisting of a W36 x 135 with a bottom cover plate 10 x \( \frac{3}{8} \) in. is investigated for the region of maximum positive moment. Properties are computed for the steel section alone, the composite section with \( n = 8 \), and the composite section with \( 3n = 24 \).
Steel Section, 28 Ft from End Support

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>W36 × 135</td>
<td>39.70</td>
<td>-17.96</td>
<td>-67.35</td>
<td>1,210</td>
<td>7,796</td>
<td>7,796</td>
</tr>
<tr>
<td>Bot. Cover Plate 10 × 3/8</td>
<td>3.75</td>
<td>-17.96</td>
<td>-67.35</td>
<td>1,210</td>
<td>7,796</td>
<td>7,796</td>
</tr>
</tbody>
</table>

43.45 in.²  
-67.35 in.³  
\[ d_s = \frac{-67.35}{43.45} = -1.55 \text{ in.} \]

\[ d_{\text{Top of steel}} = 17.78 + 1.55 = 19.33 \text{ in.} \]

\[ d_{\text{Bot. of steel}} = 17.78 + 0.38 - 1.55 = 16.61 \text{ in.} \]

\[ Z_{\text{Top of steel}} = \frac{8902}{19.33} = 460 \text{ in.}³ \]

\[ Z_{\text{Bot. of steel}} = \frac{8902}{16.61} = 536 \text{ in.}³ \]

Composite Section, 3n = 24, 28 Ft from End Support

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel section</td>
<td>43.45</td>
<td>23.15</td>
<td>567.18</td>
<td>13,130</td>
<td>100</td>
<td>9,006</td>
</tr>
<tr>
<td>Conc. 84 × 7/24</td>
<td>24.50</td>
<td>23.15</td>
<td>567.18</td>
<td>13,130</td>
<td>100</td>
<td>9,006</td>
</tr>
</tbody>
</table>

67.95 in.²  
499.83 in.³  
\[ d_s = \frac{499.83}{67.95} = 7.36 \text{ in.} \]

\[ d_{\text{Top of steel}} = 17.78 - 7.36 = 10.42 \text{ in.} \]

\[ d_{\text{Bot. of steel}} = 18.16 + 7.36 = 25.52 \text{ in.} \]

\[ Z_{\text{Top of steel}} = \frac{18557}{10.42} = 1,781 \text{ in.}³ \]

\[ Z_{\text{Bot. of steel}} = \frac{18557}{25.52} = 727 \text{ in.}³ \]

Composite Section, n = 8, 28 Ft from End Support

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel section</td>
<td>43.45</td>
<td>23.15</td>
<td>1,701.53</td>
<td>39,390</td>
<td>300</td>
<td>39,690</td>
</tr>
<tr>
<td>Conc. 84 × 3/8</td>
<td>73.50</td>
<td>23.15</td>
<td>1,701.53</td>
<td>39,390</td>
<td>300</td>
<td>39,690</td>
</tr>
</tbody>
</table>

\[ d_s = \frac{1,634.18}{116.95} = 13.97 \text{ in.} \]

\[ d_{\text{Top of steel}} = 17.78 - 13.97 = 3.81 \text{ in.} \]

\[ d_{\text{Bot. of steel}} = 18.16 + 13.97 = 32.13 \text{ in.} \]

\[ Z_{\text{Top of steel}} = \frac{25,867}{3.81} = 6,789 \text{ in.}³ \]

\[ Z_{\text{Bot. of steel}} = \frac{25,867}{32.13} = 805 \text{ in.}³ \]

\[ d_{\text{Top of conc.}} = 26.65 - 13.97 = 12.68 \text{ in.} \]

\[ Z_{\text{Top of conc.}} = \frac{25,867}{12.68} = 2,040 \text{ in.}³ \]

Check of Steel and Concrete Stresses

Stresses are checked at the top and bottom of steel and at the top of concrete. Calculations show that the W36 × 135 with bottom cover plate satisfies stress limitations. The concrete stress is well within the allowable for compression.
Bending Moments 28 Ft from End Support

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$LL + I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$, kip-ft</td>
<td>309</td>
<td>57</td>
<td>750</td>
</tr>
</tbody>
</table>

**Steel Stresses—Combination A**

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$LL + I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_h$, ksi</td>
<td>$\frac{309 \times 12}{460} = 8.06$</td>
<td>$\frac{57 \times 12}{1781} = 0.38$</td>
<td>$\frac{750 \times 12}{9778} = 1.33$</td>
</tr>
</tbody>
</table>

**Bottom of Steel (Tension)**

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$LL + I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_h$, ksi</td>
<td>$\frac{309 \times 12}{536} = 6.92$</td>
<td>$\frac{57 \times 12}{727} = 0.94$</td>
<td>$\frac{750 \times 12}{805} = 11.18$</td>
</tr>
</tbody>
</table>

**Concrete Stresses—Combination B**

<table>
<thead>
<tr>
<th></th>
<th>$DL_2$</th>
<th>$LL + I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_c$, ksi</td>
<td>$\frac{57 \times 12}{2040 \times 8} = 0.042$</td>
<td>$\frac{750 \times 12}{2040 \times 8} = 0.551$</td>
</tr>
</tbody>
</table>

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.593</td>
<td>ksi</td>
</tr>
</tbody>
</table>

**LOCATION OF COVER-PLATE CUTOFFS**

The resisting moments of the rolled section alone, and the rolled section plus 10 x 1-in. cover plates, are plotted on the curve of maximum negative moments. The theoretical cutoff point, exclusive of fatigue considerations, for the plates is determined by scaling the location at which the resisting moment of the W36 x 135 alone equals the bending moment. Since the W36 x 135 has a section modulus $Z = 438.6$-in.$^2$ and the allowable compressive stress, determined by buckling is 17.70 ksi, this resisting moment equals

$$M_R = \frac{17.70 \times 438.6}{12} = 646 \text{ kip-ft}$$

The theoretical cutoff point is about 8.75 ft from the interior support. The minimum terminal distance required by AASHO specifications sets the end of the 10 x 1-in. cover plates about 10 ft from that support.

Next, the allowable fatigue stress in the W36 x 135 adjacent to the fillet welds joining the ends of the cover plates to the flange is checked. Calculations show that the allowable fatigue stress 10 ft from the support is below the bending stress. Thus, fatigue governs the cutoff location.

Another check is made 12 ft from the support. Again, the allowable fatigue stress is lower than the bending stress. The procedure is repeated for a point 15 ft from the support and at the dead-load inflection point, 17.5 ft from the support. Fatigue requirements are not satisfied at these locations.

Allowable fatigue stresses are more highly restrictive in the vicinity of inflection points than elsewhere, because of stress reversals that occur where dead-load moments are small. A negative value of $R$ in the formula for fatigue stress, $F_v = 12000/(1 - R)$, lowers the allowable fatigue stress to a greater extent than positive values of $R$. For the design under consideration, the ratio of bending stress to allowable fatigue stress increases with distance from the interior support, the inflection point. In this region, the W36 x 135 alone is unable to resist the fatigue stresses adjacent to fillet welds across the end of a cover plate.

Hence, the 10 x 3/8-in. bottom cover plate in the positive-moment portion of the span is extended through the inflection region and butt welded to the 10 x 1-in.
bottom cover plate. Also, a 10 x 3/8-in. cover plate is added to the top flange and butt welded to the 10 x 1-in. top cover plate.

**Stresses 10 Ft from Interior Support**

Bending stresses are computed 10 ft from the interior support. Also, the allowable fatigue stress is calculated in the W36x135 flanges and is found to govern.

**Maximum Moments 10 Ft from Interior Support**

<table>
<thead>
<tr>
<th>With Positive Live-Load Moment</th>
<th>With Negative Live-Load Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DL_1$: $M = -203$</td>
<td>$DL_1$: $M = -203$</td>
</tr>
<tr>
<td>$DL_2$: $M = -39$</td>
<td>$DL_2$: $M = -39$</td>
</tr>
<tr>
<td>$LL + I$: $M = +183$</td>
<td>$LL + I$: $M = -359$</td>
</tr>
<tr>
<td>$-59$ kip-ft</td>
<td>$-601$ kip-ft</td>
</tr>
</tbody>
</table>

Bending stress in top and bottom flange is

$$f_b = \frac{601 \times 12}{438.6} = 16.44 \text{ ksi}$$

Ratio of minimum to maximum stress at cover-plate ends is

$$R = \frac{59}{601} = 0.098$$

The allowable fatigue stress in the W36x135 flanges adjacent to the fillet welds for the cover-plate ends is

$$F_r = \frac{12.0}{1 - 0.098} = 13.30 < 16.44 \text{ ksi (overstressed)}$$

Since the W36x135 alone is inadequate in fatigue, try extending the cover plates 2 ft further from the support.

**Stresses 12 Ft from Interior Support**

Investigation of bending and fatigue stresses 12 ft from the interior support shows that fatigue governs.

**Maximum Moments 12 Ft from Interior Support**

<table>
<thead>
<tr>
<th>With Positive Live-Load Moment</th>
<th>With Negative Live-Load Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DL_1$: $M = -145$</td>
<td>$DL_1$: $M = -145$</td>
</tr>
<tr>
<td>$DL_2$: $M = -28$</td>
<td>$DL_2$: $M = -28$</td>
</tr>
<tr>
<td>$LL + I$: $M = +247$</td>
<td>$LL + I$: $M = -350$</td>
</tr>
<tr>
<td>$74$ kip-ft</td>
<td>$-523$ kip-ft</td>
</tr>
</tbody>
</table>

Bending stress in top and bottom flange is

$$f_b = \frac{523 \times 12}{438.6} = 14.31 \text{ ksi}$$

Ratio of minimum to maximum stress at cover-plate ends is

$$R = \frac{74}{523} = -0.141$$

The allowable fatigue stress in the W36x135 is

$$F_r = \frac{12.0}{1 - (-0.141)} = 10.52 \text{ ksi} < 14.31 \text{ (overstressed)}$$
Since the W36×135 again is inadequate in fatigue, try extending the cover plates 3 ft further from the support.

**Stresses 15 Ft from Interior Support**

Investigation of stresses 15 ft from the interior support shows that fatigue still governs.

**Maximum Moments 15 Ft from Interior Support**

<table>
<thead>
<tr>
<th>With Positive Live-Load Moment</th>
<th>With Negative Live-Load Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DL_1$: $M = - 60$</td>
<td>$DL_1$: $M = - 60$</td>
</tr>
<tr>
<td>$DL_2$: $M = - 11$</td>
<td>$DL_2$: $M = - 11$</td>
</tr>
<tr>
<td>$LL + I$: $M = 345$</td>
<td>$LL + I$: $M = -325$</td>
</tr>
<tr>
<td>274 kip-ft</td>
<td>-396 kip-ft</td>
</tr>
</tbody>
</table>

Bending stress in top and bottom flange is

$$f_b = \frac{396 \times 12}{438.6} = 10.83 \text{ ksi}$$

Ratio of minimum to maximum stress at cover-plate ends is

$$R = \frac{274}{-396} = -0.692$$

The allowable fatigue stress in the W36×135 is

$$F_r = \frac{12.0}{1 - (-0.692)} = 7.09 \text{ ksi} < 10.83 \text{ (overstressed)}$$

Since the W36×135 still is inadequate, try extending the cover plates another 2.5 ft, to the dead-load inflection point.

**Stresses at Inflection Point**

Because of stress reversal, allowable fatigue stress is even smaller at the inflection point than nearer the support and continues to govern.

**Maximum Moments 17.5 Ft from Interior Support**

<table>
<thead>
<tr>
<th>With Positive Live-Load Moment</th>
<th>With Negative Live-Load Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DL_1$: $M = 0$</td>
<td>$DL_1$: $M = 0$</td>
</tr>
<tr>
<td>$DL_2$: $M = 0$</td>
<td>$DL_2$: $M = 0$</td>
</tr>
<tr>
<td>$LL + I$: $M = 420$ kip-ft</td>
<td>$LL + I$: $M = -310$ kip-ft</td>
</tr>
</tbody>
</table>

Bending stress in top and bottom flange is

$$f_b = \frac{420 \times 12}{438.6} = 11.49 \text{ ksi}$$

Ratio of minimum to maximum stress is

$$R = \frac{-310}{420} = -0.738$$

The allowable fatigue stress in the W36×135 is

$$F_r = \frac{12.0}{1 - (-0.738)} = 6.90 \text{ ksi} < 11.49 \text{ (overstressed)}$$
The W36×135 still is inadequate in fatigue, so extend the 10 x 3/8-in. bottom cover plate in the positive-moment region and butt weld it to the bottom 10 x 1-in. cover plate. Also, add a top cover plate 10 x 3/8-in. in the inflection-point region and butt weld it to the 10 x 1-in. top cover plate.

NEGATIVE-MOMENT TRANSITION SECTION
Fatigue limitations for weld metal or base metal adjacent to butt welds are not so restrictive as the fatigue limitations for metal adjacent to fillet welds. Trials indicate that the butt-welded joints between the 10 x 1-in. cover plates and the 10 x 3/8-in. cover plates may be located as close as 6 ft to the interior support.

### Steel Section 6 Ft from Interior Support

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>W36×135</td>
<td>39.70</td>
<td>7.50</td>
<td>17.97</td>
<td>2,422</td>
<td>7,796</td>
</tr>
<tr>
<td>2 Plates 10×3/8</td>
<td>47.20 in.²</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
I_{NA} = 10,218 \text{ in.}^4
\]

\[
Z = \frac{10,218}{18.15} = 563 \text{ in.}^3
\]

As computed previously, the allowable compressive stress, determined by lateral buckling, is 17.70 ksi. Hence, the resisting moment is

\[
M_R = \frac{17.70 \times 563}{12} = 830 \text{ kip-ft}
\]

### Maximum Moments 6 Ft from Interior Support

**With Positive Live-Load Moment**

\[
\begin{align*}
DL_1: \ M &= -330 \\
DL_2: \ M &= -61 \\
LL + I: \ M &= +85 \\
&\quad -306 \text{ kip-ft}
\end{align*}
\]

**With Negative Live-Load Moment**

\[
\begin{align*}
DL_1: \ M &= -330 \\
DL_2: \ M &= -61 \\
LL + I: \ M &= -395 \\
&\quad -786 \text{ kip-ft}
\end{align*}
\]

Bending stress in top and bottom cover plates at the butt-welded splice is

\[
f_o = \frac{786 \times 12}{563} = 16.75 \text{ ksi} < 17.70 \text{ ksi}
\]

Ratio of minimum to maximum stress in the butt weld is

\[
R = \frac{-306}{-786} = 0.389
\]

The allowable butt-weld fatigue stress in tension is

\[
F_r = \frac{17.2}{1 - 0.62(0.389)} = 22.66 \text{ ksi} > 16.75
\]

The allowable butt-weld fatigue stress in compression is

\[
F_r = \frac{0.55 \times 36}{1 - \left(\frac{0.55 \times 36 - 1}{10.6 - 1}\right)0.389} = 29.91 \text{ ksi} > 16.75
\]

Lateral buckling governs, \( F_r = 17.70 \) ksi. Since the bending stress is smaller, the W36×135 with 10 x 3/8-in. cover plates is satisfactory 6 ft from the interior support.
CUTOFF OF TOP COVER PLATE

To satisfy fatigue stresses, the 10 x 3\(\frac{3}{8}\)-in. top cover plate must extend sufficiently into the composite area that live-load positive moments at the end of the plate are carried by the composite section. Preliminary calculations indicate that shear connectors over a length of about 6 ft are required to develop the compressive stress that occurs in the concrete slab under composite action. Since shear connectors start about 17.5 ft from the interior support, a trial cutoff point 24 ft from that support is investigated.

**Maximum Moments 24 Ft from Interior Support**

<table>
<thead>
<tr>
<th>With Positive Live-Load Moment</th>
<th>With Negative Live-Load Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>(DL_1): (M = 130)</td>
<td>(DL_1): (M = +130)</td>
</tr>
<tr>
<td>(DL_2): (M = 25)</td>
<td>(DL_2): (M = + 25)</td>
</tr>
<tr>
<td>(LL+I): (M = 580)</td>
<td>(LL+I): (M = -275)</td>
</tr>
<tr>
<td></td>
<td>(120 \text{ kip-ft})</td>
</tr>
</tbody>
</table>

**Stresses at Top of Steel—Combination A**

<table>
<thead>
<tr>
<th>Compression (Composite Section)</th>
<th>Tension (Steel Only)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(DL_1): (f_b = \frac{130 \times 12}{460} = 3.39)</td>
<td>(f_b = \frac{120 \times 12}{460} = 3.13 \text{ ksi})</td>
</tr>
<tr>
<td>(DL_2): (f_b = \frac{25 \times 12}{1,781} = 0.17)</td>
<td></td>
</tr>
<tr>
<td>(LL+I): (f_b = \frac{580 \times 12}{6,789} = 1.03)</td>
<td>4.59 ksi</td>
</tr>
</tbody>
</table>

Ratio of minimum to maximum stress at top of W36 x 135 is

\[ R = \frac{3.13}{4.59} = -0.682 \]

The allowable fatigue stress at the fillet weld is

\[ F = \frac{12.0}{1 - (-0.682)} = 7.13 \text{ ksi} > 4.59 \]

Thus, fatigue requirements are satisfied if the top cover plate ends 24 ft from the interior support.

**Weld at End of Top Cover Plate**

The fillet weld connecting the top cover plate to the W36 x 135 flange within the terminal distance of the end must develop the force in the plate. The terminal distance is 1.5 times the plate width, or 15 in. for the 10-in.-wide cover plate. The weld size is determined at the theoretical cutoff point, 15 in. from the plate end, 22.75 ft from the interior support. A \(\frac{1}{16}\)-in. fillet weld 40 in. long would be adequate. Minimum requirements, rather than strength considerations govern, however. The \(\frac{13}{46}\)-in. thickness of the W36 x 135 flange requires at least a \(\frac{5}{16}\)-in. weld.

**Maximum Moments 22.75 Ft from Interior Support**

<table>
<thead>
<tr>
<th>With Positive Live-Load Moment</th>
<th>With Negative Live-Load Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>(DL_1): (M = 94)</td>
<td>(DL_1): (M = + 94)</td>
</tr>
<tr>
<td>(DL_2): (M = 20)</td>
<td>(DL_2): (M = + 20)</td>
</tr>
<tr>
<td>(LL+I): (M = 540)</td>
<td>(LL+I): (M = -287)</td>
</tr>
<tr>
<td></td>
<td>(-173 \text{ kip-ft})</td>
</tr>
</tbody>
</table>

The steel section consists of the W36 x 135 and top and bottom cover plates 10 x 3\(\frac{3}{8}\)-in. As computed for the transition section 6 ft from the interior support, the section modulus \(Z = 563 \text{ in.}^3\).
Composite Section, 3n – 24, 22.75 Ft from Interior Support

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>47.20</td>
<td>23.15</td>
<td>567.18</td>
<td>13,130</td>
<td>10,218</td>
<td>10,218</td>
</tr>
<tr>
<td>Conc. 84 × 7/24</td>
<td>24.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ d_{12} = \frac{567.18}{71.70} = 7.91 \text{ in.} \]

\[ d_{\text{Top of steel}} = 18.15 - 7.91 = 10.24 \text{ in.} \]

\[ d_{\text{Bot. of steel}} = 18.15 + 7.91 = 26.06 \text{ in.} \]

\[ Z_{\text{Top of steel}} = 18,962 \times 10.24 = 1852 \text{ in.}^3 \]

\[ Z_{\text{Bot. of steel}} = \frac{18,962}{26.06} = 728 \text{ in.}^3 \]

Composite Section, n = 8, 22.75 Ft from Interior Support

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>47.20</td>
<td>23.15</td>
<td>1,701.53</td>
<td>39.390</td>
<td>10,218</td>
<td>10,218</td>
</tr>
<tr>
<td>Conc. 84 × 3/8</td>
<td>73.50</td>
<td></td>
<td></td>
<td></td>
<td>300</td>
<td>39,900</td>
</tr>
</tbody>
</table>

\[ d = \frac{1,701.53}{120.70} = 14.10 \text{ in.} \]

\[ d_{\text{Top of steel}} = 18.15 - 14.10 = 4.05 \text{ in.} \]

\[ d_{\text{Bot. of steel}} = 18.15 + 14.10 = 32.25 \text{ in.} \]

\[ Z_{\text{Top of steel}} = 25,916 \times 4.05 = 6,399 \text{ in.}^3 \]

\[ Z_{\text{Bot. of steel}} = \frac{25,916}{32.25} = 804 \text{ in.}^3 \]

Stresses at Top of Steel—Combination A

**Compression (Composite Section)**

\[ DL_1: f_c = \frac{94 \times 12}{563} = 2.00 \text{ ksi} \]

\[ DL_2: f_c = \frac{20 \times 12}{1,852} = 0.13 \text{ ksi} \]

\[ LL + I: f_c = \frac{540 \times 12}{6,399} = 1.01 \text{ ksi} \]

**Tension (Steel Only)**

\[ f_s = \frac{173 \times 12}{563} = 3.69 \text{ ksi} \]

Ratio of minimum to maximum stress in top cover plate is

\[ R = \frac{-3.14}{3.69} = -0.851 \]

The allowable weld fatigue stress in shear is

\[ F_r = \frac{10.8}{1 - 0.55 \left( -0.851 \right)} = 7.36 \text{ ksi} \]

Allowable load on weld = 7.36 × 0.707 = 5.20 kips per in.

Force in cover plate = 10 × 3/8 × 3.69 = 13.84 kips

Weld size required = \[\frac{13.84}{5.20 \times 40} = 0.066\text{ in.}, \text{say 1/8 in.}\]

Use 1/8-in. fillet weld required for flange thickness.
CUTOFF OF BOTTOM COVER PLATE

The theoretical cutoff point for the 10 x \(\frac{3}{8}\)-in. bottom cover plate is located by trial at 17 ft from the end support. Fatigue limitations set the plate end at 13.5 ft from the support. There is no reversal of stress in this region of the span. The maximum stress range occurs with cycling between maximum positive live-load moment and maximum negative live-load moment.

Weld size is governed by material thickness. A check on strength is included for illustrative purposes.

**Stresses 17 Ft from End Support**

Between the end support and the theoretical cutoff point 17 ft away, the steel section consists of the W36 x 135, with section modulus \(Z = 438.6\) in.\(^3\).

**Composite Section, 3n = 24, near End Support**

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad(^2)</th>
<th>I(_c)</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>W36 x 135</td>
<td>39.70</td>
<td>24.50</td>
<td>23.15</td>
<td>567.18</td>
<td>13,130</td>
<td>7,796</td>
</tr>
<tr>
<td>Conc. 84 x 7/24</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\begin{align*}
  d_1 &= \frac{567.18}{64.20} = 8.83 \text{ in.} \\
  d_{\text{Top of steel}} &= 17.18 - 8.83 = 8.95 \text{ in.} \\
  Z_{\text{Top of steel}} &= \frac{16,018}{8.95} = 1,790 \text{ in.}^3 \\
  I_{\text{Top of steel}} &= 17.78 + 8.95 = 26.73 \text{ in.}^4 \\
  Z_{\text{Top of steel}} &= \frac{16,018}{26.73} = 599 \text{ in.}^3
\end{align*}

**Composite Section, n = 8, near End Support**

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad(^2)</th>
<th>I(_c)</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>W36 x 135</td>
<td>39.70</td>
<td>73.50</td>
<td>23.15</td>
<td>1,701.53</td>
<td>39,300</td>
<td>7,796</td>
</tr>
<tr>
<td>Conc. 84 x (\frac{3}{8})</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\begin{align*}
  d_s &= \frac{1,701.53}{113.20} = 15.03 \text{ in.} \\
  d_{\text{Top of steel}} &= 17.78 - 15.03 = 2.75 \text{ in.} \\
  Z_{\text{Top of steel}} &= \frac{21,912}{2.75} = 7,968 \text{ in.}^3 \\
  Z_{\text{Top of steel}} &= \frac{21,912}{32.81} = 688 \text{ in.}^3
\end{align*}

**Bending Moments 17 Ft from End Support**

<table>
<thead>
<tr>
<th></th>
<th>(DL_1)</th>
<th>(DL_2)</th>
<th>(LL+I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M, \text{ kip-ft})</td>
<td>274</td>
<td>50</td>
<td>640</td>
</tr>
</tbody>
</table>

**Steel Stresses—Combination A**

**Top of Steel (Compression)**

\[
\begin{align*}
  DL_1: f_b &= \frac{274 \times 12}{438.6} = 7.50 \\
  DL_2: f_b &= \frac{50 \times 12}{1,790} = 0.34 \\
  LL+I: f_b &= \frac{640 \times 12}{7,968} = 8.80 \text{ ksi}
\end{align*}

**Bottom of Steel (Tension)**

\[
\begin{align*}
  f_b &= \frac{274 \times 12}{438.6} = 7.50 \\
  f_b &= \frac{50 \times 12}{599} = 1.00 \\
  f_b &= \frac{640 \times 12}{668} = 11.50 \text{ ksi}
\end{align*}

II/3.38
Fatigue Check at Cover-Plate End

The composite section with W36 x 135 is adequate for bending stresses. Fatigue stresses, however, will govern. The 10 x ⅜-in. bottom cover plate must be extended beyond the 15-in. minimum terminal distance. Try ending the plate 13.5 ft from the end support.

Maximum Moments 13.5 Ft from End Support

<table>
<thead>
<tr>
<th>With Positive Live-Load Moment</th>
<th>With Negative Live-Load Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DL_1$: $M = 232$</td>
<td>$DL_1$: $M = +232$</td>
</tr>
<tr>
<td>$DL_2$: $M = 41$</td>
<td>$DL_2$: $M = +41$</td>
</tr>
<tr>
<td>$LL + I$: $M = 560$</td>
<td>$LL + I$: $M = -78$</td>
</tr>
<tr>
<td></td>
<td>$195$ kip-ft</td>
</tr>
</tbody>
</table>

Stresses at Bottom of Steel—Combination A

Tension (Composite Section)

$DL_1$: $f_b = \frac{232 \times 12}{438.6} = 6.35$

$DL_2$: $f_b = \frac{41 \times 12}{599} = 0.82$

$LL + I$: $f_b = \frac{560 \times 12}{668} = 10.06$

$17.23$ ksi $< 20$

The ratio of minimum to maximum stress in beam flange at cover-plate weld:

$R = \frac{5.34}{17.23} = 0.310$

The allowable fatigue stress is

$F_r = \frac{12.0}{1 - 0.310} = 17.39$ ksi $> 17.23$

Since allowable stresses are satisfied, the plate can be terminated 13.5 ft from the end support.

Weld at End of Bottom Cover Plate

The fillet weld connecting the bottom cover plate to the W36 x 135 flange within 15 in. of the end must develop the force in the plate. The weld size is determined at the theoretical cutoff point 14.75 ft from the end support. A ¾-in. fillet weld 40 in. long would be adequate, but the ⅜-in. flange of the W36 x 135 requires at least a ⅜-in. weld.

Maximum Moments 14.75 Ft from End Support

<table>
<thead>
<tr>
<th>With Positive Live-Load Moment</th>
<th>With Negative Live-Load Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DL_1$: $M = 247$</td>
<td>$DL_1$: $M = +247$</td>
</tr>
<tr>
<td>$DL_2$: $M = 45$</td>
<td>$DL_2$: $M = +45$</td>
</tr>
<tr>
<td>$LL + I$: $M = 590$</td>
<td>$LL + I$: $M = -88$</td>
</tr>
<tr>
<td></td>
<td>$204$ kip-ft</td>
</tr>
</tbody>
</table>

Stresses at Bottom of Steel—Combination A

Tension (Composite Section)

$DL_1$: $f_b = \frac{247 \times 12}{536} = 5.53$

$DL_2$: $f_b = \frac{45 \times 12}{727} = 0.74$

$LL + I$: $f_b = \frac{590 \times 12}{805} = 3.80$

$15.07$ ksi
Ratio of minimum to maximum stress in bottom cover plate is

\[
R = \frac{4.57}{15.07} = 0.303
\]

The allowable weld fatigue stress in shear is

\[
F_\gamma = \frac{10.8}{1 - 0.55(0.303)} = 12.96 \text{ ksi} > 12.4
\]

Allowable load on weld = \(12.4 \times 0.707 = 8.76\) kips per in.

Force in cover plate = \(10 \times \frac{3}{8} \times 15.07 = 56.5\) kips

Weld size required = \(\frac{56.5}{8.76 \times 40.0} = 0.161\) in., say \(\frac{3}{16}\) in.

Use \(\frac{3}{16}\) in. fillet weld, required for flange thickness.

From the preceding calculations, it is evident that the stringer design is governed to a great extent by fatigue at the cover-plate cutoff points. Two alternate designs have been made, aimed at minimizing the fatigue limitations by partially or completely eliminating cover plates. These alternate designs are discussed in detail at the end of this example.

**COMPARISON OF FATIGUE FORMULAS FOR 100,000 AND 500,000 CYCLES**

The preceding designs have been based on 500,000 cycles of loading, corresponding to fatigue conditions for heavily travelled expressways. Rolled-beam composite design, however, finds frequent application for structures carrying secondary roadways over expressways, for which fatigue limitations may be based on 100,000 applications of load. For comparison, tensile-fatigue formulas are given below for A36 steel, for butt welds and base metal adjacent to fillet welds at 100,000 and 500,000 cycles.

<table>
<thead>
<tr>
<th>Cycles</th>
<th>Butt Welds</th>
<th>Base Metal Adjacent to Fillet Welds</th>
</tr>
</thead>
<tbody>
<tr>
<td>100,000</td>
<td>(F_\gamma = \frac{20.5}{1 - 0.55R})</td>
<td>(F_\gamma = \frac{18.0}{1 - R})</td>
</tr>
<tr>
<td>500,000</td>
<td>(F_\gamma = \frac{17.2}{1 - 0.62R})</td>
<td>(F_\gamma = \frac{12.0}{1 - R})</td>
</tr>
</tbody>
</table>

The allowable fatigue stresses for 100,000 cycles are about 20 to 50% higher than those for 500,000 cycles. Thus, the extent to which fatigue controls cover-plate cutoff locations is proportionately reduced in structures carrying secondary roadways. This is illustrated in a third alternate design based on 100,000 cycles of fatigue loading, appearing at the end of the present example.

**RE-ANALYSIS BASED ON VARIABLE SECTION**

After material sizes and cover-plate cutoff locations have been determined, moments based on variable moment of inertia can be obtained. Curves of maximum moments are plotted from the data. For comparison, the moment curves based on constant moment of inertia are shown by dashed lines. The diagram shows that there is little difference between the curves for variable and constant moment of inertia.

Stresses based on the variable section are first checked at the points of maximum-negative and maximum-positive moment.

**Stresses for Maximum-Negative Moment**

The effect of variable moment of inertia at the negative-moment section is a decrease in stress from 19.50 to 18.72 ksi.
Bending Moments at Interior Support

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$LL+I$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$, kip-ft</td>
<td>-609</td>
<td>-92</td>
<td>-502</td>
<td>-1,203</td>
</tr>
</tbody>
</table>

$f_b = \frac{1,203 \times 12}{771} = 18.72$ ksi

**MAXIMUM-MOMENT CURVES—VARIABLE I**

**Stresses for Maximum Positive Moment**

The effect of variable moment of inertia at the positive-moment section is an increase in steel stress from 19.04 to 19.07 ksi and in concrete stress from 0.593 to 0.620 ksi.

**Bending Moments 28 Ft from End Support**

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$LL+I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$, kip-ft</td>
<td>286</td>
<td>60</td>
<td>783</td>
</tr>
</tbody>
</table>

Span 1 70'-0"

Sym. about $\delta$ Interior Bearing

End Bearing
Steel Stresses—Combination A

Top of Steel (Compression)

\[ DL_1: f_b = \frac{286 \times 12}{460} = 7.46 \]

\[ DL_2: f_b = \frac{60 \times 12}{1,781} = 0.40 \]

\[ LL+I: f_b = \frac{783 \times 12}{6,789} = 1.38 \text{ ksi} \]

Bottom of Steel (Tension)

\[ f_b = \frac{286 \times 12}{536} = 6.40 \]

\[ f_b = \frac{60 \times 12}{727} = 0.98 \]

\[ f_b = \frac{783 \times 12}{805} = 11.67 \text{ ksi} \]

Concrete Stresses—Combination B

\[ DL_1: f_c = \frac{60 \times 12}{2,040 \times 8} = 0.044 \]

\[ LL+I: f_c = \frac{783 \times 12}{2,040 \times 8} = 0.576 \text{ ksi} \]

Check of Fatigue Stresses

Stresses at the ends of the top and bottom cover plates, as well as at the butt weld between cover plates, are checked and exhibit only slight changes from the values based on constant moment of inertia.

Maximum Moments 24 Ft from Interior Support

With Positive Live-Load Moment

\[ DL_1: M = 95 \]

\[ DL_2: M = 30 \]

\[ LL+I: M = 630 \]

With Negative Live-Load Moment

\[ DL_1: M = + 95 \]

\[ DL_2: M = + 30 \]

\[ LL+I: M = -230 \text{ kip-ft} \]

Stresses at Top of Steel—Combination A

Compression (Composite Section)

\[ DL_1: f_b = \frac{95 \times 12}{460} = 2.48 \]

\[ DL_2: f_b = \frac{30 \times 12}{1,781} = 0.20 \]

\[ LL+I: f_b = \frac{630 \times 12}{6,789} = 1.11 \text{ ksi} \]

Tension (Steel Only)

\[ f_b = \frac{105 \times 12}{460} = 2.74 \text{ ksi} \]

Ratio of minimum to maximum stress at top of W36×135 is

\[ R = \frac{2.74}{3.79} = 0.723 \]

The allowable fatigue stress at the fillet weld is

\[ F_s = \frac{12.0}{1-(-0.723)} = 6.96 \text{ ksi} > 3.79 \]

Thus, at the cover-plate end 24 ft from the interior support, stresses are slightly lower for variable moment of inertia than for constant moment of inertia.

Maximum Moments 13.5 Ft from End Support

With Positive Live-Load Moment

\[ DL_1: M = 221 \]

\[ DL_2: M = 44 \]

\[ LL+I: M = 571 \]

With Negative Live-Load Moment

\[ DL_1: M = + 221 \]

\[ DL_2: M = + 44 \]

\[ LL+I: M = -68 \text{ kip-ft} \]
Stresses at Bottom of Steel—Combination A

Tension (Composite Section)

\[ DL_1: f_b = \frac{221 \times 12}{438.6} = 6.05 \]

\[ DL_2: f_b = \frac{44 \times 12}{599} = 0.88 \]

\[ LL+I: f_b = \frac{571 \times 12}{668} = 10.26 \text{ ksi} \]

The ratio of minimum to maximum stress in beam flange at cover-plate weld is:

\[ R = \frac{5.39}{17.19} = 0.314 \]

The allowable fatigue stress is

\[ F_r = \frac{12.0}{1-0.314} = 17.49 \text{ ksi} > 17.19 \]

At 13.5 ft from the end support, stresses at the cover-plate end for constant and variable moment of inertia differ by less than 1%.

Maximum Moments 6 Ft from Interior Support

With Positive Live-Load Moment

\[ DL_1: M = -390 \]

\[ DL_2: M = -50 \]

\[ LL+I: M = +80 \text{ kip-ft} \]

With Negative Live-Load Moment

\[ DL_1: M = -390 \]

\[ DL_2: M = -50 \]

\[ LL+I: M = -330 \text{ kip-ft} \]

Bending stress in top and bottom cover plates at the butt-welded splice is

\[ f_b = \frac{770 \times 12}{563} = 16.42 \text{ ksi} < 17.70 \]

The ratio of minimum to maximum stress in the butt weld is

\[ R = \frac{-360}{-770} = 0.467 \]

The allowable butt-weld fatigue stress in tension is

\[ F_r = \frac{17.2}{1-0.62(0.467)} = 24.19 \text{ ksi} > 16.42 \]

The allowable butt-weld fatigue stress in compression is

\[ F_r = \frac{0.55 \times 36}{1 - \left( \frac{0.55 \times 36}{10.6} - 1 \right) 0.467} = 33.28 \text{ ksi} > 16.42 \]

At 6 ft from the interior support, bending stress is smaller for variable moment of inertia, and lateral buckling still governs.

The computations illustrate that a uniform moment of inertia may be assumed with sufficient accuracy in analysis of two-span, continuous, composite stringers.

**BEARING STIFFENERS**

Bearing stiffeners are not required at the supports of rolled beams if the web shearing stress does not exceed 75% of the allowable shearing stress. Calculations indicate that shearing stresses are well below this value. Hence, stiffeners are not necessary.
Reactions at End Support

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$LL+I$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$, kips</td>
<td>23.6</td>
<td>4.3</td>
<td>57.0</td>
<td>84.9</td>
</tr>
</tbody>
</table>

\[ f_r = \frac{84.9}{0.598 \times 35.55} = 4.0 \text{ ksi} \]

The allowable shear without stiffeners is

\[ f_s = 12 \times 0.75 = 9 \text{ ksi} > 4.0 \]

No end stiffeners are required.

Shears at Interior Support

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$LL+I$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$, kips</td>
<td>39.3</td>
<td>7.2</td>
<td>62.1</td>
<td>108.6</td>
</tr>
</tbody>
</table>

\[ f_r = \frac{108.6}{0.598 \times 35.55} = 5.1 \text{ ksi} < 9 \text{ ksi} \]

No stiffeners are required at the interior support.

**DESIGN OF SHEAR CONNECTORS**

Welded stud shear connectors, $\frac{3}{8}$ in. in diameter and 4 in. high, are provided in the composite region of the span and in the negative-moment region adjacent to the dead-load inflection point. The allowable load on the shear connectors is calculated according to AASHO Specifications for an $H/d$ ratio greater than 4.0. Service behavior under repeated load governs the spacing. The required spacing is calculated for points along the span, and the theoretical spacing curve is plotted. The actual, stepped spacing diagram is enclosed within this theoretical spacing curve. The shear connector pitch is changed in 3-in. increments.

Concrete: $f'_c = 4,000$ psi; $n = 8$

Studs: $\frac{3}{8}$-in.-dia, 4-in.-high, $H/d = 4.0/0.875 = 4.6 > 4.0$

The ultimate strength of a shear connector equals

\[ Q_s = 0.93d^2\sqrt{f'_c} = 0.93(0.875)^2\sqrt{4,000} = 45.0 \text{ kips per stud} \]

With $\alpha$ given as 10.6 for 500,000 cycles of load in AASHO specifications, the load range per shear connector is

\[ Z_r = \alpha d^2 = 10.6(0.875)^2 = 8.11 \text{ kips per stud} \]

**Shear Connectors—Strength Requirements**

At midspan, the maximum compressive stress in the concrete is

\[ H_1 = A_s F_y = [39.70 + (10.0 \times 0.375)]36.0 = 1,564 \text{ kips (governs)} \]

\[ H_2 = 0.85f'_c b t = 0.85 \times 4.0 \times 84.0 \times 7.0 = 1,989.2 \text{ kips} \]

The number of studs required from point of maximum moment ($0.4L$) to end support and to dead-load inflection point is

\[ N = \frac{H_1}{\phi Q_s} = \frac{1,564}{0.85 \times 45.0} = 40.8 \]

where $\phi =$ reduction factor $= 0.85$
At the cover-plate cutoffs, 13.5 ft from the end support:

\[ H_1 = A_s F_y = 39.70 \times 36.0 = 1,429 \text{ kips (governs)} \]

\[ H_2 = 1,999.2 \text{ kips} \]

The number of studs required between the cover-plate cutoff and the end bearing is

\[ N = H_1 \frac{1}{\phi_Q} = \frac{1,429}{0.85 \times 45.0} = 37.4 \]

**Shear-Connector Spacing for Service Behavior (Fatigue)**

The required spacing of shear connectors under repeated loads is calculated first for the end support. There, shear range for the live load \( V_p = 57.0 - (-6.0) = 63.0 \text{ kips.} \)

For \( n = 8 \), the horizontal shear per linear inch is

\[ S_r = \frac{V_p Q}{I} = \frac{63.0 \times (73.5 \times 8.12)}{21,912} = 1.72 \text{ kips per in.} \]

Spacing required (3 studs) \( \frac{3Z_r}{S_r} = \frac{3 \times 8.11}{1.72} = 14.1 \text{ in.} \)

Next, the shear-connector spacing required at each tenth point is computed. Then, the theoretical spacing envelope is plotted. The stepped spacing diagram for fabrication is drawn within the envelope and provides 66 studs from the end support to the 0.4L point and 57 studs from there to the dead-load inflection point. From the end support to the cover-plate end, 39 studs are provided. The number of studs supplied exceed the number required for strength and fatigue.

**SHEAR-CONNECTOR SPACING**

**Shear Connectors Required for Slab Reinforcement**

To develop the slab reinforcement as recommended by AASHO specifications, 6 extra studs are required in the negative-moment region adjacent to the inflection point.

For longitudinal reinforcement within the effective flange width in the negative-moment region, 14 No. 5 bars are specified. These provide an area

\[ A_s = 14 \times 0.31 = 4.34 \text{ in.}^2 \]

and have a yield strength of

\[ H_2 = A_s F_y = 4.34 \times 40.0 = 173.6 \text{ kips} \]
This is equivalent to

\[ N = \frac{H_2}{\phi Q_e} = \frac{173.6}{0.85 \times 45.0} = 4.5 \text{ studs} \]

Live load plus impact, however, requires

\[ N = \frac{A f_c}{Z'} = \frac{4.34 \times 10.0}{8.11} = 5.3 \text{ studs} \]

Use 6 extra studs adjacent to the dead-load inflection point to develop the slab reinforcement in the negative-moment region.

**Shear Connectors Required Beyond End of Top Cover Plate**

At the end of the 10 x \( \frac{3}{8} \)-in. top cover plate, 24 ft from the interior support, full composite action must be insured. Thus, enough shear connectors must be provided between plate end and the inflection point to develop the maximum force in the concrete slab at the plate end. A 15 in. pitch is used over this distance.

**Moments 24 Ft from Interior Support**

<table>
<thead>
<tr>
<th></th>
<th>( DL_2 )</th>
<th>( LL+I )</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M, \text{ kips} )</td>
<td>25</td>
<td>580</td>
<td>605</td>
</tr>
</tbody>
</table>

**Concrete Stresses—Combination B**

**Top of Slab (Compression)**

\[ DL_2: f_c = \frac{25 \times 12}{2040 \times 8} = 0.018 \text{ ksi} \]

\[ LL+I: f_c = \frac{580 \times 12}{2040 \times 8} = 0.426 \text{ ksi} \]

**Bottom of Slab (Compression)**

\[ f_c = \frac{25 \times 12 \times 5.68}{25,867 \times 8} = 0.008 \text{ ksi} \]

\[ f_c = \frac{580 \times 12 \times 5.68}{25,867 \times 8} = 0.191 \text{ ksi} \]

The average stress in the concrete slab is \( \frac{1}{2} (0.444 + 0.199) = 0.321 \text{ ksi} \) and the total force in the slab is \( 84 \times 7 \times 0.321 = 189 \text{ kips} \). With a factor of safety of 3.0 for stud strength, the number of studs required to develop 189 kips is

\[ N = \frac{189}{45.0 / 3} = 12.6 \]

Use 5 rows of studs at 15-in. pitch = 15 studs over a length of 5 ft.

**WELDED FIELD SPLICE**

A full-penetration butt weld is used to make a field splice for the two-span stringer at one dead-load inflection point, 17.5 ft from the interior support. The field splice is designed as a full-strength welded splice, as shown below. All welds are to be ground smooth. Hence, there are no fatigue restrictions.
BOLTED FIELD SPICE

A bolted field splice is investigated as an alternate design. The splice, made with 3/4-in.-dia, high-strength friction bolts, must meet static strength requirements. It also must satisfy requirements for fatigue in base metal adjacent to friction-type fasteners.

For static strength, the splice material is proportioned to carry the greater of:

1. 75% of the moment capacity of the net section.
2. The average of the actual maximum moment and the moment capacity of the net section.

The net section is the gross section on the weaker side of a splice less bolt holes.

Fatigue need not be considered when calculating bolt stresses, since no reduction in allowable bolt stress is required by AASHO specifications, regardless of the stress range or number of stress repetitions. Fatigue in base metal adjacent to friction-type connectors, however, should be considered in the design of splice plates. A fatigue design moment, defined as follows, assures that splice-plate stresses will be less than the allowable fatigue stress.

Fatigue design moment = actual maximum moment \times \frac{\text{allowable tensile stress}}{\text{allowable fatigue stress}}

If greater than moments (1) and (2), the fatigue design moment controls the splice plate design. In this example, (2) governs, because it is greater than (1) and the fatigue design moment. In other designs, the fatigue design moment may govern.

The shear to be used in design of a splice is not as well standardized as is the moment. Some designs are made for 75% of the shear capacity of the web or the average of the actual shear and the shear capacity of the web, whichever is greater. Such a design is usually quite conservative, particularly for a rolled beam with a heavy web and high shear capacity. Since both shear and moment are directly related to applied load, it would appear reasonable to use a design shear increased by the same proportion as the design moment. Accordingly, the field splice is designed for shear determined as follows:

Design shear = actual maximum shear \times \frac{\text{design moment}}{\text{actual maximum moment}}

Design calculations for the splice begin with tabulation of maximum shear and moment at a point 17.5 ft from the interior support, and computation of net-section properties.

\[ DL_1: V = 23.0 \]
\[ DL_2: V = 4.5 \]
\[ LL+I: V = 48.0 \]
\[ 75.5 \text{ kips} \]

**Maximum Shear**

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>( A_d^2 )</th>
<th>( I_s )</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>W36 x 135</td>
<td>39.70</td>
<td>7.50</td>
<td>17.97</td>
<td>7796</td>
<td>7796</td>
</tr>
<tr>
<td>2 Plates 10 x 3/4</td>
<td>47.20</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( I = \frac{10,218}{1} \text{ in.}^4 \)

Use 3/4-in.-dia, high-strength bolts (ASTM A325), with 1-in. hole diameter.

*Flange hole area = 1 \times 1.169 = 1.169 \text{ in.}^2*

*Web hole area = 1 \times 0.598 = 0.598 \text{ in.}^2*
Required Moment and Shear Capacity of Splice

Fatigue is investigated but is found not to govern the design of splice material. Moments for design are determined by the average of the maximum moment and the moment capacity of the net section.

Ratio of minimum to maximum stress in the cover plate is

$$R = \frac{-310}{420} = -0.738$$

The allowable fatigue stress in tension is

$$F_r = \frac{20.5k_1}{1-0.55R} = \frac{20.5 \times 1}{1-0.55(-0.738)} = 14.58 \text{ ksi}$$

The allowable fatigue stress in compression is

$$F_c = \frac{0.55F_r}{1-\left(\frac{0.55F_r}{13.3} - 1\right)R} = \frac{0.55 \times 36}{1-\left(\frac{0.55 	imes 36}{13.3} - 1\right)(-0.738)} = 14.55 \text{ ksi} \ (\text{governs})$$

For the splice plates, then, the fatigue design moment is

$$M_{fat} = \frac{420 \times 20}{14.55} = 578 \text{ kip-ft}$$

The moment capacity of the net section is

$$M_{net} = \frac{20 \times 8,331}{18.15 \times 12} = 765 \text{ kip-ft}$$

75% $$M_{net} = 0.75 \times 765 = 574 \text{ kip-ft}$$

The average moment capacity is

$$M_{av} = \frac{420 + 765}{2} = 593 \text{ kip-ft} > 578$$

Because $$M_{av}$$ is larger than the fatigue design moment and 75% of $$M_{net}$$, the splice is designed for 593 kip-ft.

The shear capacity corresponding to the average-moment capacity is

$$V = 75.5 \times \frac{593}{420} = 107 \text{ kips}$$
Web-Splice Design

The web splice is designed to carry the total shear at the section, the moment due to the eccentricity of this shear, and a portion of the total moment on the section. The share of total moment to be resisted by the web is obtained by multiplying the total moment by the ratio of the net moment of inertia of the web to the net moment of inertia of the entire section.

A check is made of the shear in the extreme bolt and extreme fiber stress of the web splice plate. Computations show that both values are within allowable limits.

Deduction of twice the flange thickness from the depth of the W36 x 135 yields the web depth: 35.55 - 2 x 0.795 = 33.96 in. The net moment of inertia of the web then is

\[ I_w = \frac{0.598(33.96)^2}{12} - 444 = 1,508 \text{ in.}^4 \]

**Web Moment**

\[ M_w = \frac{107 \times 3.25}{12} = 29 \]

\[ M_w = \frac{1,508}{8,331} \times 593 = 108 \text{ kip-ft} \]

**Moment of Inertia of Bolts**

\[ I_{x-x} = 2 \times 2 \times 371.25 = 1,485 \]

\[ I_{y-y} = 20(1.5)^2 = 45 \]

\[ 1,530 \text{ in.}^4 \]

Load per bolt due to shear is

\[ P_s = \frac{107}{20} = 5.35 \text{ kips} \]

Load on the outermost bolt due to moment is

\[ P_m = \frac{137 \times 12 \times 13.58}{1,530} = 14.59 \text{ kips} \]
The vertical component of this load is

\[ P_v = \frac{14.59 \times 1.5}{13.58} = 1.61 \text{ kips} \]

And the horizontal component is

\[ P_h = \frac{14.59 \times 13.5}{13.58} = 14.50 \text{ kips} \]

Hence, the total load on the outermost bolt is the resultant

\[ P = \sqrt{(5.35 + 1.61)^2 + (14.50)^2} = 16.08 \text{ kips} \]

Allowable double shear on \( \frac{7}{8}\)-in.-dia, \( H S \) bolt = 16.2 kips > 16.08. The bolts and bolt arrangement are satisfactory.

**Web Splice Plates—Design for Average Moment**

For the web splice, assume two plates 12\( \frac{1}{2} \) x \( \frac{3}{8} \)-in. by 2 ft 6 in. long.

\[ I = 2 \times 0.375 \left( \frac{30}{12}^3 - 742.5 \right) = 1,131 \text{ in.}^4 \]

These plates resist the portion of the bending moment carried by the web, 108 kip-ft. This produces a maximum stress

\[ f = \frac{108 \times 12 \times 15}{1,131} = 17.19 \text{ ksi} < 20 \text{ (allowable)} \]

The assumed plates are satisfactory.

**Flange-Splice Design**

The flange splice carries that portion of the total moment not carried by the web. The splice plates transmit this moment across the splice as a couple in axial tension and compression, and into the cover plates and W36 x 135 flanges by double shear on \( \frac{7}{8}\)-in.-dia bolts. Two rows of bolts are used, with \( \frac{9}{16} \) and \( \frac{5}{8} \)-in.-thick splice plates.

**Flange Bolts Required**

The required average-moment capacity of the flange splice is

\[ M_f = 593 - 108 = 485 \text{ kip-ft} \]

Compressive and tensile forces in the flanges form a couple that supply this capacity.

\[ P_f = \frac{485 \times 12}{36.30 - 1.17} = 166 \text{ kips} \]

Bolts required = \( \frac{166}{16.2} = 11 \). Use 12.

**Flange Splice Plates—Design for Average Moment**

To carry the 166-kip force on the flange, the flange splice plates must have an area of at least

\[ A = \frac{166}{20} = 8.30 \text{ in.}^2 \]

Use on each flange one plate 10 x \( \frac{3}{8} \) in. and two plates 4\( \frac{1}{2} \) x \( \frac{5}{8} \) in.

The net area supplied is

\[ (10 - 2) \frac{3}{8} = 4.50 \]

\[ (9 - 2) \frac{5}{8} = 4.38 \]

Net \( A = 8.88 \text{ in.}^2 > 8.30 \)
FLANGE SPLICE

SECTION

When the field splice is welded, the 10 x 3\(\frac{3}{8}\)-in. top cover plate extends past the splice to the cutoff point 24 ft from the interior support. This cutoff location is governed by allowable fatigue stress in the metal adjacent to the fillet weld across the end of the cover plate. When the field splice is bolted, the top cover plate may be cut off at the splice, since, between the first line of bolts on each side of the splice, no stress exists either in the cover plate or rolled beam. The stress is carried solely by the splice plates across this region and transferred gradually back into the stringer flange through the bolts. Thus, no welding across the end of the cover plate is required, nor is there any notch effect from the abrupt ending of the plate.

A 10 x 3\(\frac{3}{8}\)-in. fill plate is used beneath the top splice plate beyond the cover-plate cutoff. Elimination of the fillet weld there also renders it unnecessary to fully develop the slab at 24 ft from the interior support. Adjacent to the splice, 15-in. shear-connector pitch is used and is terminated short of the flange splice plates. Two extra rows of shear connectors are still required to develop the slab reinforcement in the negative-moment region and are placed on the corresponding side of the splice at 6-in. pitch.

DEFLECTIONS

The final step in this design is calculation of dead-load camber and live-load deflection. Deflections due to dead load are computed by the formula given earlier, at the splice, midway between the end support and splice, and midway between splice and interior support. These ordinates provide sufficient data for the fabricator to camber the beams. The moment of inertia at midspan is assumed constant throughout the span in these calculations.

Deflections Due to Dead Loads

\[ DL_1: w = 0.900 \text{ kips per ft} \quad M_R = 551 \text{ kip-ft} \]

\[ DL_2: w = 0.165 \text{ kips per ft} \quad M_R = 101 \text{ kip-ft} \]

Uniformly Loaded Span

\[
\Delta = \frac{72wL^4}{E,I} ab[1+ab-4C_R(1+a)]
\]
where \( \Delta \) - deflection, in., at distance \( aL \) from end support
\( b = 1 - a \)
\( w = \) dead load, kips per ft
\( L = \) span, ft
\( E_s = \) modulus of elasticity of the steel = 29(10)\(^3\) ksi
\( I = \) moment of inertia at midspan, in.\(^4\)
\( C_n = \frac{M_n}{wL^2} \)
\( M_n = \) bending moment at interior support, kip-ft

**Deflections Under DL\(_1\)**

\[
\Delta = \frac{72x0.900(70)^4}{29(10)^3\times8,902}ab[1+ab-4\times0.125(1+a)] = 6.03ab[1+ab-0.5(1+a)]
\]

At \( a = \frac{3}{8} \),
\[ \Delta = 6.03(0.375)(0.625)[1+0.375(0.625) - 0.5(1.375)] = 0.772 \text{ in.} \]

At \( a = \frac{3}{4} \),
\[ \Delta = 6.03(0.750)(0.250)[1+0.75(0.25) - 0.5(1.750)] = 0.353 \text{ in.} \]

At \( a = \frac{7}{8} \),
\[ \Delta = 6.03(0.875)(0.125)[1+0.875(0.125) - 0.5(1.875)] = 0.113 \text{ in.} \]

**Deflections Under DL\(_2\)**

\[
\Delta = \frac{72x0.165(70)^4}{29(10)^3\times18,557}ab[1+ab-4\times0.125(1+a)] = 0.530ab[1+ab-0.5(1+a)]
\]

At \( a = \frac{3}{8} \),
\[ \Delta = 0.530(0.375)(0.625)[1+0.375(0.625) - 0.5(1.375)] = 0.068 \text{ in.} \]

At \( a = \frac{3}{4} \),
\[ \Delta = 0.530(0.750)(0.250)[1+0.75(0.25) - 0.5(1.75)] = 0.031 \text{ in.} \]

At \( a = \frac{7}{8} \),
\[ \Delta = 0.530(0.875)(0.125)[1+0.875(0.125) - 0.5(1.875)] = 0.010 \text{ in.} \]

**Total DL Deflections**

At \( a = \frac{3}{8} \), \( \Delta = 0.772 + 0.068 = 0.840 \text{ in.} \), say \( \frac{7}{8} \) in.

At \( a = \frac{3}{4} \), \( \Delta = 0.353 + 0.031 = 0.384 \text{ in.} \), say \( \frac{3}{4} \) in.

At \( a = \frac{7}{8} \), \( \Delta = 0.113 + 0.010 = 0.123 \text{ in.} \), say \( \frac{1}{8} \) in.

**Camber Diagram**

![Camber Diagram](image-url)
Deflection Due To Live Load + Impact

Live load for deflection consists of two lanes of truck loading, which are distributed equally to the four stringers. The approximate formula given earlier for deflection at the 0.4 point of end spans of continuous beams due to HS truck loading is used. The moment of inertia at midspan is assumed constant throughout the span. Calculations show that live-load deflections are less than half the allowable value.

The live-load deflection, in., 28 ft from the end support is given by

\[ \Delta = \frac{300}{E_sI} [P_T(L^2 + 3.89L^2 - 680L + 5,910) - 0.32M_HL^3] \]

where
- \( P_T \) = weight of front truck wheel \( \times \) distribution factor, plus impact, kips
- \( I \) = moment of inertia at midspan, in.\(^4\)
- \( L \) = span, ft
- \( E_s = 29(10)^3 \) ksi
- \( M_H \) = bending moment due to live load plus impact at the interior support, kip-ft

Assume that two lanes of live load (four wheels abreast) plus 25.6% impact are equally distributed over four stringers.

\[ P_T = 4 \times 4 \times 1.256 = 20.1 \text{ kips} \]

\[ I = 4 \times 25,867 = 103,468 \text{ in.}^4 \]

The moment \( M_H \) at the interior support can be computed in any of several ways; for example, by influence coefficients, as shown below. (See References 3 and 4 at the end of the chapter.)

\[
\begin{align*}
20.1 \times 70 \times 0.0480 &= 68 \\
80.4 \times 70 \times 0.0840 &= 473 \\
80.4 \times 70 \times 0.0960 &= 540 \\
M_H &= 1,081 \text{ kip-ft}
\end{align*}
\]

\[ M_R = 1,081k' \]

14'-0"  14'-0"  14'-0"  28'-0"  42'-0"
28'-0"  70'-0"

Girder Loaded For Maximum Deflection

The maximum live-load deflection therefore is

\[
\Delta = \frac{300}{29(10)^3 \times 103,468} \left[ 20.1[(70)^3 + 3.89(70)^2 - 680(70) + 5,910] - 0.32(1,081)(70)^3 \right]
\]

\[
= \frac{300 \times 4,744,449}{29(10)^3 \times 103,468} = 0.472 \text{ in.}
\]

The ratio of live-load deflection to span is

\[
\frac{0.472}{70 \times 12} = \frac{1}{1,780} < \frac{1}{800}
\]

**FINAL DESIGN**

An elevation of the two-span, continuous, composite stringer is shown on the next page. See also the detail drawing at the end of this chapter. Alternate designs will be examined next.
Design III—Alternate A

An alternate design for the two span, continuous, composite stringer employs a W36×160 beam with 10 1/2 x 3/8-in. cover plates added to the top and bottom flanges at the interior support. This rolled beam was selected because it is the lightest section that requires no bottom cover plate in the positive-moment region. It also permits the cover plates to end between the interior support and the inflection point without exceeding the allowable fatigue stress.

Properties are computed for the steel section alone in the maximum-negative-moment region, and for the composite section with \( n = 8 \) and \( 3n \) equal to 24 in the maximum-positive-moment region. Bending moments and shears are computed for constant moment of inertia in both spans. Stresses are calculated at the top and bottom of steel and at the top of concrete.

**MAXIMUM NEGATIVE MOMENT**

From the maximum-moment curves, the total moment at the interior support is \(-1,253\) kip-ft. This moment must be taken by the W36×160 with top and bottom cover plates. For the W36×160, the moment of inertia is \(9,739\) in.\(^4\), the half-depth \(c = 18\) in., and the section modulus \(Z = 541\).

**Steel Section at Interior Support**

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad(^2)</th>
<th>(I_o)</th>
<th>(I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W36×160</td>
<td>47.09</td>
<td>18.31</td>
<td>240.35</td>
<td>4,401</td>
<td>9,739</td>
<td>9,739</td>
</tr>
<tr>
<td>2 Plates 10(\frac{1}{2}) x 3/8</td>
<td>13.13</td>
<td>18.31</td>
<td>240.35</td>
<td>4,401</td>
<td>9,739</td>
<td>9,739</td>
</tr>
</tbody>
</table>

\(I_{NA} = 14,140\) in.\(^4\)

\(Z = \frac{14,140}{18.63} = 759\) in.\(^3\)

Maximum stress \(f_e = \frac{1,253 \times 12}{759} = 19.80\) ksi < 20

**Allowable Compressive Stress Near Interior Support**

\(F_e = 20,000 - 7.5\left(\frac{I}{b}\right)^2 = 20,000 - 7.5\left(\frac{17.5 \times 12}{12}\right)^2 = 17.70\) ksi
Because of continuity, AASHO specifications permit a 20% increase in allowable stresses up to 20 ksi at the interior support.

\[ F' \equiv 17.73 \times 1.20 = 21.28 \text{ ksi. Use } 20 \text{ ksi.} \]

**MAXIMUM POSITIVE MOMENT**
The W36 x 160 is investigated for the region of maximum positive moment. Properties are computed for the composite section with \( n = 8 \) and \( 3n = 24 \).

### Composite Section, \( 3n = 24 \), 28 Ft from End Support

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>Iₙ</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>W36 x 160</td>
<td>47.09</td>
<td>23.10</td>
<td>565.95</td>
<td>13,073</td>
<td>9.739</td>
<td>9.739</td>
</tr>
<tr>
<td>Conc. 84 x 7/24</td>
<td>24.50</td>
<td>23.10</td>
<td>565.95</td>
<td>13,073</td>
<td>100</td>
<td>13,173</td>
</tr>
</tbody>
</table>

\[ d_{xx} = \frac{565.95}{71.59} = 7.91 \text{ in.} \quad 565.95 \text{ in.}^3 \quad -7.91 \times 565.95 = -4,477 \quad I_{NA} = \frac{18,435}{4} \text{ in.}^4 \]

\[ d_{\text{Top of steel}} = 18.00 - 7.91 = 10.09 \text{ in.} \quad d_{\text{Bot. of steel}} = 18.00 + 7.91 = 25.91 \text{ in.} \]

\[ Z_{\text{Top of steel}} = \frac{18,435}{10.09} = 1,827 \text{ in.}^3 \quad Z_{\text{Bot. of steel}} = \frac{18,435}{25.91} = 712 \text{ in.}^3 \]

### Composite Section, \( n = 8 \), 28 Ft from End Support

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>Iₙ</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>W36 x 160</td>
<td>47.09</td>
<td>23.10</td>
<td>1,698</td>
<td>39,224</td>
<td>9,739</td>
<td>9,739</td>
</tr>
<tr>
<td>Conc. 84 x 3/4</td>
<td>73.50</td>
<td>23.10</td>
<td>1,698</td>
<td>39,224</td>
<td>300</td>
<td>39,524</td>
</tr>
</tbody>
</table>

\[ d_s = \frac{1,698}{120.59} = 14.08 \text{ in.} \quad 1,698 \text{ in.}^3 \quad -14.08 \times 1,698 = -23,908 \quad I_{NA} = \frac{25,355}{25} \text{ in.}^4 \]

\[ d_{\text{Top of steel}} = 18.00 - 14.08 = 3.92 \text{ in.} \quad d_{\text{Bot. of steel}} = 18.00 + 14.08 = 32.08 \text{ in.} \]

\[ Z_{\text{Top of steel}} = \frac{25,355}{3.92} = 6,468 \text{ in.}^3 \quad Z_{\text{Bot. of steel}} = \frac{25,355}{32.08} = 790 \text{ in.}^3 \]

\[ d_{\text{Top of conc.}} = 26.38 - 14.08 = 12.30 \text{ in.} \]

\[ Z_{\text{Top of conc.}} = \frac{25,355}{12.30} = 2,061 \text{ in.}^3 \]

**Check of Steel and Concrete Stresses**

Stresses are checked at top and bottom of steel and at the top of concrete. Calculations show that the W36 x 160 satisfies stress limitations. The concrete stress is well within the allowable for compression.

### Bending Moments 28 Ft from End Support

<table>
<thead>
<tr>
<th>M, kip-ft</th>
<th>DL₁</th>
<th>DL₂</th>
<th>LL + I</th>
</tr>
</thead>
<tbody>
<tr>
<td>309</td>
<td>57</td>
<td>750</td>
<td></td>
</tr>
</tbody>
</table>
Steel Stresses—Combination A

**Top of Steel (Compression)**

\[
DL: \quad f_b = \frac{309 \times 12}{541} = 6.85
\]

\[
DL_2: \quad f_b = \frac{57 \times 12}{1,827} = 0.37
\]

\[
LL + I: \quad f_b = \frac{750 \times 12}{6,468} = 1.39 \quad \text{ksi}
\]

**Bottom of Steel (Tension)**

\[
\begin{align*}
DL & : \quad f_b = \frac{309 \times 12}{541} = 6.85 \\
DL_2 & : \quad f_b = -\frac{57 \times 12}{712} = 0.96 \\
LL + I & : \quad f_b = \frac{750 \times 12}{790} = 11.39 \quad \text{ksi}
\end{align*}
\]

Concrete Stresses—Combination B

\[
\begin{align*}
DL_2: \quad f_c &= \frac{57 \times 12}{2,061 \times 8} = 0.042 \\
LL + I: \quad f_c &= \frac{750 \times 12}{2,061 \times 8} = 0.546 \\
& \quad 0.588 \text{ ksi}
\end{align*}
\]

**LOCATION OF COVER-PLATE CUTOFFS**

Stresses are checked at theoretical cutoff points for the 10\(\frac{1}{2}\) x 9\(\frac{1}{6}\)-in. cover plates 6.0 ft from the interior support and found to be less than allowable values. The actual ends, 7 ft 4 in. from the support are then investigated for fatigue strength at the fillet welds. Finally, the size of weld connecting the ends of the cover plates to the flanges is determined.

**Maximum Moments 6 Ft from Interior Support**

**With Positive Live-Load Moment**

\[
\begin{align*}
DL_1: & \quad M = -330 \\
DL_2: & \quad M = -60 \\
LL + I: & \quad M = +85 \quad -305 \text{ kip-ft}
\end{align*}
\]

**With Negative Live-Load Moment**

\[
\begin{align*}
DL_1: & \quad M = -330 \\
DL_2: & \quad M = -60 \\
LL + I: & \quad M = -400 \quad -790 \text{ kip-ft}
\end{align*}
\]

For the W36×160 alone, with \(Z = 541 \text{ in.}^3\), the maximum stress is

\[
f_b = \frac{790 \times 12}{541} = 17.52 \text{ ksi}
\]

Since this is less than 17.70 ksi, the allowable compressive stress in the bottom flange for lateral buckling, the theoretical cutoff 6 ft from the interior support is satisfactory.

The required terminal distance is at least 1.5 x 10\(\frac{1}{2}\) = 15.75 in.

Try cutoff of the cover plates 6 ft + 1 ft 4 in. = 7 ft 4 in. from the interior support.

**Fatigue Check at Cover-Plate End**

Bending stresses are computed 7 ft 4 in. from the interior support and are found to be less than the allowable fatigue stress in the W36×160 flanges.

**Maximum Moments 7 Ft 4 In. from Interior Support**

**With Positive Live-Load Moment**

\[
\begin{align*}
DL_1: & \quad M = -285 \\
DL_2: & \quad M = -50 \\
LL + I: & \quad M = +115 \quad -220 \text{ kip-ft}
\end{align*}
\]

**With Negative Live-Load Moment**

\[
\begin{align*}
DL_1: & \quad M = -285 \\
DL_2: & \quad M = -50 \\
LL + I: & \quad M = -380 \quad -715 \text{ kip-ft}
\end{align*}
\]

Bending stress in top and bottom flanges is

\[
f_b = \frac{715 \times 12}{541} = 15.86 \text{ ksi}
\]
Ratio of minimum to maximum stress at cover-plate ends is

\[ R = \frac{-220}{-715} = 0.308 \]

The allowable fatigue stress in the W36 x 160 flanges adjacent to the fillet welds for the plate ends is

\[ F_c = \frac{12.0}{1 - 0.308} = 17.34 \text{ ksi} > 15.86 \]

**Welds at Cover-Plate Ends**

The weld size is determined by the force in each cover plate at the theoretical cutoff point. 6 ft from the interior support. Computations show that a \( \frac{3}{4} \)-in. fillet weld 42.5 in. long in the terminal region of each plate would be satisfactory. For the 1-in.-thick flanges of the W36 x 160, however, a \( \frac{5}{16} \)-in. weld is required.

The stress in the cover plates at theoretical cutoff is

\[ f_b = \frac{790 \times 12}{759} = 12.49 \text{ ksi} \]

Ratio of minimum to maximum stress in the cover plates 6 ft from the interior support is

\[ R = \frac{-305}{-790} = 0.386 \]

The allowable weld fatigue stress in shear is

\[ F_v = \frac{10.8}{1 - 0.55(0.386)} = 13.7 \text{ ksi} > 12.4 \text{ ksi} \]

Allowable load on weld = 12.4 \( \times \) 0.707 = 8.76 kips per in.

Force in cover plate = 10.5 \( \times \) \( \frac{5}{8} \) \( \times \) 12.49 = 81.9 kips

Weld size required = \( \frac{81.9}{8.76 \times 42.5} \) = 0.22 in., say \( \frac{1}{4} \) in.

Use \( \frac{5}{16} \)-in. fillet weld, required for flange thickness.

**FINAL DESIGN—ALTERNATE A**

An elevation of the two-span, composite Alternate A is shown below. The alternate designs will be discussed later.

---

**BEAM ELEVATION DESIGN III—ALTERNATE A**
Design III—Alternate B

A second alternate design utilizes a W36 × 160 butt welded to a W33 × 230. This arrangement eliminates cover plates and the low allowable fatigue stresses at the ends of cover plates.

Section properties and stresses are computed for the maximum-negative and maximum-positive-moment regions. Again, bending moments and shears are calculated for constant moment of inertia in both spans. The stress is also checked at the butt-welded field splice and found to be less than the allowable fatigue stress.

MAXIMUM NEGATIVE MOMENT

From the maximum-moment curves, the total moment at the interior support is −1,253 kip-ft. This moment must be taken by the W36 × 230, with section modulus $Z = 835.5$ in.³. The maximum stress is

$$f_v = \frac{1.253 \times 12}{835.5} = 18.00 \text{ ksi} < 20$$

Allowable Compressive Stress Near Interior Support

The length $L$ for lateral buckling is 17.5 ft, the distance from interior bearing to dead-load inflection point.

$$F_s = 20,000 - 7.5 \left( \frac{L}{b} \right)^2 = 20,000 - 7.5 \left( \frac{17.5 \times 12}{16.47} \right)^2 = 18.78 \text{ ksi}$$

Because of continuity, AASHO specifications permit a 20% increase in allowable stress up to 20 ksi at the interior support.

$$F_s' = 1.20 \times 18.78 = 22.54 \text{ ksi}. \text{ Use 20 ksi.}$$

MAXIMUM POSITIVE MOMENT

The W36 × 160 is investigated for the region of maximum positive moment. Properties are computed for the composite section with $n = 8$ and $3n = 24$. For the W36 × 160, the section modulus $Z = 541$.

### Composite Section, 3n = 24, 28 Ft from End Support

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>W36 × 160</td>
<td>47.09</td>
<td>24.50</td>
<td>549.7</td>
<td>12,335</td>
<td>9,739</td>
<td>9,739</td>
</tr>
<tr>
<td>Conc. 84 × 7 24</td>
<td>22.44</td>
<td>100</td>
<td>12,435</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$$d_{1,1} = \frac{549.7}{71.59} = 7.68 \text{ in.}$$

$$d_{Top \ of \ steel} = 18.00 - 7.68 = 10.32 \text{ in.}$$

$$d_{Bot \ of \ steel} = 18.00 + 7.68 = 25.68 \text{ in.}$$

$$Z_{Top \ of \ steel} = \frac{17,952}{10.32} = 1,740 \text{ in.}³$$

$$Z_{Bot \ of \ steel} = \frac{17,952}{25.68} = 699 \text{ in.}³$$
Composite Section, \( n = 8, \) 28 Ft from End Support

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>( Ad^2 )</th>
<th>( I_o )</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>W36 × 160</td>
<td>47.09</td>
<td>73.50</td>
<td>22.44</td>
<td>1,649.3</td>
<td>9,739</td>
<td>9,739</td>
</tr>
<tr>
<td>Conc. 84 × ( \sqrt{3} )</td>
<td>120.59 in.²</td>
<td>1,649.3 in.³</td>
<td>-13.68 × 1649.3 = -22,562</td>
<td>( I_{N/A} ) = 24,487 in.⁴</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( d_{Top \ of \ steel} = 18.00 - 13.68 = 4.32 \text{ in.} \)
\( d_{Bot, \ of \ steel} = 18.00 + 13.68 = 31.68 \text{ in.} \)

\( Z_{Top \ of \ steel} = \frac{24,487}{4.32} = 5,668 \text{ in.}² \)
\( Z_{Bot, \ of \ steel} = \frac{24,487}{31.68} = 773 \text{ in.}³ \)

\( d_{Top \ of \ conc.} = 25.94 - 13.68 = 12.26 \text{ in.} \)

\( Z_{Top \ of \ conc.} = \frac{24,487}{12.26} = 1,997 \text{ in.}³ \)

Check of Steel and Concrete Stresses

Stresses are checked at top and bottom of steel and at the top of concrete. Calculations show that the W36 × 160 satisfies stress limitations. The concrete stress is well within the allowable for compression.

Bending Moments 28 Ft from End Support

<table>
<thead>
<tr>
<th>M, kip-ft</th>
<th>( DL_1 )</th>
<th>( DL_2 )</th>
<th>( LL + I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>309</td>
<td>57</td>
<td>750</td>
<td></td>
</tr>
</tbody>
</table>

Steel Stresses—Combination A

Top of Steel (Compression)
\( DL_1: f_b = \frac{309 \times 12}{541} = 6.85 \)
\( DL_2: f_b = \frac{57 \times 12}{1,740} = 0.39 \)
\( LL + I: f_b = \frac{750 \times 12}{5,668} = 1.59 \), \( 8.83 \text{ ksi} \)

Bottom of Steel (Tension)
\( f_b = \frac{309 \times 12}{541} = 6.85 \)
\( f_b = \frac{57 \times 12}{699} = 0.98 \)
\( f_b = \frac{750 \times 12}{773} = 11.64 \), \( 19.47 \text{ ksi} \)

Concrete Stresses—Combination B

Top of Concrete (Compression)
\( DL_1: f_c = \frac{57 \times 12}{1,997 \times 8} = 0.043 \)

\( LL + I: f_c = \frac{750 \times 12}{1,997 \times 8} = 0.563 \)

Fatigue Check of Butt-Welded Field Splice

The W36 × 230 and W36 × 160 are spliced at the dead-load inflection point, 17.5 ft from the interior support. Stresses in the W36 × 160 are computed at this location and found to be less than the allowable fatigue stress for the butt-welded splice.

\( DL_1 + DL_2: M = 0 \text{ kip-ft} \)
\( LL + I: M = 420 \text{ kip-ft} \)

\( DL_1 + DL_2: M = 0 \text{ kip-ft} \)
\( LL + I: M = -310 \text{ kip-ft} \)

II/3.59
Actual stress in top and bottom flanges is
\[ f_b = \frac{420 \times 12}{541} = 9.32 \text{ ksi} \]

Ratio of minimum to maximum stress in butt-welded flange splice is
\[ R = \frac{310}{420} = -0.738 \]

The allowable fatigue stress in tension is
\[ F_c = \frac{17.2}{1 - 0.62(-0.738)} = 11.80 \text{ ksi} > 9.32 \]

The allowable fatigue stress in compression is
\[ F_c = \frac{0.55 \times 36}{1 - \left[ \frac{0.55 \times 36}{10.6} - 1 \right](-0.738)} = 12.06 \text{ ksi} > 9.32 \]

**FINAL DESIGN—ALTERNATE B**
An elevation of the two-span, composite Alternate B is shown below. The alternate designs will be discussed next.

![Beam Elevation Design III—Alternate B](image)

**BEAM ELEVATION DESIGN III—ALTERNATE B**

**Design III—Alternates A and B: Economic Considerations**

Design III and its Alternates A and B are different solutions to the same structural problem. Design III is presented in considerable detail as an example that illustrates the most likely conditions to be encountered by a designer. It may not necessarily be the most advantageous of the three designs from an economic standpoint. The relative economies achieved by Design III, Alternate A, and Alternate B are dependent on the quantity of steel required for each design and amount of fabrication involved.

Alternate A requires about 7¼% more steel, and Alternate B about 16⅔% more steel than Design III. But Alternate A has less than a third as much fillet weld as Design III and only about a fifth as much length of plate shearing. Alternate B eliminates fillet welds and shearing entirely, except for minor details, but requires a major butt weld of the two rolled sections.

In a comparison of the two alternate designs with Design III, the smaller amount of fabrication in Alternate A tends to offset its greater weight. The even smaller amount of fabrication of Alternate B tends to offset its still greater weight. The relative importance of steel quantity versus amount of fabrication in deciding the economy
of a structure varies from one part of the country to another, from one time to another, and with local conditions. Furthermore, the most economic design arrangements may change with span. For slightly longer spans, lighter rolled sections with cover plates may prove superior to heavier rolled sections with a minimum of cover-plate material.

**Design III—Alternate C**

Many bridges carrying minor highways and streets are designed for 100,000 cycles of maximum stress. The design procedure is similar to that used for 500,000 cycles of maximum stress, and the same rolled beam and cover plates may be used at the points of maximum moments. However, allowable fatigue stresses in base metal adjacent to fillet welds are considerably higher for 100,000 cycles of maximum stress. When a light rolled beam is used, the heavy cover plates required for maximum negative moment may be cut off a few feet each side of the interior support. Also, the bottom cover plate required for maximum positive moment may be cut off between the point of maximum positive moment and the inflection point.

For 100,000 stress cycles, Design III—Alternate C uses the same W36 x 135 beam and cover plates at the points of maximum moment as were used for Design III. For Alternate C, however, the 10 x 1-in., negative-moment cover plates can be cut off near the interior support and fillet welded across their ends. As for Design III, the theoretical cutoff point is 8.75 ft from the interior support, but, for Alternate C, the allowable fatigue stress at the fillet weld is satisfied at only 10 ft from the support. Thus, the 10 x 1-in. top and bottom cover plates are ended at this point, rather than butt welded to 10 x ¾-in. plates as in Design III.

**Maximum Moments 8.75 Ft from Interior Support**

<table>
<thead>
<tr>
<th>With Positive Live-Load Moment</th>
<th>With Negative Live-Load Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DL_1$: $M = -240$</td>
<td>$DL_1$: $M = -240$</td>
</tr>
<tr>
<td>$DL_2$: $M = -40$</td>
<td>$DL_2$: $M = -40$</td>
</tr>
<tr>
<td>$LL + I$: $M = +155$</td>
<td>$LL + I$: $M = -365$</td>
</tr>
<tr>
<td>$-125$ kip-ft</td>
<td>$-645$ kip-ft</td>
</tr>
</tbody>
</table>

For the W36 x 135 alone, with $Z=438.6$, the maximum stress is

$$f_b = \frac{645 \times 12}{438.6} = 17.65 \text{ ksi}$$

Since this is less than 17.70 ksi, the allowable compressive stress in the bottom flange for lateral buckling, the theoretical cutoff 8.75 ft from the interior support is satisfactory. The required terminal distance is at least $1.5 \times 10 = 15$ in.

Try cutoff of the cover plates 8.75 + 1.25 = 10 ft from the interior support.

**Fatigue Check for 100,000 Cycles**

Bending stresses are computed 10 ft from the interior support and found to be less than the allowable fatigue stress in the W36 x 135 flanges.

**Maximum Moments 10 Ft from Interior Support**

<table>
<thead>
<tr>
<th>With Positive Live-Load Moment</th>
<th>With Negative Live-Load Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DL_1$: $M = -203$</td>
<td>$DL_1$: $M = -203$</td>
</tr>
<tr>
<td>$DL_2$: $M = -39$</td>
<td>$DL_2$: $M = -39$</td>
</tr>
<tr>
<td>$LL + I$: $M = +183$</td>
<td>$LL + I$: $M = -359$</td>
</tr>
<tr>
<td>$-59$ kip-ft</td>
<td>$-601$ kip-ft</td>
</tr>
</tbody>
</table>
Bending stress in top and bottom flanges is

\[
f_b = \frac{601 \times 12}{438.6} = 16.44 \text{ ksi}
\]

Ratio of minimum to maximum stress in cover-plate ends is

\[
R = \frac{59}{601} = 0.098
\]

The allowable fatigue stress in the W36 x 135 flanges adjacent to the fillet welds at the plate ends is

\[
F_r = \frac{18.0}{1 - 0.098} = 19.96 \text{ ksi} > 16.44
\]

**Welds at Cover-Plate Ends**

The weld size is determined by the force in each cover plate at the theoretical cutoff point, 8 ft 9 in. from the interior support. Computations show that the minimum permissible size for the \(\frac{3}{16}\)-in.-thick flanges of the W36 x 135, a \(\frac{5}{16}\)-in. fillet weld, is adequate. Length of weld is 40 in.

The stress in the cover plates at theoretical cutoff is

\[
f_b = \frac{645 \times 12}{771} = 10.04 \text{ ksi}
\]

Ratio of minimum to maximum stress in the cover plates 8 ft 9 in. from the interior support is

\[
R = \frac{125}{645} = 0.194
\]

The allowable weld fatigue stress in shear is

\[
F_r = \frac{12.0}{1 - 0.5 (0.194)} = 13.29 \text{ ksi} > 12.4
\]

Allowable load on weld = 12.4 \times 0.707 = 8.76 kips per in.

Force in cover plate = 10 \times 1 \times 10.04 = 100.4 kips

Weld size required = \(\frac{100.4}{8.76 \times 40} = 0.29\) in.

Use minimum \(\frac{5}{16}\)-in. fillet weld.

**CUTOFFS OF POSITIVE-MOMENT COVER PLATE**

A 10 x \(\frac{3}{8}\)-in. cover plate is used along the bottom flange in the positive-moment region. A location 40 ft from the end support is investigated as the theoretical cutoff point nearest the interior support.

**Stresses 40 Ft from End Support**

The composite section between the end of the 10 x \(\frac{3}{8}\)-in. cover plate and the deadload inflection point consists of the W36 x 135 and the 7-in. concrete slab. As computed in Design III, \(Z = 1,790\) in.\(^3\) at top of steel and 599 in.\(^3\) at bottom of steel for \(3n = 24\), and \(Z = 7,968\) in.\(^3\) at top of steel and 668 in.\(^3\) at bottom of steel for \(n = 8\).

**Bending Moments 40 Ft from End Support**

<table>
<thead>
<tr>
<th>(M, \text{kip-ft})</th>
<th>(DL_1)</th>
<th>(DL_1)</th>
<th>(LL+I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>225</td>
<td>45</td>
<td>690</td>
<td></td>
</tr>
</tbody>
</table>
Steel Stresses—Combination A

Top of Steel (Compression)
\[ DL_1: f_b = \frac{225 \times 12}{438.6} = 6.16 \]
\[ DL_2: f_b = \frac{45 \times 12}{1,790} = 0.30 \]
\[ LL+I: f_b = \frac{690 \times 12}{7,968} = 1.04 \text{ ksi} \]

Bottom of Steel (Tension)
\[ f_b = \frac{225 \times 12}{438.6} = 6.16 \]
\[ f_b = \frac{45 \times 12}{599} = 0.90 \]
\[ f_b = \frac{690 \times 12}{668} = 12.40 \text{ ksi} < 20 \]

Fatigue Check 45 Ft from End Support—100,000 Cycles

Fatigue requirements are fulfilled if the end of the cover plate is set at 45 ft from the end support. Stresses are computed for both maximum-positive and maximum-negative moments and found to be less than the allowable fatigue stress. Weld size is governed by material thickness (computations are not shown).

Maximum Moments 45 Ft from End Support

With Positive Live-Load Moment
\[ DL_1: M = 145 \]
\[ DL_2: M = 28 \]
\[ LL+I: M = 600 \]
\[ 773 \text{ kip-ft} \]

With Negative Live-Load Moment
\[ DL_1: M = 145 \]
\[ DL_2: M = 28 \]
\[ LL+I: M = -270 \]
\[ -97 \text{ kip-ft} \]

Stresses at Bottom of Steel—Combination A

Tension (Composite Section)
\[ DL_1: f_b = \frac{145 \times 12}{438.6} = 3.97 \]
\[ DL_2: f_b = \frac{28 \times 12}{599} = 0.56 \]
\[ LL+I: f_b = \frac{600 \times 12}{668} = 10.78 \]
\[ 16.31 \text{ ksi} \]

Tension (Steel Only)
\[ f_b = \frac{97 \times 12}{438.6} = 2.65 \text{ ksi} \]

Negative Moment

Ratio of minimum to maximum stress in beam flange at cover-plate weld:
\[ R = \frac{-2.65}{16.31} = -0.173 \]

The allowable fatigue stress is
\[ F_r = \frac{18.0}{1 - (-0.173)} = 15.34 \text{ ksi} > 15.31 \]

Since allowable stresses are satisfied, the plate can be terminated 45 ft from the end support.

Fatigue Check at Cutoff 15 Ft 9 In. from End Support—100,000 Cycles

The theoretical cutoff point nearest the end support for the bottom cover plate is 17 ft from that support, as established in Design III. Fatigue requirements based on 100,000 cycles are satisfied if the cover plate ends 15.75 ft from the end support.
Maximum Moments 15.75 Ft from End Support

With Positive Live-Load Moment

\[ DL_i: M = 260 \]
\[ DL_c: M = 46 \]
\[ LL + I: M = 615 \]
\[ LL + I: M = 93 \]
\[ 92\text{ kip-ft} \]
\[ 213 \text{ kip-ft} \]

With Negative Live-Load Moment

\[ DL_i: M = 260 \]
\[ DL_c: M = 46 \]

Stresses at Bottom of Steel—Combination A

Tension (Composite Section)

\[ DL_i: f_b = \frac{260 \times 12}{438.6} = 7.11 \]
\[ DL_c: f_b = \frac{46 \times 12}{599} = 0.92 \]
\[ LL + I: f_b = \frac{615 \times 12}{668} = 11.05 \]
\[ 19.08 \text{ ksi} \]

Tension (Steel Only)

\[ f_b = \frac{213 \times 12}{438.6} = 5.83 \text{ ksi} \]

Ratio of minimum to maximum stress in beam flange at cover-plate weld:

\[ R = \frac{5.83}{19.08} = 0.305 \]

The allowable fatigue stress is

\[ F_r = \frac{18.0}{1 - 0.305} = 25.90 \text{ ksi} > 19.08 \]

Since allowable stresses are satisfied, the plate can be ended 15.75 ft from the end support.

FINAL DESIGN—ALTERNATE C

An elevation of the two span, continuous, composite Alternate C is shown below.
Design IV—Two-Span Continuous Beam (70-70 Ft) Composite For Positive And Negative Moment

In Design III, composite action is developed only for positive bending moments. The design procedure for this example is similar to that for Design III, except that longitudinal reinforcing bars in the concrete slab are considered to act compositely with the beam in the negative-moment region. Concrete is assumed to be cracked under tensile stress and not acting as part of the composite section, except to transfer shear from the reinforcing bars to the beam. Reinforcing bars in the direction of the stringers are assumed to be effective if embedded in a width of slab not exceeding the following:

1. One-fourth the span of the stringer.
2. Distance center to center of stringers.
3. Twelve times the least thickness of the slab.

MAXIMUM NEGATIVE MOMENT
A steel section consisting of a W36 x 135, a 10 x 5/8-in. top cover plate, and a 10 x 1-in. bottom cover plate is selected to work in conjunction with the reinforcing bars at the maximum-negative-moment section. Properties of the steel section alone and of the composite section are computed and stresses checked at the interior support. The rolled beam with cover plates resists moments due to $DL_1$. The composite section composed of the rolled beam with cover plates and the longitudinal reinforcing bars resists moments due to $DL_2$ and $LL + I$. Since stud shear connectors are welded to the tension flange in the negative-moment region, the allowable fatigue stress for base metal in the top cover plate adjacent to the connectors is computed. Fatigue, however, does not control at the support in this example. Use of slab reinforcement in composite action results in a 3/8-in. reduction in top-cover-plate thickness from that used in Design III.

![Diagram of composite beam section](image-url)
Steel Section at Interior Support

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>W36 x 135</td>
<td>39.70</td>
<td>−18.28</td>
<td>−182.8</td>
<td>3,342</td>
<td>7,796</td>
<td>7,796</td>
</tr>
<tr>
<td>Bottom Plate 10 x 1</td>
<td>10.00</td>
<td>18.09</td>
<td>113.1</td>
<td>2,045</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top Plate 10 x 5 6</td>
<td>6.25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ d_x = \frac{-69.7}{55.95} = -1.25 \text{ in.} \]

\[ I_{x,1} = \frac{13,096}{13,906} = 1.53 \text{ in.} \]

\[ d_{\text{top, of steel}} = 17.78 + 0.62 + 1.25 = 19.65 \text{ in.} \]

\[ d_{\text{bot, of steel}} = 17.78 + 1.00 - 1.25 = 17.53 \text{ in.} \]

\[ Z_{\text{top, of steel}} = \frac{13,096}{19.65} = 666 \text{ in.}^3 \]

\[ Z_{\text{bot, of steel}} = \frac{13,096}{17.53} = 747 \text{ in.}^3 \]

Composite Section at Interior Support

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>55.95</td>
<td>23.34</td>
<td>−69.7</td>
<td>2,364</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reinf. Steel 14 No. 5</td>
<td>4.34</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ d_x = \frac{31.6}{60.29} = 0.52 \text{ in.} \]

\[ I_{x,1} = \frac{15,531}{22.82} = 681 \text{ in.}^3 \]

\[ d_{\text{reinf.}} = 23.34 - 0.52 = 22.82 \text{ in.} \]

\[ Z_{\text{reinf.}} = \frac{15,531}{22.82} = 681 \text{ in.}^3 \]

\[ d_{\text{top, of steel}} = 17.78 + 0.62 - 0.52 = 17.88 \text{ in.} \]

\[ d_{\text{bot, of steel}} = 17.78 + 1.00 + 0.52 = 19.30 \text{ in.} \]

\[ Z_{\text{top, of steel}} = \frac{15,531}{17.88} = 869 \text{ in.}^3 \]

\[ Z_{\text{bot, of steel}} = \frac{15,531}{19.30} = 805 \text{ in.}^3 \]

Bending Moments (Constant I)

<table>
<thead>
<tr>
<th></th>
<th>DL₁</th>
<th>DL₂</th>
<th>LL + I</th>
</tr>
</thead>
<tbody>
<tr>
<td>M, kip-ft</td>
<td>−551</td>
<td>−101</td>
<td>−601</td>
</tr>
</tbody>
</table>

Steel Stresses for Maximum Negative Moment

Top of Stringer (Tension)

\[ DL₁: f_b = \frac{551 \times 12}{666} = 9.93 \]

\[ DL₂: f_b = \frac{101 \times 12}{869} = 1.39 \]

\[ LL + I: f_b = \frac{601 \times 12}{869} = 8.30 \]

Bottom of Stringer (Compression)

\[ f_b = \frac{551 \times 12}{747} = 8.85 \]

\[ f_b = \frac{101 \times 12}{805} = 1.51 \]

\[ f_b = \frac{601 \times 12}{805} = 8.96 \]

The stress in the reinforcing bars is

\[ f_b = \frac{(101 + 601) \times 12}{681} = 12.37 \text{ ksi} \]

II/3.66
Allowable Compressive Stress

As for Design III, length $L$ for lateral buckling is 17.5 ft, the distance from interior support to dead-load inflection point.

$$F_1 = 20,000 - 7.5 \left( \frac{L}{6} \right)^2 = 20,000 - 7.5 \left( \frac{17.5 \times 12}{12} \right)^2 = 17.70 \text{ ksi}$$

Because of continuity, the allowable stress at the interior support may be increased 20%, up to 20 ksi.

$$F'_1 = 17.70 \times 1.20 = 21.2 \text{ ksi. Use 20 ksi.}$$

Since the bending stresses are less than 20 ksi, the assumed section is satisfactory.

Fatigue Stress in Top Flange

Since shear connectors are welded to the tension flange of the stringer, fatigue must be considered. The allowable fatigue stress for base metal adjacent to welded-stud shear connectors is calculated and found not to govern.

Minimum Cover-Plate Stress

$$DL_1: f_b = 9.93$$

$$DL_2: f_b = 1.39 \quad 11.32 \text{ ksi}$$

Ratio of minimum to maximum stress in the cover plate is

$$R = \frac{11.32}{19.62} = 0.577$$

The allowable fatigue stress in tension is

$$F_r = \frac{16.5}{1 - 0.65 (0.577)} = 26.40 \text{ ksi} > 20 \text{ ksi}$$

LOCATION OF COVER-PLATE CUTOFFS

For positive moment, the same section is required as for Design III—a W36 x 135 with a 10 x $\frac{3}{8}$-in. bottom cover plate. Near the inflection point, bottom-flange stresses and the variation in these stresses are nearly the same for Designs III and IV. As was shown in Design III, the 10 x $\frac{3}{8}$-in. bottom cover plate cannot be cut off, because of fatigue restrictions, but must be extended through the inflection point and butt welded to the 10 x 1-in. bottom cover plate required at the interior support. Calculations show that this change in cover-plate thickness can be made 5 ft from the interior support and that the top cover plate can be cut off there.

Steel Section 5 Ft from Interior Support

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>Iₒ</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>W36 x 135</td>
<td>39.70</td>
<td>-17.96</td>
<td>-67.35</td>
<td>1,210</td>
<td>7,796</td>
<td>7,796</td>
</tr>
<tr>
<td>Bottom Plate 10 x $\frac{3}{8}$</td>
<td>3.75</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$$d_r = \frac{-67.35}{43.45} = -1.55$$

$$d_{Top ~ of ~ steel} = 17.78 + 1.55 = 19.33 \text{ in.}$$

$$Z_{Top ~ of ~ steel} = \frac{8,902}{19.33} = 460 \text{ in.}^3$$

$$d_{Bot ~ of ~ steel} = 17.78 + 0.38 - 1.55 = 16.61 \text{ in.}$$

$$Z_{Bot ~ of ~ steel} = \frac{8,902}{16.61 - 536 \text{ in.}^3}$$
Composite Section 5 Ft from Interior Support

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>L.</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>43.45</td>
<td>4.34</td>
<td>67.35</td>
<td>101.30</td>
<td>2,364</td>
<td>9,006</td>
</tr>
<tr>
<td>Reinf. Steel 14 No. 5</td>
<td>4.34</td>
<td>23.34</td>
<td>111.30</td>
<td>2,364</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ d_s = \frac{33.95}{47.79} = 0.71 \text{ in.} \]
\[ 33.95 \text{ in.}^3 \]
\[ d_{\text{Top of steel}} = 17.78 - 0.71 = 17.07 \text{ in.} \]
\[ d_{\text{Bot. of steel}} = 17.78 + 0.38 + 0.71 = 18.87 \text{ in.} \]
\[ Z_{\text{Top of steel}} = \frac{11,346}{17.07} = 665 \text{ in.}^3 \]
\[ Z_{\text{Bot. of steel}} = \frac{11,346}{18.87} = 601 \text{ in.}^3 \]

Maximum Moments 5 Ft from Interior Support, Kip-Ft

With Positive Live-Load Moment
\[ DL_1: M = -360 \]
\[ DL_2: M = -67 \]
\[ LL + I: M = +65 \]

With Negative Live-Load Moment
\[ DL_1: M = -360 \]
\[ DL_2: M = -67 \]
\[ LL + I: M = -412 \]

Stresses at Bottom of Steel

Minimum (Compression)
\[ DL_1: f_b = \frac{360 \times 12}{536} = 8.06 \text{ ksi} \]
\[ DL_2 + LL + I: f_b = \frac{2 \times 12}{601} = 0.04 \text{ ksi} \]

Maximum (Compression)
\[ f_b = \frac{360 \times 12}{536} = 8.06 \text{ ksi} \]
\[ f_b = \frac{479 \times 12}{601} = 9.56 \text{ ksi} < 17.70 \text{ ksi} \]

Ratio of minimum to maximum stress in bottom cover plate is

\[ R = \frac{-8.10}{-17.62} = 0.460 \]

The allowable fatigue stress in compression is

\[ F_r = \frac{19.8}{1 - \left[ \frac{19.8}{10.6 - 1} \right] 0.461} = 32.96 \text{ ksi} > 17.70 \text{ ksi} \]

Lateral buckling governs.

CUTOFF OF TOP COVER PLATE

Since stresses due to \( DL_2 \) and \( LL + I \) are resisted by a composite section including the reinforcing bars, top-flange stresses and the variation in these stresses are sufficiently small that it is possible to cut off the top cover plate near the interior support and not exceed allowable fatigue stresses in base metal at the end of the cover plate.

The theoretical cutoff point for the 10 x \( \frac{3}{8} \)-in. top cover plate is 3.5 ft from the interior support. Fatigue is not critical. The plate may be ended 5 ft from the support. Thus, when composite action is developed in the negative-moment region, the top cover plate is decreased in thickness over the interior support and can be ended a few feet from the interior support.
Steel Section 3.5 Ft from Interior Support

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>W36 × 135</td>
<td>39.70</td>
<td>10.00</td>
<td>-18.28</td>
<td>-182.80</td>
<td>7,796</td>
<td>7,796</td>
</tr>
<tr>
<td>Bottom Plate 10 × 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ d_s = \frac{182.80}{49.70} = 3.68 \text{ in.} \]

\[ d_{\text{Top of steel}} = 17.78 + 3.68 = 21.46 \text{ in.} \]

\[ Z_{\text{Top of steel}} = \frac{10,465}{21.46} = 488 \text{ in.}^3 \]

Composite Section 3.5 Ft from Interior Support

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>49.70</td>
<td>4.34</td>
<td>23.34</td>
<td>-182.80</td>
<td>101.30</td>
<td>2,364</td>
</tr>
<tr>
<td>Reinf. Steel 14 No. 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ d_s = \frac{-81.50}{54.04} = -1.51 \text{ in.} \]

\[ d_{\text{Top of steel}} = 17.78 - 1.51 = 19.29 \text{ in.} \]

\[ Z_{\text{Top of steel}} = \frac{13,379}{19.29} = 694 \text{ in.}^3 \]

Bending Moments 3.5 Ft from Interior Support

<table>
<thead>
<tr>
<th></th>
<th>DL₁</th>
<th>DL₂</th>
<th>LL + I</th>
</tr>
</thead>
<tbody>
<tr>
<td>M, kip-ft</td>
<td>-415</td>
<td>-75</td>
<td>-465</td>
</tr>
</tbody>
</table>

Stresses at Top of Steel (Tension)

\[ DL₁: f_s = \frac{415 \times 12}{488} = 10.20 \]

\[ DL₂ + LL + I: f_s = \frac{540 \times 12}{694} = 9.34 \]

Fatigue Check 5 Ft from Interior Support

A theoretical cutoff of the top cover plate 3.5 ft from the interior support is satisfactory, but for fatigue considerations, the plate must be extended at least 1.5 × 10 = 15 in. For convenience, the plate is ended 5 ft from the support. Computations show that allowable fatigue stress does not govern there.

Maximum Moments 5 Ft from Interior Support, Kip-Ft

With Positive Live-Load Moment

\[ DL₁: M = -360 \]

\[ DL₂: M = 67 \]

\[ LL + I: M = +65 \]

With Negative Live-Load Moment

\[ DL₁: M = -360 \]

\[ DL₂: M = -67 \]

\[ LL + I: M = -412 \]
Stresses at Top of Steel

Minimum (Tension)

\[ DL_t: f_t = \frac{360 \times 12}{460} = 9.39 \]

Maximum (Tension)

\[ f_t = \frac{360 \times 12}{460} = 9.39 \]

\[ DL_t + LL + I: f_t = \frac{2 \times 12}{665} = 0.04 \text{ ksi} \]

\[ f_t = \frac{479 \times 12}{665} = 8.64 \text{ ksi} \]

Ratio of minimum to maximum stress in the top flange of the W36 × 135 is

\[ R = \frac{9.43}{18.03} = 0.523 \]

The allowable fatigue stress is

\[ F_r = \frac{12.0}{1 - 0.523} = 25.16 \text{ ksi} > 18.03 \]

Weld at End of Top Cover Plate

Size of the weld required to develop the 10 × \(\frac{3}{8}\)-in. top cover plate at the theoretical cutoff point, 3.5 ft from the interior support, is determined. The section is the same as that at the interior support. A \(\frac{3}{4}\)-in. fillet weld 46 in. long would be adequate for strength. The \(\frac{13}{16}\)-in. thickness of the W36 × 135 flange, however, requires a \(\frac{5}{16}\)-in. weld.

Steel Stresses 3.5 Ft from Interior Support

Top of Steel (Tension)

\[ DL_t: f_t = \frac{415 \times 12}{666} = 7.48 \]

\[ DL_t + LL + I: f_t = \frac{540 \times 12}{869} = 7.46 \text{ ksi} \]

Force in cover plate = 10 × \(\frac{3}{8}\) × 14.94 = 9.34 kips

Since the cutoff is close to the support, where the stress range is small, fatigue does not control. The allowable load on the fillet weld is 12.4 × 0.707 = 8.76 kips per in.

Weld size required = \(\frac{9.34}{8.76 \times 46} = 0.23 \text{ in.}, \) say \(\frac{3}{8}\) in.

Use \(\frac{5}{16}\)-in. fillet weld, required for flange thickness.

SHEAR-CONNECTOR SPACING

Shear connectors are required the full length of the stringer. Sample calculations for shear connector spacing with three studs per space, at the interior support are shown.

The range of shear for live load at the interior support is \(V_c = 62.1\) kips. The statical moment of the reinforcing steel (area = 4.34 in.\(^2\)) about the neutral axis is

\[ Q = 4.34 \times 22.82 = 99.0 \text{ in.}^3 \]

\[ S_c = \frac{V_c \cdot Q}{I} = \frac{62.1 \times 99.0}{15,531} = 0.40 \text{ kips per in.} \]

Spacing required (3 studs) = \(\frac{3 \times 8.11}{0.40} = 60.8 \text{ in.} \)

Use the maximum allowable spacing = 24 in.

The tensile force in the reinforcing bars is

\[ H_3 = A_r F_y = 4.34 \times 40.0 = 173.6 \text{ kips} \]
The number of studs required between point of maximum negative moment and
dead-load inflection point is

\[ N = \frac{H_i}{\phi Q_i} = \frac{173.6}{0.85 \times 45.0} = 4.5 \]

Use the maximum allowable spacing of 24 in. throughout the negative-moment
region except over the support. There, use 48 in. spacing. This spacing provides more
than enough studs to satisfy strength requirements.

**FINAL DESIGN**

An elevation of the two-span beam, composite for positive and negative moment,
is shown below.

---

**BEAM ELEVATION—DESIGN IV**

**Design V—Four-Span Continuous Beam (70-90-90-70 Ft)**

**Composite For Positive Moment Only**

This design is included to show the advantages of multi-span continuous construction.
The design procedure is similar to that for the two-span continuous beam of Design III.
When the end spans are held at 70 ft, as in the two-span design, and the interior
spans are increased to 90 ft, the same section—a W36 × 135 with a 10 x 3⁄8-in. bottom
cover plate—suffices for maximum positive moment in both end and interior spans.
Negative moments over the first interior support and the center support are slightly
higher than the negative moment for Design III. Maximum economy is obtained by
extending the W36 × 135, with the maximum permissible cover plate (10 x 1 3⁄8-in.),
over the first interior support and then introducing a short length of W36 × 150 with
10 x 1 1⁄4-in. cover plates over the center support.

Cover-plate cutoff locations are controlled by fatigue limitations. The bottom
cover plates cannot be ended near inflection points, because allowable fatigue stresses
are lower than bending stresses. Hence, the bottom cover plate must extend along the
stringer, except for a short length near each end support. Although the bottom cover-
plate transition, from 10 x 1 3⁄8-in. to 10 x 3⁄8-in., in the second span near the first
interior support can be made at 14'-0" from the support, the thicker plate is continued
to the splice point. This is done to avoid the additional butt-welded splice which
would be required in the bottom cover plate. The bottom cover-plate transition over
the center support is made at 19 ft from the center support.

Investigation shows that the top cover plates may be cut off 14 ft from each interior
support, if enough shear connectors to develop the force in the slab are placed on the
cover plates adjacent to the cutoffs, to insure composite action of steel section and
concrete slab. Six studs in two rows are sufficient to develop this force.

Field splices, which are generally located at dead-load inflection points, should be
kept to a minimum, consistent with the longest practical shipping lengths. For this
design example field splices are made 18 ft right of the first interior support and 19
ft right of the center support.

For the spans considered, the weight of stringers and framing details per square
foot of roadway are nearly the same for two-span and four-span construction. Thus,
three- or four-span continuous construction permits the use of longer interior spans at
about the same steel weight per square foot of roadway. Four-span construction also
is more economical because fewer bearing assemblies and expansion joints are required.
These are expensive to fabricate. Furthermore, fewer expansion joints result in a
better riding roadway.

Maximum-moment curves and an elevation of the four-span stringer, with either
welded or bolted field splices, are shown on the following pages.

REFERENCES

1. I. M., Viest, R. S. Fountain and R. C. Singleton, “Composite Construction in
2. “Alpha Composite Construction Engineering Handbook,” Porete Manufac-
turing Company, North Arlington, N. J.
3. “Moments, Shears and Reactions for Continuous Highway Bridges,” American
Institute of Steel Construction, New York.
4. George Anger, “Ten-Division Influence Lines for Continuous Beams,” Frederick
Ungar, New York.
Book Company, New York.
Cliffs, N. J.
New York.
Society of Civil Engineers, New York.
40'-0" c. to c. Bearings

**Beams**

8'-4"

28'-0" Roadway

6'-4"

8'-4"

33'-4" c. to c.

2'-8"

14'-0"

2'-8"

28'-0" Clear Roadway

Sym. about Roadway

7" Slab

Use 3, 3/8" Ø ASTM A325
High Strength Bolts (Typical all diaphragms)

Diaphragm Plate 8" x 1/8" (Typ.)

Half Section at End Bearing

Half Section at Intermediate Diaphragm

Typical Cross Section

Detail at Shear Connectors

Cover Plate 8" x 1/8"

4" x 4" Granular Flux-filled Studs

1/8" Ø x 4" Non-Interference

Cover-Plate Welding

20'-0"

10 @ 12"

= 10'-0"

16 @ 12"

= 15'-0"

Shear Connector Spacing

Spacing

Diaphragm Spaceing

Elevation of Stringer

Cover Plate 8" x 1/8"

7'-6"

40'-0"

25'-0"

7'-6"

Note: Turn mill camber up.
No other camber required.

Note:

Total Wt. = 17,454 lb
Wt. per sq ft (0.1 to 0.5 slab) = 13.1 lb per sq ft

All material ASTM A36

*Wt. does not include bearing shoes, railing or studs.

Composite Design Example
Simple-Span Rolled Beams
Design II

II/3.75
Design III

**Composite Design Example**

2-Span Continuous Rolled Beams

**Elevation of Girder**

- **Shear Connectors**
  - @ 12" to here
  - Plate 10" x 1/4"
  - Top and Bottom
  - 10" x 4/5" Fill

- **Alternate Bolted Field Splice**
  - All bolts shall be 5/8" H.S. Bolts ASTM A325

- **Welded Field Splice**

**Cross Section**

- **Cover Plate**
  - 10" x 1/4"
  - Top and Bottom

- **Cover Plate Splice**

**Note:**
- All materials ASTM A36
- Total Wt. = 90,940 lb
- Wt. per sq ft (0.0 to 0. slab) = 19.5 lb per sq ft
- Wt. does not include bearing shoes, railing or studs.
Framing Plan

<table>
<thead>
<tr>
<th>End Bearing</th>
<th>1st Interior Bearing</th>
<th>Center Bearing</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 @ 12&quot; = 12'-0&quot;</td>
<td>11'-0&quot;</td>
<td>14'-0&quot;</td>
</tr>
<tr>
<td>9 @ 15&quot; = 18'-0&quot;</td>
<td>12'-0&quot;</td>
<td>14'-0&quot;</td>
</tr>
<tr>
<td>23'-4&quot;</td>
<td>22'-6&quot;</td>
<td>22'-6&quot;</td>
</tr>
<tr>
<td>Cover Plate 10&quot; x 14&quot;</td>
<td>Cover Plate 10&quot; x 14&quot;</td>
<td>Cover Plate 10&quot; x 14&quot;</td>
</tr>
<tr>
<td>13'-0&quot;</td>
<td>14'-0&quot;</td>
<td>19'-0&quot;</td>
</tr>
<tr>
<td>Shop Splice</td>
<td>Field Splice</td>
<td>Field Splice</td>
</tr>
<tr>
<td>72'-0&quot; Span 1</td>
<td>92'-0&quot; Span 2</td>
<td>91'-0&quot;</td>
</tr>
</tbody>
</table>

Elevation of Stringer

- 33'-4" o. to o., 26'-0" Clear Roadway
- Use 3. 5/8" ASTM A325 High Strength Bolts (Typical at Diaphragms)
- 7" Slab

Field Splice

- Shop Splice
- End Bearing
- 1st Interior Support
- Center Support

Composite Design Example
4-Span Continuous Rolled Beams
Design V

Note:
1. All material ASTM A36
2. Details not shown are similar to details on Design III.

*Total Wt. = 216,422 lb
Wt. per sq ft (0 to 0, slab) = 20.3 lb per sq ft
*Wt. does not include bearing shoes, railing or studs.
Composite: Wide-Flange Beam Load Factor Design

Introduction
Chapter 3 illustrates the design of a composite, wide-flange beam bridge by the working stress method. This chapter illustrates load factor design for the same type of construction.

The example presented in this chapter deals with the design of a two-span, continuous beam (70 ft.—70 ft.), composite for positive and negative moments similar to Design IV of Chapter 3. The load factor design is in accordance with the 1973 Standard Specifications for Highway Bridges of the American Association of State Highway and Transportation Officials and their Interim Specifications dated 1974, 1975 and 1976. These specifications will be referred to for brevity as AASHTO followed by an article and section reference. USS COR-TEN B (ASTM A588, Grade A) is used for the steel portion of the composite beam. This is a high-strength low-alloy structural steel that is widely used in unpainted bridges where its enhanced atmospheric corrosion resistance is desired to help minimize maintenance costs.

The procedures for dead-load distribution, lateral distribution of live load, computation of reactions, shears, moments and deflections, determination of effective slab widths, section properties (except for plastic section modulus and related properties) and stresses in composite sections are the same for load factor and working stress designs. Descriptive text and illustrative calculations similar to those presented in Chapter 3 are not repeated but the similarity is pointed out.

General Design Considerations
Members designed by the Load Factor method are proportioned for multiples of the design loads. They are required to meet certain criteria for three theoretical load levels: 1) Maximum Design Load 2) Overload and 3) Service Load. The Maximum Design Load and Overload requirements are based on multiples of the service loads with certain other coefficients necessary to insure the required capabilities of the structure. Service loads are defined as the same loads as used in working stress design.

The Maximum Design Load criteria insures the structure's capability of withstanding a few passages of exceptionally heavy vehicles (simultaneously in more than one lane), in times of extreme emergency, that may induce significant permanent deformations.

The Overload criteria insures control of permanent deformations in a member, caused by occasional overweight vehicles equal to 5/3 the design live and impact loads (simultaneously in more than one lane), that would be objectionable to riding quality of the structure.
The Service Load criteria insures that the live load deflection and fatigue life (for assumed fatigue loading) of a member are controlled within acceptable limits.

Moments, shears and other forces are determined by assuming elastic behavior of the structure except for a continuous beam of compact section where negative moments over supports, determined by elastic analysis, may be reduced by a maximum of 10%. This reduction, however, must be accompanied by an increase in the maximum positive moment equal to the average decrease of the negative moments in the span.

**DESIGN LOADS**
The moments, shears or forces to be sustained by a stress-carrying steel member are computed from the following formulas for the three loading levels.

- **Service Load:** \( D + (L + I) \)
- **Overload:** \( D + \frac{5}{3}(L + I) \)
- **Maximum Design Load:** \( 1.30 \left[ D + \frac{5}{3}(L + I) \right] \)

where \( D = \) dead load
\( L = \) live load
\( I = \) impact load

The factor 1.30 is included to compensate for uncertainties in strength, theory, loading, analysis, and material properties and dimensions. The factor 5/3 is incorporated to allow for overloads. Factors for other group loading combinations are given in AASHTO Art. 1.7.123.

**COMPACT SECTIONS**
Compact sections are able to form plastic hinges which rotate at near constant moment.

A steel section is considered compact when its geometry is such that the fully plastic bending-moment capacity can be reached without local buckling or lateral torsional buckling. Prevention of lateral buckling requires the presence of adequate bracing of the compression flange at suitable intervals. The following criteria define compactness:

1. **Width-thickness ratio** of compression flange projection should not exceed

\[
\frac{b'}{t} = \frac{1,600}{\sqrt{F_y}}
\]

where \( b' = \) width of projecting compression flange element
\( t = \) flange thickness
\( F_y = \) specified minimum yield point or yield strength, psi, of the type of steel being used

2. **Depth-thickness ratio** of the web should not exceed

\[
\frac{d}{t_w} = \frac{13,300}{\sqrt{F_y}}
\]

where \( d = \) beam depth
\( t_w = \) web thickness
3. The compression flange should be supported laterally by adequate bracing at intervals not exceeding either of the following:

\[ L_b = \frac{7,000r_y}{\sqrt{F_v}} \text{ when } M_i \geq 0.7M_1 \]

\[ L_b = \frac{12,000r_y}{\sqrt{F_v}} \text{ when } M_i < 0.7M_1 \]

where \( r_y \) = radius of gyration with respect to Y-Y axis

\( M_1 \) = larger of the bending moments at two adjacent braced points

\( M_2 \) = smaller of the bending moments at those braced points

The displacement or twisting of beams called lateral buckling may also be prevented by embedment of the top and sides of the compression flange in concrete.

4. Axial compression should not exceed

\[ P = 0.15F_v A \]

where \( A \) = beam cross-sectional area

5. Shear should not exceed

\[ V = 0.55F_v d t_w \]

**DESIGN FOR MAXIMUM DESIGN LOADS**

For a compact section, the maximum strength or maximum moment capacity is given by

\[ M_u = F_v Z \]

where \( Z \) = plastic section modulus

\( M_u \) must be equal to or greater than moment induced in the beam by the maximum design load, that is,

\[ F_v Z \geq 1.30 \left[ D + \frac{5}{3}(L+I) \right] \]

Here, \( D \), \( L \) and \( I \) represent moments induced by the service loads.

If a compact section is provided to carry negative moments at supports of a continuous beam, the negative moments determined by elastic theory may be reduced 10%. This reduction, however, must be accompanied by an increase in the maximum positive moment equal to the average decrease of the negative moments in the span. This redistribution of moments is the only exception in load-factor design to elastic theory for analysis of structures.

Adequately braced, noncompact sections may be used, but with lower moment capacity. With adequate bracing, the maximum strength of a symmetrical, noncompact section may be computed from

\[ M_u = F_v S \]

where \( S \) = elastic section modulus

The section consequently must be proportioned so that

\[ F_v S \geq 1.30 \left[ D + \frac{5}{3}(L+I) \right] \]

For this relationship to be permitted, the following criteria must be satisfied:

1. Width-thickness ratio of the compression flange projection, when the bending
moment induced by the Maximum Design Load, \( M \), equals the maximum strength, \( M_u \), should not exceed

\[
\frac{b'}{t} = \frac{2.200}{\sqrt{F_v}}
\]

When \( M < M_u \), \( b'/t \) may be increased in the ratio \( \sqrt{M_u/M} \).

2. Depth-thickness ratio of the web should not exceed

\[
\frac{D}{t_w} = 150
\]

where \( D \) = clear unsupported distance between flange components

3. Spacing of lateral bracing of the compression flange should not exceed

\[
L_b = \frac{20,000,000 A_f}{F_v d}
\]

where \( A_f \) = cross-sectional area of compression flange

4. As for a compact section, axial compression should not exceed

\[
P = 0.15 F_v A
\]

5. Shear should not exceed either of the following values

\[
V = \frac{3.5 E t_w}{D}
\]

\[
V = 0.58 F_v D t_w
\]

where \( E \) = steel modulus of elasticity

For sections with geometric properties falling between the limits for compact sections and those for braced, noncompact sections, maximum strength may be calculated by straight-line interpolation between the moment capacities of the two types of sections. Web thickness, however, must satisfy Criterion 2 for compact sections.

When a member does not meet Criterion 3 for spacing of lateral bracing of braced, noncompact sections, it is called an unbraced section, and the AASHTO lateral buckling equation for maximum strength is

\[
M_u = F_v S \left[ 1 - \frac{3F_v}{4E \pi^2} \left( \frac{L_b}{b'} \right)^2 \right]
\]

When \( M_u < 0.7 M_i \), this value of \( M_u \) may be increased 20% but may not exceed \( F_i S \).

For sections unsymmetrical about the \( X-X \) axis but symmetrical about the \( Y-Y \) axis, maximum strength may be computed from the formula for \( M_u \), given, except that when this formula is used, \( b' \) should be replaced by \( 0.9b' \).

The AASHTO lateral buckling equation for maximum strength, \( M_u \), was developed for prismatic compression flanges. When a compression flange cover plate is terminated within an unbraced length, the compression flange section throughout this length is no longer prismatic and the AASHTO lateral buckling requirements are not directly applicable.

However, it can be shown that by a modification of application the AASHTO lateral buckling formula can be applied conservatively to cover-plated beams.* This can be done by rearranging the AASHTO formula and computing the critical buckling stress of the braced panel, in which the cover plate terminates, as that of the beam without cover plates. This stress may then be increased by 20% providing the ratio of compression flange axial forces at the ends of the braced panel are equal to or less than 0.7.

---

*United States Steel Research reviewed the basis for the AASHTO requirements and analyzed the buckling loads of stepped columns with various geometries. Based on these results a design procedure was developed which relates the strength of a stepped flange to that of a prismatic flange. For additional information on this procedure contact a USS Construction Representative through the nearest USS Sales Office.
The critical buckling stress, $F_{cr}$, is determined from the following rearrangement of the AASHTO buckling formula:

$$F_{cr} = \frac{M_u}{S} = F_y \left[ 1 - \frac{3F_y(L_b)}{4\pi^2E(b')^2} \right]$$

where $b'$ = projecting compression flange width of the beam
$S$ = section modulus of the steel section without the terminated cover plate

The maximum strength at any point in the panel is expressed as:

$$M_u = F_{cr}S_e$$

where $S_e$ = section modulus at the point considered

For composite beams, those in which a concrete slab assists a steel section in resisting bending moments, the method for computing maximum strength depends on whether or not the steel section satisfies Criteria 2 and 5 previously given for compact sections and the stress-strain diagram for the steel exhibits a yield plateau followed by a strain-hardening range. If these criteria are satisfied, the beam is considered a compact, composite section.

Maximum strength in the positive-moment regions of a compact section with concrete slab on the top flange is computed for a fully plastic stress distribution on the section. This moment capacity equals the sum of the moments about the neutral axis of all compressive and tensile forces acting on the section.

**SECTION**

**STRESS DISTRIBUTION**

**COMPACT-COMPOSITE-BEAM STRESSES AT MAXIMUM DESIGN LOAD**

The compressive force in the concrete slab is the smallest of the values of $C$ computed from the following formulas.

1. Capacity of slab and its longitudinal steel reinforcement in the compression zone:

   $$C = 0.85f'c_b + (AF_y)_c$$

   where $f'c$ = specified 28-day compressive strength of concrete, psi
   $b$ = effective width of slab
   $t_s$ = slab thickness
   $(AF_y)_c$ = product of area and yield point of that part of slab reinforcement parallel to the beam and lying in compression zone

2. Capacity of the steel section:

   $$C = (AF_y)_{sf} + (AF_y)_{sf} + (AF_y)_{sf} + \Sigma(AF_y) = \Sigma(AF_y)$$

   where $(AF_y)_{sf}$ = product of area and yield point of bottom flange of steel section, including cover plate, if any
3. Capacity of shear connectors:

\[ C = \Sigma Q_u \]

where \( \Sigma Q_u \) = sum of ultimate strengths of shear connectors located between section under consideration and nearest section of zero moment.

The depth of the assumed rectangular stress block (uniform stress distribution) for the slab is determined from the compressive force in the slab:

\[ a = \frac{C - (AF_y)_{1f}}{0.85f'_c} \]

When the compressive force in the slab is less than \( C \) computed for the capacity of the steel section, there will be a compressive force in the top portion of the steel section. This force is given by

\[ C' = \frac{\Sigma (AF_y) - C}{2} \]

The distance \( \bar{y} \) of the neutral axis below the top of the steel section can be computed from one of the following formulas:

\[ \bar{y} = \frac{C'}{(AF_y)_{1f}} t_{1f} \text{ when } C' < (AF_y)_{1f} \]

\[ \bar{y} = t_{1f} + \frac{C' - (AF_y)_{1f} d_w}{(AF_y)_{w}} \text{ when } C' \geq (AF_y)_{1f} \]

where \( t_{1f} \) = thickness of steel top flange

\( d_w \) = clear distance between flanges of steel section.

The total tensile force acting on the section must equal the total compressive force for a beam subject only to bending and shear.

All quantities needed for computing the maximum bending strength \( M_u \) of the compact, composite section are thus determined. The section then must be so proportioned that

\[ M_u \geq 1.30 \left[ D + \frac{5}{3} (L + I) \right] \]

where again \( D, L \) and \( I \) are the moments induced in the member by the Service Loads.

When the steel section of a composite beam does not satisfy compactness requirements, maximum strength should be taken as the moment at first yielding. In other words, in a composite, noncompact section designed for Maximum Design Loads, the elastic stresses caused by multiples of the initial dead load, superimposed dead load, and live plus impact loads cannot exceed the yield stress. These stresses can be calculated by the usual elastic theory for composite beams, taking into account whether the construction is shored or unshored when the slab is cast.

**DESIGN FOR OVERLOAD**

To guard against objectionable deformation under occasional overloads, the following moment relationship must be observed for noncomposite sections:

\[ 0.8F_y S \geq \left[ D + \frac{5}{3} (L + I) \right] \]

For the same reason, composite sections in positive bending must satisfy the relationship

\[ 0.95F_y S \geq \left[ D + \frac{5}{3} (L + I) \right] \]

**DESIGN FOR SERVICE LOADS**

Fatigue is investigated in the same manner as in working stress design, using Service Loads and the provisions of AASHTO Art. 1.7.3. If the longitudinal reinforcing steel
in tension over the negative moment region is considered in computing section properties, the stress range in the reinforcing steel is limited to 20,000 psi.

**SHEAR CONNECTORS**

Requirements for shear connectors in load factor design are identical to requirements for working-stress design, which are applied in Chapters 3 and 4.

**Design Example—Two-Span Continuous Beam (70-70 Ft) Composite for Positive and Negative Moment**

To illustrate the load factor method, an interior stringer of a two-span bridge, similar to Design IV of Chapter 3, will be designed. The stringer consists of a rolled steel beam, with cover plates as required, that acts compositely with the concrete bridge deck in both positive-moment and negative-moment regions. For this purpose, in the negative-moment region, the section consists of the steel beam, cover plates, and longitudinal reinforcement in the slab. The following data apply to this design:

**Roadway Section:** The same as that shown for Design I in Chapter 3.

**Specifications:** 1973 AASHTO Standard Specifications for Highway Bridges, Interims 1974, 1975 and 1976

**Loading:** HS20-44

**Structural Steel:** ASTM A588, with $F_y = 50,000$ psi

**Concrete:** $f'_c = 4,000$ psi, modular ratio $n = 8$

**Slab Reinforcing Steel:** ASTM A615, Grade 40 with $F_y = 40,000$ psi

**Loading Conditions:**

Case 1—Weight of stringer and slab ($DL_1$) supported by the steel stringer alone.

Case 2—Superimposed dead load ($DL_2$) (curbs and railings) supported by the composite section, with the increased modular ratio $3n = 3 \times 8 = 24$.

Case 3—Live load plus impact ($L + I$) supported by the composite section with the modular ratio $n = 8$.

**Fatigue Considerations:** 100,000 cycles of maximum stress for both truck and lane load as required for a secondary street or highway. (If the structure were considered located on a freeway, expressway, or major highway or street, it might be governed by a different number of stress cycles . . . see AASHTO Art. 1.7.3.)

**LOADS, SHEARS AND MOMENTS**

Moments and shears are determined by elastic theory with the initial assumption of uniform moment of inertia for both spans of the stringer.

The dead load $DL_1$ carried by the steel consists of the weight of the 7-in.-thick concrete slab and an assumed weight of 0.170 kips per ft for the stringer and framing details. The dead load $DL_2$ carried by the composite section comprises the proportional weight of the curbs and railings. Live load is HS20-44 truck loading, with impact for a 70-ft span.
Dead Load Carried by Steel
Slab = 7/12 x 8.33 x 0.150 = 0.730
Steel beam, details, haunches, diaphragms = 0.170
\[ DL_1 \text{ per stringer} = \frac{0.900}{4} = 0.225 \text{ k/ft} \]

Dead Load Carried by Composite Section *
Curbs and railings, \[ DL_2 = \frac{0.660}{4} = 0.165 \text{ k/ft} \]

Live Load
Live-load distribution = \[ \frac{8.33}{5.5} = 1.51 \text{ wheels} = 0.755 \text{ axle} \]
Impact = \[ \frac{50}{70+125} = 0.256 \]

The curves shown for maximum moment and maximum shear may be calculated by any convenient method.

MAXIMUM-MOMENT CURVES—CONSTANT I

*No future wearing surface is anticipated for this bridge. If a future wearing surface will be required, its weight must be included in the dead load carried by the composite section and distributed equally to all stringers.
MAXIMUM-SHEAR CURVES—CONSTANT I

DESIGN OF STRINGER SECTION
A W30×108 beam of COR-TEN B (A588, Grade A) steel is considered as the basic section. In positive moment regions, the top flange is in compression. Since the concrete slab provides local and lateral buckling restraint, only criterion 2 and 5 of the previously stated compact section criteria are applicable. In negative-moment regions, the bottom flange is in compression and without continuous buckling restraint. Therefore all compact section criteria must be investigated.

CHECK OF BEAM PROPERTIES
Actual width-thickness ratio of the projecting compression flange element (bottom flange in negative-moment regions) is

\[ \frac{b'}{t} = \frac{(10.484 - 0.548)/2}{0.760} = 6.54 \]

The allowable width-thickness ratio is

\[ \frac{b'}{t} = \frac{1.600}{\sqrt{50,000}} = 7.16 > 6.54 \]

Web depth-thickness ratio is

\[ \frac{d}{t_w} = \frac{29.82}{0.548} = 54.4 \]
The allowable depth-thickness ratio is
\[
\frac{d}{t_w} = \frac{13,300}{\sqrt{50,000}} = 59.5 \times 54.4
\]

Diaphragms must be placed at the support and at a minimum of two points in each span, in accordance with AASHTO Art. 1.7.21, which limits spacing of diaphragms to a maximum of 25 ft. Assume two intermediate diaphragms in each span. For equal spacing, the distance between diaphragms would be
\[
L_b = \frac{70}{3} = 23.3 \text{ ft}
\]

For positive moments, the concrete slab braces the compression flange. In the negative-moment region, however, the bottom flange is in compression, and the compression is assumed to extend a distance of 18 ft from the pier to the dead-load inflection point. Consequently, the unsupported length of the compression flange for negative moments is \(L_b = 18 \text{ ft}\).

For the W30 \times 108, \(r_s = 2.15\). The moment at 18 ft from the support will be less than 0.7 the moment at the support. Hence, for a compact section, the maximum unsupported length of the compression flange in the negative-moment region may not exceed
\[
L_b = \frac{12,000 \times 2.15}{\sqrt{50,000}} = 115 \text{ in. = 9.62 ft < 18 ft}
\]

The maximum shear allowed for a compact section is
\[
V = 0.55F_vd_{tw} = 0.55 \times 50 \times 29.82 \times 0.548 = 449 \text{ kips}
\]

The actual shear for Maximum Design Load is
\[
V = 1.30(39.3 + 7.2 + \frac{5}{3} \times 62.1) = 195 < 449 \text{ kips}
\]

All requirements for compactness are satisfied, except the requirement for spacing of lateral bracing of the compression flange in the negative-moment region. The section over the pier is checked next as a braced, noncompact section.

\[
L_b = \frac{20,000,000A_s}{F_vd} = \frac{20,000,000 \times 10.484 \times 0.760}{50,000 \times 29.82} = 106.9 \text{ in. = 8.91 ft < 18 ft}
\]

The 18-ft unbraced length of the compression flange from the support to the inflection point exceeds the allowable for a braced noncompact section. Hence, the section must be considered an unbraced section.

**STRENGTH OF UNBRACED SECTION IN THE NEGATIVE-MOMENT REGION**

For the unbraced W30 \times 108 section, the maximum strength \(M_v\) is reduced from \(F_vS\) by the reduction formula previously given. Because the beam with reinforcing steel is unsymmetrical about the X-X axis, 0.9\(b'\) is used in the formula instead of \(b'\),

\[
0.9b' = 0.9(10.484 - 0.548)/2 = 4.47 \text{ in.}
\]

\[
M_v = F_vS\left[1 - \frac{3F_v}{4\pi^2L_b(0.9b')}\right] = F_vS\left[1 - \frac{3 \times 50}{4\pi^2 \times 29,000 \left(\frac{18 \times 12}{4.47}\right)^2}\right]
\]

\[
= F_vS(1 - 0.306) = 0.694F_vS
\]

It is anticipated that the beam alone will suffice over part of the negative-moment region but that cover plates will be required in the immediate vicinity of the support.
TRIAL SECTION FOR MAXIMUM NEGATIVE MOMENT

The W30×108 beam with a $\frac{3}{4} \times 8$ in. top cover plate and a 1×12-in. bottom cover plate is tried as the section for maximum negative moment, where the slab contains 14 No. 6 longitudinal bars at 6-in. spacing. Since the cover plates will be cut off somewhere within the 18 ft unbraced length adjacent to the support, and since the beam section alone qualifies only as an unbraced, noncompact section in this region, the cover-plated beam at the support also will be taken as an unbraced, noncompact section. Consequently, just as for the W30×108 alone, maximum strength with respect to the compression flange is defined by

$$M_u = F_y S \left[ 1 - \frac{3F_y}{4\pi^2E} \left( \frac{L_0}{0.9b'} \right)^2 \right]$$

or, dividing through by $S$, the critical lateral buckling stress for the compression flange is expressed as

$$F_{cr} = \frac{M_u}{S} = F_y \left[ 1 - \frac{3F_y}{4\pi^2E} \left( \frac{L_0}{0.9b'} \right)^2 \right]$$

Because the cover plates are cut off, the section is nonprismatic and the $b'$ in the reduction formula is taken as that for the beam alone in accordance with the procedure outlined on page 3A.4. Thus, $0.9b' = 4.47$ in., from the preceding page, and $F_{cr} = 0.694F_y$. This stress may be increased 20 percent, however, because by inspection the compression flange force $M_y/d$ at 18 ft from the support is less than 0.7 of the compression flange force $M_1/d$ at the support.

$$F_{cr} = 1.20 \times 0.694F_y = 0.833F_y = \text{allowable stress for compression flange}$$

The allowable stress for the tension flange is $F_y$. 

---

**Diagram:**

- Beam section labeled with dimensions:
  - 12 in. x 7 in. = 84 in.
  - Use 14 No. 6 Bars @ 6 in.
- Cover Plate: $\frac{3}{4} \times 8$ in.
- Neutral Axis for W30×108
- Neutral Axis for Steel and Reinf.
- Neutral Axis for Steel Section
- Cover Plate: 1 in. x 12 in.
The preceding investigation indicates that the design relationship for the negative-moment section compression flange is

\[ 1.30 \left[ D + \frac{5}{3} \left( L + I \right) \right] \leq 0.833 F_{\Sigma} S \]

and that the design relationship for the tension flange is

\[ 1.30 \left[ D + \frac{5}{3} \left( L + I \right) \right] \leq F'_{\Sigma} S \]

It can be seen that the relationship for Overload

\[ D + \frac{5}{3} \left( L + I \right) \leq 0.95 F'_{\Sigma} S \]

does not govern.

Section properties are calculated for the cover-plated beam alone and for this beam plus the longitudinal reinforcing bars in the concrete slab. The allowable stresses in the cover-plated beam are

\[ F_{\Sigma} = 0.833 F'_{\Sigma} = 0.833 \times 50 = 41.6 \text{ ksi for compression flange} \]
\[ F_{\Sigma} = 50 \text{ ksi for tension flange} \]

### Steel Section at Interior Support

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>Iₙ</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>W30 x 108</td>
<td>31.8</td>
<td>12.0</td>
<td>15.41</td>
<td>2,850</td>
<td>4,470</td>
<td>4,470</td>
</tr>
<tr>
<td>Bottom Plate 1 x 12</td>
<td>12.00</td>
<td>3.00</td>
<td>15.10</td>
<td>684</td>
<td>684</td>
<td></td>
</tr>
</tbody>
</table>

\[ d_s = \frac{-139.6}{46.80} = -2.98 \text{ in.} \]
\[ d_{Top} = 14.91 + 0.38 + 2.98 = 18.27 \text{ in.} \]
\[ s_{Top} = \frac{7,588}{18.27} = 415 \text{ in.} \]

### Section with Reinforcing Steel at Interior Support

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>Iₙ</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>46.80</td>
<td>6.16</td>
<td>19.58</td>
<td>2,362</td>
<td>8,004</td>
<td>8,004</td>
</tr>
<tr>
<td>Reinf. Steel 14 No. 6</td>
<td>6.16</td>
<td>52.96</td>
<td>19.0</td>
<td>0.36</td>
<td>10,359</td>
<td>10,359</td>
</tr>
</tbody>
</table>

\[ d_s = \frac{-19.0}{52.96} = -0.36 \text{ in.} \]
\[ d_{Reinf.} = 15.65 + 1.0 + 3.30 = 19.95 \text{ in.} \]
\[ S_{Reinf.} = \frac{10,359}{19.95} = 519 \text{ in.} \]
\[ d_{Top \text{ of steel}} = 14.91 + 0.38 + 0.36 = 15.65 \text{ in.} \]
\[ d_{Bot \text{ of steel}} = 14.91 + 1.00 - 0.36 = 15.55 \text{ in.} \]
\[ S_{Top \text{ of steel}} = \frac{10,359}{15.65} = 662 \text{ in.} \]
\[ S_{Bot \text{ of steel}} = \frac{10,359}{15.55} = 666 \text{ in.} \]

### Bending Moments (Constant I)

<table>
<thead>
<tr>
<th>M, kip-ft</th>
<th>DL₁</th>
<th>DL₂</th>
<th>LL + I</th>
</tr>
</thead>
<tbody>
<tr>
<td>-551</td>
<td>-101</td>
<td>-601</td>
<td></td>
</tr>
</tbody>
</table>
Steel Stresses for Maximum Negative Moment
Due to Maximum Design Load

Top of Steel (Tension)  Bottom of Steel (Compression)

For $DL_1$: $F_t = \frac{551 \times 12}{415} \times 1.30 = 20.7 \quad F_s = \frac{551 \times 12}{587} \times 1.30 = 14.6$

For $DL_2$: $F_t = \frac{101 \times 12}{662} \times 1.30 = 2.4 \quad F_s = \frac{101 \times 12}{662} \times 1.30 = 2.4$

For $L+I$: $F_t = \frac{601 \times 12}{662} \times 1.30 \times \frac{5}{3} = 23.6 \quad F_s = \frac{601 \times 12}{662} \times 1.30 \times \frac{5}{3} = 23.5$

$46.7 < 50.0 \text{ ksi} \quad \overline{40.5 < 41.6 \text{ ksi}}$

Reinforcing Steel Stress (Tension)

$F_t = \frac{1.3 \times 12 (101 + 1.667 \times 601)}{519} = 33.1 < 40 \text{ ksi}$

The trial section is satisfactory for maximum negative moment. Fatigue considerations limit the stress range in the reinforcement to 20,000 psi. The actual stress range is well within this value:

$\text{Range} = \frac{601 \times 12}{519} = 13.9 < 20.0 \text{ ksi}$

MAXIMUM POSITIVE MOMENT
The stringer is next investigated for maximum positive moment, which occurs 28 ft from the end support. There, the section consists of the W30×108 beam acting compositely with the slab and is compact. Properties are listed for the steel section alone and computed for the composite section with $n=8$ and $n=24$.

Steel Section W30×108

$I_s = 4,470 \text{ in.}^4 \quad S_{Top} = S_{Bot} = 300 \text{ in.}^3 \quad A = 31.8 \text{ in.}^2$

![Diagram of POSITIVE-MOMENT SECTION]
Composite Section, $3n=24$, 28 Ft from End Support

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_s$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>W30×108</td>
<td>31.8</td>
<td>19.79</td>
<td>484.9</td>
<td>9,595</td>
<td>4,470</td>
<td>4,470</td>
</tr>
<tr>
<td>Conc. 84×7/24</td>
<td>24.5</td>
<td>19.79</td>
<td>484.9</td>
<td>9,595</td>
<td>100</td>
<td>9,695</td>
</tr>
</tbody>
</table>

\[ d_s = \frac{484.9}{56.3} = 8.61 \text{ in.} \]
\[ 56.3 \text{ in.}^2 \quad 484.9 \text{ in.}^3 \quad 14,165 \]
\[ I_{NA} = \frac{-8.61 \times 484.9}{9,990} = 4,175 \text{ in.}^4 \]
\[ d_{\text{Top of steel}} = 14.91 - 8.61 = 6.30 \text{ in.} \]
\[ d_{\text{Bot of steel}} = 14.91 + 8.61 = 23.52 \text{ in.} \]
\[ S_{\text{Top of steel}} = \frac{9,990}{6.30} = 1,586 \text{ in.}^3 \]
\[ S_{\text{Bot of steel}} = \frac{9,990}{23.52} = 425 \text{ in.}^3 \]

Composite Section, $n=8$, 28 Ft from End Support

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_s$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>W30×108</td>
<td>31.8</td>
<td>19.79</td>
<td>1,454.6</td>
<td>28,786</td>
<td>4,470</td>
<td>4,470</td>
</tr>
<tr>
<td>Conc. 84×3/8</td>
<td>73.5</td>
<td>19.79</td>
<td>1,454.6</td>
<td>28,786</td>
<td>300</td>
<td>29,086</td>
</tr>
</tbody>
</table>

\[ d_s = \frac{1,454.6}{105.3} = 13.81 \text{ in.} \]
\[ 105.3 \text{ in.}^2 \quad 1,454.6 \text{ in.}^3 \quad 33,556 \]
\[ I_{NA} = \frac{-13.81 \times 1,454.6}{13,468} = 20,088 \text{ in.}^4 \]
\[ d_{\text{Top of steel}} = 14.91 - 13.81 = 1.10 \text{ in.} \]
\[ d_{\text{Bot of steel}} = 14.91 + 13.81 = 28.72 \text{ in.} \]
\[ S_{\text{Top of steel}} = \frac{13,468}{1.10} = 12,244 \text{ in.}^3 \]
\[ S_{\text{Bot of steel}} = \frac{13,468}{28.72} = 469 \text{ in.}^3 \]

Check of Steel Stresses for Overload

Stresses are then calculated for Overload according to the relationship

\[ D + \frac{5}{3} (L + I) \leq 0.95 F_s S \]

Thus, the allowable stress for Overload is $F_s = 0.95 \times 50 = 47.5$ ksi. The results show that the relationship is satisfied by the composite section.

Bending Moments 28 Ft from End Support

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$L + I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$, kip-ft</td>
<td>309</td>
<td>57</td>
<td>750</td>
</tr>
</tbody>
</table>

Steel Stresses for Overload—Combination A

Top of Steel (Compression) Bottom of Steel (Tension)

For $DL_1$: $F_c = \frac{309 \times 12}{300} = 12.4$

For $DL_2$: $F_c = \frac{57 \times 12}{1,586} = 0.4$

For $L + I$: $F_c = \frac{750 \times 12}{12,244} \times 5 = 1.2$

\[ 14.0 \text{ ksi} \]

For $F_b = \frac{750 \times 12}{469} \times 5 = 32.0$

\[ 46.0 < 47.5 \text{ ksi} \]
Check of Maximum Strength Under Maximum Design Loads

The strength of the composite section is checked for maximum positive moment with the relationship

\[ 1.30 \left[ D + \frac{5}{3} (L + I) \right] \leq F_v Z \]

It has previously been established that the W30 \times 108 beam and slab is a compact, composite section, since the compression flange is adequately braced by the slab.

Consequently, the maximum strength of the composite section is determined by the fully plastic moment capacity of the section. This is found by determining the compressive force in the slab, locating the neutral axis and calculating the total moment about the neutral axis of all forces acting on the section. The compressive force in the slab, if adequate shear connectors are provided, is the smaller value of either the fully plastic force developable by the steel section or the compressive strength of the slab. In this case, the plastic capacity of the steel section governs, as indicated below.

The capacity of the concrete slab, with concrete strength \( f'_c = 4 \text{ ksi} \) and reinforcing steel with \( F_v = 40 \text{ ksi} \), is

\[ C_s = 0.85 f'_c b t + (AF_v) = 0.85 \times 4 \times 84 \times 7 + 6.16 \times 40 = 2,246 \text{ kips} \]

The plastic force developable by the W30 is

\[ C_s = 31.8 \times 50 = 1,590 \text{ kips (controls)} \]

The neutral axis lies within the slab. The distance of the axis below the top of the slab, in this case, equals the depth \( a \) of the compressive stress block for the slab.

\[ a = \frac{C_s - (AF_v)}{0.85 f'_c b} = \frac{1,590 - 6.16 \times 40}{0.85 	imes 4 \times 84} = 4.70 \text{ in.} \]

For this location of the neutral axis, the longitudinal reinforcement of the slab lies in the compression zone and therefore inclusion of the force in the bars in the calculation of \( a \) was warranted.

With the neutral axis located, the fully plastic moment \( M_u \) for the section can now be calculated.

\[ M_u = \frac{0.85 \times 4 \times 84 (4.70)^2}{2} + 6.16 \times 40 (4.70 - 3.70) + 1,590 [14.91 + 1.0 + 0.38 + (7.00 - 4.70)] = 32,960 \text{ kip-in.} \]

The maximum positive moment induced by the Maximum Design Load is

\[ M = 1.30 \left[ DL_1 + DL_2 + \frac{5}{3} (L + I) \right] = 1.30 \left[ 309 + 57 + \frac{5}{3} \times 750 \right] \times 12 = 25,214 < 32,960 \text{ kip-in.} \]

Therefore, the composite section with the W30 \times 108 is satisfactory for maximum strength in positive moment.

**TOP-COVER-PLATE CUTOFF LOCATION**

The next task in the design of the stringer is determining where the cover plates in the negative-moment region can be cut off. Since the top flange is in tension, the termination of the top cover plate is governed by the maximum strength relationship

\[ 1.30 \left[ D + \frac{5}{3} (L + I) \right] \leq F_v S \]

Section properties are first calculated without the top cover plate but including the longitudinal reinforcing of the concrete slab. The cutoff location then is determined by trial.
Steel Section with Bottom Plate Only

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_s$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>W30 × 108</td>
<td>31.8</td>
<td>15.41</td>
<td>-184.9</td>
<td>2,850</td>
<td>4,470</td>
<td>4,470</td>
</tr>
<tr>
<td>Bottom Plate 1 × 12</td>
<td>12.00</td>
<td>15.41</td>
<td>-184.9</td>
<td>2,850</td>
<td>4,470</td>
<td>4,470</td>
</tr>
</tbody>
</table>

$d_s = \frac{-184.9}{43.80} = -4.22$ in.  
$-4.22 \times 184.9 = -780$  
$I_{NA} = 6,540$ in.\(^4\)

$d_{Top} = 14.91 + 4.22 = 19.13$ in.  
d_{Bot.} = 14.91 + 1.00 - 4.22 = 11.69$ in.

$S_{Top} = \frac{6,540}{19.13} = 342$ in.\(^3\)  
$S_{Bot.} = \frac{6,540}{11.69} = 559$ in.\(^3\)

Steel Section with Bottom Plate and Reinforcing Steel

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_s$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>43.80</td>
<td>19.58</td>
<td>-184.9</td>
<td>2,362</td>
<td>7,320</td>
<td></td>
</tr>
<tr>
<td>Reinf. Steel 14 No. 6</td>
<td>6.16</td>
<td>19.58</td>
<td>120.6</td>
<td>2,362</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$49.96$ in.\(^2\)  
-64.3 in.\(^2\)  
$I_{NA} = 9,599$ in.\(^4\)

$d_{Top} = 14.91 + 1.29 = 16.20$ in.  
d_{Bot.} = 14.91 + 1.00 - 1.29 = 14.62$ in.

$S_{Top} = \frac{9,599}{16.20} = 593$ in.\(^3\)  
$S_{Bot.} = \frac{9,599}{14.62} = 657$ in.\(^3\)

$d_{Reinf.} = 16.20 + 0.38 + 1.0 + 3.30 = 20.88$ in.

$S_{Reinf.} = \frac{9,599}{20.88} = 460$ in.\(^3\)

Try a theoretical cutoff of the top plate 3.0 ft from the pier. The actual cutoff is 1.5 times the plate width away from the theoretical cutoff or 4.0 ft from the pier. The total length of plate satisfies the minimum cover plate length criteria of $(2D + 3.0)$ ft where $D$ = depth of the beam in ft.

$L_{min} = 2 \times 2.5 + 3.0 = 8.0$ ft

The allowable stress, from the strength relationship is

$F_b = 50$ ksi

Bending Moments 3.0 Ft from Pier

<table>
<thead>
<tr>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$-(L + I)$</th>
<th>$+(L + I)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$, kip-ft</td>
<td>-438</td>
<td>-78</td>
<td>-470</td>
</tr>
</tbody>
</table>

Steel Stresses 3.0 Ft from Pier Due to Maximum Design Loads

**Top of Steel (Tension)**

For $DL_1$: $F_b = \frac{438 \times 12}{342} \times 1.30 = 20.0$

For $DL_2$: $F_b = \frac{78 \times 12}{593} \times 1.30 = 2.1$

For $L + I$: $F_b = \frac{470 \times 12}{593} \times 1.30 \times \frac{5}{3} = 20.6$

$42.7 < 50.0$ ksi
The stress at the theoretical cutoff point is within the allowable.

Check the base metal stress adjacent to the fillet weld at the actual cutoff 4.0 ft from the pier.

<table>
<thead>
<tr>
<th>Bending Moments 4.0 Ft from Pier</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DL_1$</td>
</tr>
<tr>
<td>$M$, kip·ft</td>
</tr>
</tbody>
</table>

The allowable fatigue stress range for base metal at the end of cover plates falls into AASHTO fatigue category E. For 100,000 loading cycles, the allowable range of stress in the top flange of the W30 adjacent to the fillet weld at the end of the cover plate is

$$F_{sr} = 21 \text{ ksi}$$

The stress range induced by service loads at this point is

$$f_{sr} = \frac{(440 + 51) \times 12}{593} = 9.9 < 21 \text{ ksi}$$

Basic stress and fatigue are then checked in the longitudinal reinforcing steel at the actual cutoff 4.0 ft from the pier.

For $DL_2$: \(F_b = \frac{72 \times 12}{460} \times 1.30 = 2.4\)

For $L+I$: \(F_b = \frac{440 \times 12}{460} \times 1.30 \times \frac{5}{3} = 24.9\)

$$\frac{27.3}{3} < 40.0 \text{ ksi}$$

Stress range in the reinforcing steel is limited to 20 ksi. The actual stress range is determined to be

$$f_{sr} = \frac{(440 + 51) \times 12}{460} = 12.8 < 20 \text{ ksi}$$

Since all stresses are within the allowable and the fatigue stress range is within satisfactory limits, the top plate can be terminated 4 ft from the pier.

**BOTTOM-PLATE CUTOFF LOCATION**

The cutoff of the bottom cover plate is governed by the relationship for an unbraced section with the 20% increase in stress because the ratio of flange forces at the braced ends is less than 0.7:

$$1.30 \left[ D + \frac{5}{3} (L+I) \right] \leq 1.2 \times 0.694 F_p S = 0.833 F_p S$$

Section properties are listed for the W30 × 108 and computed for the section composed of the W30 and the slab reinforcement.

**Steel Section W30 × 108**

$$I_v = 4,470 \text{ in.}^4 \quad S_{Top} = S_{Bot} = 300 \text{ in.}^3 \quad A = 31.8 \text{ in.}^2$$
Section with W30 and Reinforcing Steel

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>W30×108</td>
<td>31.8</td>
<td>6.16</td>
<td>19.58</td>
<td>120.6</td>
<td>2,362</td>
<td>4,470</td>
</tr>
<tr>
<td>Reinf. Steel 14 No. 6</td>
<td>37.96</td>
<td>120.6 in.²</td>
<td>37.96</td>
<td>120.6 in.²</td>
<td>6,832</td>
<td></td>
</tr>
<tr>
<td>dₘ = 120.6 in.²</td>
<td>37.96</td>
<td>3.18 in.</td>
<td>-3.18×120.6 = -384</td>
<td>Iₙₐ = 6,448 in.⁴</td>
<td></td>
<td></td>
</tr>
<tr>
<td>dₕ of steel = 14.91 - 3.18 = 11.73 in.</td>
<td>dₕ of steel = 14.91 + 3.18 = 18.09 in.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sₕ of steel = 6.448 in.³</td>
<td>Sₕ of steel = 6.448 in.³</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>dₘall = 11.73 + 0.38 + 1.0 + 3.30 = 16.41 in.</td>
<td>Sₘall = 6.448 in.³</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A theoretical cutoff of the bottom plate is tried 9.5 ft from the pier. Steel stresses at the bottom of the W30 govern and are checked first. The allowable stress for Maximum Design Load is

\[ F_b = 0.833 F_y = 0.833 \times 50 = 41.6 \text{ ksi} \]

**Bending Moments 9.5 Ft from Pier**

<table>
<thead>
<tr>
<th>M, kip-ft</th>
<th>DL₁</th>
<th>DL₂</th>
<th>-(L+I)</th>
<th>+(L+I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-219</td>
<td>-41</td>
<td>-361</td>
<td>171</td>
<td></td>
</tr>
</tbody>
</table>

**Steel Stresses 9.5 Ft from Pier Due to Maximum Design Loads**

**Bottom of Steel (Compression)**

For \( DL₁ \): \[ F_b = \frac{-219 \times 12}{300} \times 1.30 = 11.4 \]

For \( DL₂ \): \[ F_b = \frac{-41 \times 12}{356} \times 1.30 = 1.8 \]

For \( L+I \): \[ F_b = \frac{-361 \times 12}{356} \times 1.30 \times \frac{5}{3} = 26.4 \]

39.6 < 41.6 ksi

The stress at the theoretical cutoff point of the bottom plate is within the allowable.

The actual cutoff is 1.5 times the plate width beyond the theoretical cutoff, or 11 ft from the pier. Fatigue stress range is checked there for base metal adjacent to a fillet weld.

**Bending Moments 11 Ft from Pier**

<table>
<thead>
<tr>
<th>M, kip-ft</th>
<th>DL₁</th>
<th>DL₂</th>
<th>-(L+I)</th>
<th>+(L+I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-174</td>
<td>-34</td>
<td>-354</td>
<td>215</td>
<td></td>
</tr>
</tbody>
</table>

**Stresses at Bottom of Steel 11 Ft from Pier**

With Negative Live-Load Moment With Positive Live-Load Moment

For \( DL₁ \): \[ f_b = \frac{-174 \times 12}{300} = -7.0 \] \[ f_b = 215 \times 12 = +7.2 \]

For \( DL₂ \): \[ f_b = \frac{-34 \times 12}{356} = -1.1 \]

For \( L+I \): \[ f_b = \frac{-354 \times 12}{356} = -11.9 \]

\[ -20.0 \text{ ksi} \]

\[ -0.9 \text{ ksi} \]
Since there is no tension in the W30 section at the cutoff, there is no limitation on the stress range.

Reinforcing steel stresses are checked at the actual cutoff location 11.0 ft from the pier.

Reinforcing Steel Stresses 11.0 Ft from Pier (Tension)

For \( DL_b \): \( F_b = \frac{34 \times 12}{393} \times 1.30 = 1.3 \)

For \( L+I \): \( F_b = \frac{354 \times 12}{393} \times 1.30 \times \frac{5}{3} = 23.4 \)

\( \frac{24.7}{<40 \text{ ksi}} \)

Reinforcing Steel Fatigue Stress Range 11.0 Ft from Pier

Fatigue stress range in the reinforcing steel is limited to 20 ksi. The actual stress range is determined to be

\( f_u = \frac{(354+215)12}{393} = 17.4 <20 \text{ ksi} \)

All stresses are within the allowable, and the stress range is satisfactory for fatigue. Therefore, the bottom plate can be cut off 11 ft from the pier.

COVER-PLATE WELDS

The welds connecting the cover plates to the beam must be able to develop the computed forces in the cover plates at the theoretical cutoffs within the terminal length. The welds also must be able to resist the horizontal shear between the beam and the plate along the length of the plate. Usually, the former condition is more critical.

As with other components of the structure, both maximum strength under Maximum Design Loads and fatigue under service loads must be checked.

E70 XXX electrodes, with \( F_u = 70 \text{ ksi} \), will be used to make the welds. ASTM A588 steel also has a tensile strength \( F_u = 70 \text{ ksi} \).

Weld strength = \( 0.45 \times F_u = 0.45 \times 70 = 31.5 \text{ ksi} \)

The permissible load on a fillet weld then is

\( P_c = 0.707 \times 31.5 = 22.3 \text{ kips per in. for a 1 in. fillet weld} \)

WELD AT END OF BOTTOM PLATE

In the following calculations, the bottom-cover-plate weld is investigated for maximum strength and fatigue under service loads at the theoretical cutoff.

Maximum Strength

Stresses in Bottom Plate Due to Maximum Design Loads (Compression)

For \( DL_1 \): \( F_b = \frac{219 \times 12}{559} \times 1.30 = 6.1 \)

For \( DL_5 \): \( F_b = \frac{41 \times 12}{657} \times 1.30 = 1.0 \)

For \( L+I \): \( F_b = \frac{361 \times 12}{657} \times 1.30 \times \frac{5}{3} = 14.3 \)

\( \frac{21.4}{\text{ksi}} \)

Force in cover plate = \( 21.4 \times 1 \times 12 = 257 \text{ kips} \)
The terminal development length, or distance along the plate edges available for making the terminal weld, is

\[
L = 2 \times 1.5 \times 12 + 10.5 = 46.5 \text{ in.}
\]

Weld size required = \( \frac{257}{46.5 \times 22.3} = 0.248 \text{ in.} \)

The fillet weld connecting the 1 in. cover plate to the 0.76 in. thick W30 flange is limited by AASHTO Art. 1.7.26 to a minimum size of \( \frac{5}{8} \) in. Since the \( \frac{5}{8} \) in. minimum exceeds the 0.248 in. weld size required for maximum strength, a \( \frac{5}{6} \) in. weld will be used.

**Service Loads—Fatigue**

The range of stress in the bottom flange cover plate at the theoretical cutoff is

\[
f_{\nu} = \frac{(361 + 171) \times 12}{667} = 9.72 \text{ ksi}
\]

The range of force in the flange cover plate then becomes

\[
P_{\text{range}} = 9.72(1 \times 12) = 117 \text{ kips}
\]

Shear stress on the throat of fillet welds falls into AASHTO fatigue stress category F, and for 100,000 cycles, the allowable stress range cannot exceed 15 ksi. For the development length of 46.5 in. and a weld size of \( \frac{5}{6} \) in., the actual weld shear stress range is

\[
f_{\nu} = \frac{117}{46.5(0.707)(\frac{5}{6})} = 11.4 < 15 \text{ ksi}
\]

**WELD AT END OF TOP PLATE**

The top-cover-plate weld is investigated for maximum strength under maximum design loads and fatigue under Service Loads in the same manner as the bottom-cover-plate weld.

**Maximum Strength**

**Stress in Top Plate Due to Maximum Design Loads (Tension)**

For \( DL_1 \): \[
F_x = \frac{438 \times 12}{415} \times 1.3 = 16.5
\]

For \( DL_2 \): \[
F_x = \frac{78 \times 12}{662} \times 1.3 = 1.8
\]

For \( L+I \): \[
F_x = \frac{470 \times 12}{662} \times 1.3 \times \frac{5}{3} = 18.5
\]

Force in cover plate = \( 36.8 \times 8 \times 0.38 = 112 \text{ kips.} \)

The distance along the plate edges available for making the terminal weld is

\[
L = 2 \times 1.5 \times 8 + 8 = 32 \text{ in.}
\]

Weld size required = \( \frac{112}{32 \times 22.3} = 0.16 \text{ in.} \)

The 0.76 in. thick W30 flange requires a \( \frac{5}{6} \) in. minimum fillet weld. Therefore, investigate fatigue on the \( \frac{5}{6} \) in. weld.

The range of stress in the top flange cover plate at the theoretical cutoff is

\[
f_{\nu} = \frac{(470 + 35)}{662} \times 12 = 9.15 \text{ ksi}
\]
The range of force in the flange cover plate then becomes
\[ P_{\text{range}} = 9.15(0.38 \times 8) = 27.5 \text{ kips} \]

The actual weld shear stress range is
\[ f_{\nu} = \frac{27.5}{32(0.707)(5/6)} = 3.89 \text{ ksi} < 15 \text{ ksi} \]

**FATIGUE AT STUD WELDS**

Fatigue must be investigated for base metal adjacent to stud shear connectors on the tension flange. The following calculations show that the fatigue stress range is not critical at either the maximum negative-moment section over the support or at cover-plate cutoffs.

**Stud-weld Fatigue Stress Range at Pier Location**

Tensile stress in the top cover plate over the support falls into AASHTO fatigue stress category C. For 100,000 cycles, the allowable stress range is 32 ksi. The live-load stress range in the top cover plate at the pier is determined to be
\[ f_{\nu} = \frac{(601 - 0) \times 12}{662} = 10.9 < 32 \text{ ksi} \]

Studs therefore may be welded to the top cover plate over the support without any reduction in strength of the section.

**Stud-Weld Fatigue Stress Range at Top-Plate Cutoff**

The maximum stress range in the top flange of the W30 for the section 4 ft from the pier, where studs may be welded to the flange near the end of the top cover plate, was previously computed to be 9.9 ksi since this stress range is less than the 32 ksi allowable, studies may be welded to the top flange of the W30 near the top-plate cutoff without a reduction in the strength of the section.

**Stud-Weld Fatigue Stress Range at Bottom-Plate Cutoff**

The maximum stress range in the top flange of the W30 for the section 11.0 ft from the pier where the bottom plate is terminated is determined to be
\[ f_{\nu} = \frac{(354 + 215) \times 12}{550} = 12.4 < 32 \text{ ksi} \]

Studs, therefore, may be welded to the top flange of the W30 at the bottom-plate cutoff without a reduction in strength of the section.

**SHEAR-CONNECTOR SPACING**

Studs \( \frac{7}{8} \) in. diameter and 4 in. long are selected for shear connectors. Three studs are placed per row across the top flange or cover plate, for embedment in the concrete slab. Spacing of the rows is determined with the same criteria used for working-stress design and illustrated in Chapter 3. To avoid repetition, detailed calculations are not given here.

The spacing, shown in a diagram, satisfies the requirements for fatigue under Service Loads in the positive-moment region, and maximum spacing of 24 in. in the negative-moment region. However, the number of connectors must also be checked to insure that ultimate strength of the composite section can be achieved. The ultimate strength of welded studs is determined by the formula
\[ S_u = 0.4d^2(f' e E_s)^{1/4} \quad \text{where } E_s = 57,000 \sqrt{f'} \]

Therefore
\[ S_u = 0.4d^2 \sqrt[4]{57,000 (f' e)^2} = 0.404(8.75)^2 \sqrt[4]{57,000(4000)^2} = 36,800 \text{ psi} \]
The number of connectors required between the point of maximum positive moment and the end support is determined by the formula

\[ N_1 = \frac{P}{0.85 \times S_u}\]

where \( P \) is the smaller of the two values:

\[ P_1 = A_rF_s = 31.8(50) = 1590^e \text{ (controls)} \]
\[ P_2 = 0.85f'_c \beta = 0.85(4)(84)(7) = 1999^e \]

Thus, the number of connectors required is

\[ N_1 = \frac{1590}{0.85(36.8)} = 50.8 \text{ or } \frac{50.8}{3} = 17 \text{ rows} \]

Service load design provides 22 rows of shear connectors between the end support and the maximum positive-moment location. The ultimate strength requirement for this region is therefore satisfied.

The number of connectors required between the point of maximum positive moment and the interior support is determined by the formula

\[ N_2 = \frac{(P + P_3)}{0.85S_u} \]

where \( P_3 = A_rF_s = 6.16 \times 40 = 246^e \)

Therefore, the number of connectors required is

\[ N_2 = \frac{(1590 + 246)}{0.85(36.8)} = 58.7 \text{ or } \frac{58.7}{3} = 20 \text{ rows} \]

The number of rows furnished by service load design between the maximum positive-moment location and the interior support is determined to be 29. This number satisfies the ultimate strength requirement.

**WELDED FIELD SPLICE**

As in Chapter 3, the stringer is spliced near the dead-load inflection point, 17.5 ft from the pier, in one span. A full-strength welded splice is to be used, with all welds ground smooth. Therefore, only fatigue stresses need be investigated.

Fatigue in the butt-welded splice falls into AASHTO fatigue stress category B because the parts joined have the same width and thickness and the welds will be finished smooth and flush. For 100,000 cycles, the allowable range of stress in the base metal is 45 ksi.
Because of the splice location, the dead-load moments $DL_1$ and $DL_2$ are both zero. The live-load maximum positive moment is 420 kip-ft and the maximum negative moment is $-310$ kip-ft. The service-load stress range is calculated to be

For positive moment $f_s = \frac{420(12)}{469} = +10.7$

For negative moment $f_s = \frac{310(12)}{356} = -10.4$

$< 21.1$ ksi range

$< 45$ ksi

BOLTED FIELD SPLICE

As in Chapter 3, a bolted field splice is designed as an alternative to the welded splice. The splice is to be a friction-type connection made with $\frac{3}{8}$-in.-dia A325 bolts.

### Shears 17.5 Ft from Pier, Kips

<table>
<thead>
<tr>
<th></th>
<th>For Service Loads</th>
<th>Factor</th>
<th>For Overload</th>
<th>Factor</th>
<th>Max Design Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DL_1$:</td>
<td>23.0</td>
<td>1</td>
<td>23.0</td>
<td>1.30</td>
<td>29.9</td>
</tr>
<tr>
<td>$DL_2$:</td>
<td>4.5</td>
<td>1</td>
<td>4.5</td>
<td>1.30</td>
<td>5.8</td>
</tr>
<tr>
<td>$L+I$:</td>
<td>48.0</td>
<td>5/3</td>
<td>80.0</td>
<td>1.30</td>
<td>104.1</td>
</tr>
<tr>
<td></td>
<td>75.5</td>
<td></td>
<td>107.5</td>
<td></td>
<td>139.8</td>
</tr>
</tbody>
</table>

### Moments 17.5 Ft from Pier, Kip-Ft

<table>
<thead>
<tr>
<th></th>
<th>For Service Loads</th>
<th>Factor</th>
<th>For Overload</th>
<th>Factor</th>
<th>Max Design Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DL_1 + DL_2$:</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1.30</td>
<td>0</td>
</tr>
<tr>
<td>$-(L+I)$:</td>
<td>$-310$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$+(L+I)$:</td>
<td>420</td>
<td>5/3</td>
<td>700</td>
<td>1.30</td>
<td>910</td>
</tr>
<tr>
<td>Maximum:</td>
<td>420</td>
<td></td>
<td>700</td>
<td></td>
<td>910</td>
</tr>
</tbody>
</table>

For load factor design of a bolted field splice, AASHTO Specifications require that splice material be proportioned for Maximum Design Loads and resistance to fatigue under service loads. Friction connections must resist slip deformation under Overload, therefore the fasteners must be designed for an allowable stress $F_s = 21$ ksi for Overload $D + (5/3)(L+I)$. The allowable bolt load in double shear is

$$P = 2 \times 0.6013 \times 21 = 25.3 \text{ kips per bolt}$$

For design of the splice material for Maximum Design Loads, the design moment is calculated as the greater of:

- Average of the calculated moment on the section and maximum moment capacity of the section.
- $75\%$ of maximum capacity of the section.

The calculated moment is that induced by the Maximum Design Load $1.30(D + (5/3)(L+I))$. Splice material should have a capacity equal at least to the design moment. The section capacity is based on the gross section minus any flange-area loss due to bolt holes in excess of $15\%$ of each flange area.

Base metal fatigue should be investigated at the gross beam section near friction type fasteners.
Section Properties at Splice
Design of the splice begins with calculation of the section properties, section capacity, and design moment and shear.

Properties of W30 x 108

\[ I = 4,470 \text{ in.}^4 \]
\[ t_f = 0.76 \text{ in.} \]
\[ b_f = 10.484 \text{ in.} \]
\[ A_f = 31.80 \text{ in.}^2 \]
\[ t_w = 0.548 \text{ in.} \]
\[ D = 29.82 \text{ in.} \]

Area of Flange Holes

\[ A_H = 2 \times 1.0 \times 0.76 = 1.52 \text{ in.}^2 \]

Area of Flange

\[ A_f = 10.484 \times 0.76 = 7.97 \text{ in.}^2 \]

The bolt holes remove

\[ \% \text{ of flange} = \frac{1.52}{7.97} \times 100 = 19.07\% \]

Therefore, \( 19.07\% - 15\% = 4.07\% \) of the flange area must be deducted for determination of the net section. With this deduction, the net moment of inertia is

\[ I_{net} = 4,470 - 2 \times 0.0407 \times 7.97(14.53)^2 = 4,333 \text{ in.}^4 \]

Design Moments and Shears

For \( F_v = 50 \text{ ksi} \), the net-section moment capacity is

\[ M_{net} = \frac{50 \times 4,333}{14.91 \times 12} = 1,211 \text{ kip-ft} \]

75% \( M_{net} = 0.75 \times 1,211 = 908 \text{ kip-ft} \)

The average of the calculated moment for the Maximum Design Loads and the net capacity of the section is

\[ M_{av} = \frac{910 + 1,211}{2} = 1,061 > 908 \text{ kip-ft} \]

The design moment therefore is 1,061 kip-ft.

The design shear is obtained by multiplying the calculated shear for the Maximum Design Loads by the ratio of the design moment to the calculated moment on the section.

Design shear \( = 139.8 \times \frac{1,061}{910} = 163.0 \text{ kips} \)

Web-Splice Design

The web splice plates are proportioned to carry the design shear, a moment \( M_w \) due to the eccentricity of this shear and a portion \( M_v \) of the design moment on the section. The share of the design moment to be resisted by the web is obtained by multiplying the design moment by the ratio of the moment of inertia of the web \( I_w = 1,035 \text{ in.}^4 \) to the net moment of inertia of the entire section \( I_{net} = 4,333 \text{ in.}^4 \).
Web Moments for Design Loads

\[ M_v = 163.0 \times \frac{5}{12} = 68 \]
\[ M_w = 1,061 \times \frac{1,035}{4,333} = 253 \frac{321}{321} \text{ kip-ft} \]

Try two \( \frac{3}{8} \times 26\frac{1}{2} \)-in. web splice plates with a gross area of 19.88 sq in. Their gross moment of inertia is

\[ I = \frac{2 \times 0.375 (26.5)^4}{12} = 1,163 \text{ in.}^4 \]

Try three rows of bolts with seven bolts per row on each side of the joint. The area of one hole is \( 0.375 \times 1 = 0.375 \) sq in. The holes remove

\[ \% \text{ of plate} = \frac{7 \times 0.375}{0.375 \times 26.5} \times 100 = 26.4 \% \]

Consequently, the fraction of the hole area that must be deducted in determination of the net section is

\[ \frac{26.4 - 15}{26.4} = 0.432 \]

\[ \sum d^2 \text{ for Holes} \]

<table>
<thead>
<tr>
<th>Diameter (in)</th>
<th>Area (sq in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>8</td>
<td>64</td>
</tr>
<tr>
<td>12</td>
<td>144</td>
</tr>
<tr>
<td>( \sum d^2 )</td>
<td>224</td>
</tr>
</tbody>
</table>

The hole area of the web is

\[ 0.432 \times 4 \times 0.375 \times 224 = 145 \text{ in.}^4 \]

The net moment of inertia of the web splice plates then is

\[ I_{net} = 1,163 - 145 = 1,018 \text{ in.}^4 \]

Hence, the maximum bending stress in the plates for design loads is

\[ f_b = \frac{321 \times 12 \times 13.25}{1,018} = 50.1 \text{ at } 50 \text{ ksi} \]

The plates are satisfactory for bending. The allowable shear stress is

\[ F_s = 0.55 F_p = 0.55 \times 50 = 27.5 \text{ ksi} \]

The shear stress for the design shear is

\[ f_v = \frac{163.0}{19.88} = 8.20 < 27.5 \text{ ksi} \]

The \( \frac{3}{8} \times 26.5 \)-in. web splice plates are satisfactory for strength requirements. The plates are next checked for fatigue under service loads.

Web Bending Stress Range for Service Loads

The range of moment carried by the web equals

\[ M_w = (420 + 310) \times \frac{1,035}{4,333} = 174.4 \text{ kip-ft} \]

The maximum bending stress range in the gross section of the web splice plate is

\[ f_b = \frac{174.4 \times 12 \times 13.25}{1,163} = 23.8 \text{ ksi} \]
Allowable Fatigue Stress Range for Splice Material

Fatigue in base metal adjacent to friction-type fasteners is classified by AASHTO as category B. For 100,000 cycles, the associated allowable stress range is 45 ksi. The actual fatigue stress range is within the allowable and the web splice plates are satisfactory.

Use two \( \frac{3}{8} \times 26\frac{1}{2} \)-in. web splice plates.

Web Bolts

The 21 bolts in the web splice must carry the vertical shear, the moment due to the eccentricity of this shear about the centroid of the bolt group, and the portion of the beam moment taken by the web. These forces are induced by the Overload \( D+\left(\frac{5}{3}\right)(L+I) \). The allowable load in double shear was previously computed to be \( P=25.3 \) kips per bolt.

The polar moment of inertia of the bolt group about its centroid is

\[
I = 6 \times 224 + 14(3)^2 = 1,470 \text{ in.}^4
\]

The distance from the centroid to the outermost bolt is

\[
d = \sqrt{12^2 + 3^2} = 12.38 \text{ in.}
\]

Web Moments for Overload

\[
M_s = 107.5 \times \frac{5}{12} = 44.8
\]
\[
M_w = 700 \times \frac{1,035}{2,333} = 167.2
\]
\[
\frac{212.0}{\text{kip-ft}}
\]

The load per bolt due to shear is

\[
P_s = \frac{107.5}{21} = 5.1 \text{ kips}
\]

The load on the outermost bolt due to moment is

\[
P_m = \frac{212 \times 12 \times 12.38}{1,470} = 21.4 \text{ kips}
\]

The vertical component of this load is

\[
P_v = 21.4 \times \frac{3}{12.38} = 5.2 \text{ kips}
\]

The horizontal component is

\[
P_h = 21.4 \times \frac{12}{12.38} = 20.8 \text{ kips}
\]

Hence, the total load on the outermost bolt is the resultant

\[
P = \sqrt{20.8^2 + (5.2+5.1)^2} = 23.2 < 25.3 \text{ kips}
\]

Use 21 \( \frac{3}{8} \)-in.-dia A325 bolts in three columns.

Flange-Splice Design

The flange splice plates are proportioned for design loads and checked for fatigue in a similar manner to that for the web plates. The flange splice carries that portion of the design moment not carried by the web. Deducting the moment taken by the web from the design moment on the section, the flanges must be designed for

\[
M_f = 1,061 - 253 = 808 \text{ kip-ft}
\]

Compressive and tensile forces in the flanges form a couple that supply this capacity. Each force equals

\[
P_f = \frac{808 \times 12}{29.82 - 0.76} = 334 \text{ kips}
\]
To carry this force, the splice plate on each flange must have an area of at least

\[ A_f = \frac{334}{50} = 6.68 \text{ in.}^2 \]

Try a \( \frac{3}{8} \times 10 \)-in. plate on the outer surface of each flange and two \( \frac{3}{8} \times 4 \)-in. plates on the inner surface of each flange. The net area of these plates after deduction of holes for two bolts is

\[ A_f = (\frac{3}{8} \times 10) + (2 \times \frac{3}{8} \times 4) - [(2 \times 1 \times \frac{3}{8}) + (2 \times 1 \times \frac{3}{8}) - 0.15\{ (\frac{3}{8} \times 10) + (2 \times \frac{3}{8} \times 4) \}] = 6.71 > 6.68 \text{ in.}^2 \]

Use one \( \frac{3}{8} \times 10 \)-in. plate and two \( \frac{3}{8} \times 4 \)-in. plates. The plates are then checked for fatigue under service load.

The range of moment carried by the flanges equals

\[ M_f = (420 + 310)\left(1 - \frac{10.35}{43.38}\right) = 555 \text{ kip-ft} \]

The range of force in each flange is computed to be

\[ P_f = \frac{555 \times 12}{29.82 - 0.76} = 229 \text{ kips} \]

The maximum stress range in the gross section of the flange splice plate then is

\[ f_f = \frac{229}{(\frac{3}{8} \times 10 + 2 \times \frac{3}{8} \times 4)} = 31.6 \text{ ksi} < 45 \text{ ksi} \]

**Flange Bolts**

The number of bolts required in the flange splice is determined by the capacity needed for transmitting the flange force under the Overload \( D + (5/3)(L+I) \). The flange moment is the moment on the section less the moment carried by the web:

\[ M_f = 700 - 167 = 533 \text{ kip-ft} \]

Use 10 bolts in two rows. Details of the splice are shown below.
For this moment, the force in the flange is

\[ P_f = \frac{533 \times 12}{29.82 - 0.76} = 220 \text{ kips} \]

Bolts required \( \frac{220}{25.3} \approx 8.7 \)

**DEFLECTIONS**

Dead-load and live-load deflections are computed for Service Loads, in the same way as for working-stress design.

**CAMBER DIAGRAM**

Live-load deflection is 0.902 in. The deflection-span ratio is 1/930, well within the allowable value of 1/800.

**FINAL DESIGN**

An elevation of the two-span, continuous, composite stringer is shown below. See also the detail drawing at the end of this chapter. An alternate design is examined next.

**BEAM ELEVATION**

**Alternate Design**

In the negative-moment region, the stringer as designed in the first part of this chapter does not qualify as a compact section. Although satisfying every other compactness requirement, it does not meet the restrictions on unbraced length of compression flange with only two equally spaced lines of diaphragms per span. If the
unbraced length is decreased, by adding another line of diaphragms in each span, to make the negative-moment section compact, a lighter section results. If the savings in beam material thus achieved is greater than the cost of the additional diaphragms, the structure with the compact section would be more economical. This possibility is investigated for an alternate design in which compactness is obtained in the negative-moment region, with the W30×108 plus cover plates, by adding a line of diaphragms 9 ft from the center support. Since it was shown on page 3A.9 that the W30×108 alone satisfies compactness requirements for an unbraced length of up to 9.62 ft, it is clear that a diaphragm line 9 ft from the support will insure compactness for the W30×108 plus cover plates over a part of the 9 ft length.

For the compact, negative-moment section, the moments from the original analysis by elastic theory may be redistributed. They may be reduced by 10% in the negative-moment region and increased by 4% in the positive-moment region, as permitted for compact sections. This redistribution is equivalent to applying at the ends of each span at the center support, a positive moment equal to 10% of the maximum negative moment by elastic theory (see diagram).

**MOMENT REDISTRIBUTION FOR COMPACT SECTION**

If the bending moments in each span are then superimposed on the original moments, a 10% reduction in the maximum negative moment and a 4% increase in the maximum positive moment results.

**MAXIMUM POSITIVE MOMENT**

Because of the moment redistribution, the positive-moment section of the stringer must be investigated to determine whether the W30×108 alone is sufficient to carry the increase in maximum positive moment.

**Redistributed Positive Moments, Kip-Ft**

\[
DL_1: M = 309 + 0.04 \times 551 = 331 \\
DL_2: M = 57 + 0.04 \times 101 = 61 \\
L+I: M = 750 + 0.04 \times 601 = 774
\]

Stresses induced by the redistributed positive moments are determined with the section moduli previously computed. As before, the allowable stress for the Overload \(D+(5/3)(L+I)\) is 47.5 ksi.
Steel Stresses for Overload—Combination A

Bottom of Steel (Tension)

For \( DL_1: \) \( F_b = \frac{331 \times 12}{300} = 13.2 \)

For \( DL_2: \) \( F_b = \frac{61 \times 12}{425} = 1.7 \)

For \( L+I: \) \( F_b = \frac{774 \times 12}{469} \times \frac{5}{3} = 33.0 \)

\[
\frac{47.9}{47.5} \text{ ksi}
\]

The section is overstressed 1% but is considered satisfactory for Overload.

Maximum strength is checked next, to determine if the fully plastic moment capacity of the section \( M_s = 32,990 \) kip-in. is adequate. The Maximum Design Loads induce a maximum positive moment after redistribution equal to

\[
M_s = 1.30 \left[ DL_1 + DL_2 + \frac{5}{3}(L+I) \right] = 1.30 \left( 331 + 61 + \frac{5}{3} \times 774 \right)
\]

\[
= 2,187 \text{ kip-ft} = 26,244 \text{ kip-in.} < 32,990
\]

Consequently, the section has adequate strength for maximum positive moment.

MAXIMUM NEGATIVE MOMENT

Next, the section at the pier is proportioned for the redistributed maximum negative moment. A trial section consisting of the W30 \( \times 108 \) with a \( \frac{5}{16} \times 7 \)-in. top cover plate and a \( \frac{1}{2} \times 9\frac{1}{2} \)-in. bottom cover plate is chosen.

Redistributed Negative Moments at Pier, Kip-Ft

\( DL_1: \) \( M = 0.9 \times 551 = 496 \)

\( DL_2: \) \( M = 0.9 \times 101 = 91 \)

\( L+I: \) \( M = 0.9 \times 601 = 541 \)

Properties are calculated for the cover-plated beam alone and for this beam plus the longitudinal reinforcing in the concrete slab.

Steel Section at Interior Support

<table>
<thead>
<tr>
<th>Material</th>
<th>( A )</th>
<th>( d )</th>
<th>( Ad )</th>
<th>( Ad^2 )</th>
<th>( I_o )</th>
<th>( I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>W30 ( \times 108 )</td>
<td>31.8</td>
<td></td>
<td></td>
<td></td>
<td>4,470</td>
<td>4,470</td>
</tr>
<tr>
<td>Top Plate ( \frac{5}{16} \times 7 )</td>
<td>2.19</td>
<td>15.07</td>
<td>33.0</td>
<td>497</td>
<td>497</td>
<td></td>
</tr>
<tr>
<td>Bottom Plate ( \frac{1}{2} \times 9\frac{1}{2} )</td>
<td>4.75</td>
<td>-15.16</td>
<td>-72.0</td>
<td>1,092</td>
<td>1,092</td>
<td></td>
</tr>
</tbody>
</table>

\( d_r = \frac{39.0}{38.74} = 1.007 \text{ in.} \)

\( I_{NA} = \frac{-1.007 \times 39.0}{39} = 6,020 \text{ in.}^4 \)

\( d_{Top} = 14.91 + 0.31 + 1.01 = 16.23 \text{ in.} \)

\( d_{Bot} = 14.91 + 0.50 - 1.01 = 14.40 \text{ in.} \)

\( S_{Top} = \frac{6,020}{16.23} = 371 \text{ in.}^3 \)

\( S_{Bot} = \frac{6,020}{14.40} = 418 \text{ in.}^3 \)
Steel Section at Interior Support with Reinforcing Steel

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>38.74</td>
<td>-39.0</td>
<td>120.2</td>
<td>2,347</td>
<td>6,059</td>
<td></td>
</tr>
<tr>
<td>Reinf. Steel 14 No. 6</td>
<td>6.16</td>
<td>19.52</td>
<td></td>
<td>2,347</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
d_s = \frac{31.2}{44.90} = 1.81 \text{ in.}
\]

\[
I_{NA} = \frac{14.91 \times 0.50 + 1.81}{17.22} = 8,259 \text{ in.}^4
\]

\[
d_{\text{Top of steel}} = 14.91 + 0.31 - 1.81 = 13.41 \text{ in.}
\]

\[
d_{\text{Bot. of steel}} = 14.91 + 0.50 + 1.81 = 17.22 \text{ in.}
\]

\[
S_{\text{Top of steel}} = \frac{8,259}{13.41} = 616 \text{ in.}^3
\]

\[
S_{\text{Bot. of steel}} = \frac{8,259}{17.22} = 480 \text{ in.}^3
\]

\[
d_{\text{Reinf.}} = 19.52 - 1.81 = 17.71 \text{ in.}
\]

\[
S_{\text{Reinf.}} = \frac{8,259}{17.71} = 466 \text{ in.}^3
\]

**Check at Overload**

Because the section at the support is made compact by introducing an additional line of diaphragms to brace the compression flange, the Overload condition governs the design. The moment relationship is

\[
\left[ D + \frac{5}{3}(L + I) \right] \leq 0.80F_s S
\]

The allowable stress is thus \((0.80)(50) = 40 \text{ ksi}\) for the beam and \((0.80)(40) = 32 \text{ ksi}\) for the reinforcing steel.

**Steel Stresses for Overload—Combination A**

<table>
<thead>
<tr>
<th>Top of Steel (Tension)</th>
<th>Bottom of Steel (Compression)</th>
</tr>
</thead>
<tbody>
<tr>
<td>For (DL_1): (F_b = \frac{496 \times 12}{371} = 16.0)</td>
<td>(F_b = \frac{496 \times 12}{418} = 14.2)</td>
</tr>
<tr>
<td>For (DL_2): (F_b = \frac{91 \times 12}{616} = 1.8)</td>
<td>(F_b = \frac{91 \times 12}{480} = 2.3)</td>
</tr>
<tr>
<td>For (L + I): (F_b = \frac{541 \times 12}{616} \times \frac{5}{3} = 17.6)</td>
<td>(35.4 &lt; 40 \text{ ksi})</td>
</tr>
<tr>
<td></td>
<td>(F_b = \frac{541 \times 12}{480} \times \frac{5}{3} = 22.5)</td>
</tr>
</tbody>
</table>

**Reinforcing Steel Stress (Tension)**

\[
DL_1: F_b = \frac{91 \times 12}{466} = 2.3
\]

\[
L + I: F_b = \frac{541 \times 12}{466} \times \frac{5}{3} = 23.2
\]

\[
25.5 < 32 \text{ ksi}
\]

The trial section is satisfactory for maximum negative moment for Overload.

**Fatigue Check**

Fatigue in the reinforcing steel is checked next for a live-load moment range from 0 to 601 kip-ft, without the 10% reduction.

\[
f_{rs} = \frac{601(12)}{466} = 15.5 < 20 \text{ ksi}
\]

The trial section therefore is satisfactory for Overload.
COVER-PLATE CUTOFF
The cover plates are cut off 5 ft from the pier. Stresses for Overload are calculated at the theoretical cutoff location 1.5 times the plate width closer to the pier (5.00 – 1.5 × 9.5/12 = 3.81 ft = 3 ft-9¼ in. from the pier). Fatigue is investigated at the actual termination.

**Section with W30 and Reinforcing Steel**

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>W30 × 108</td>
<td>31.8</td>
<td>19.52</td>
<td>120.2</td>
<td>2,347</td>
<td>4,470</td>
<td>4,470</td>
</tr>
<tr>
<td>Reinf. Steel 14 No. 6</td>
<td>6.16</td>
<td>6.16</td>
<td>6.16</td>
<td>6.16</td>
<td>6.16</td>
<td>6.16</td>
</tr>
</tbody>
</table>

\[
d_s = \frac{120.2}{37.96} = 3.17 \text{ in.}
\]

\[
-3.17 \times 120.2 = -381
\]

\[
I_{NA} = 6,436 \text{ in.}^4
\]

\[
d_{Top} = 14.91 - 3.17 = 11.74 \text{ in.}
\]

\[
d_{Bot} = 14.91 + 3.17 = 18.08 \text{ in.}
\]

\[
S_{Top} = \frac{6,436}{11.74} = 548 \text{ in.}^3
\]

\[
S_{Bot} = \frac{6,436}{18.08} = 356 \text{ in.}^3
\]

\[
d_{Reinf.} = 19.52 - 3.17 = 16.35 \text{ in.}
\]

\[
S_{Reinf.} = \frac{6,436}{16.35} = 394 \text{ in.}^3
\]

**Bending Moments 3.81 Ft from Pier**

<table>
<thead>
<tr>
<th></th>
<th>DL₁</th>
<th>DL₂</th>
<th>(L+I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M, kip-ft</td>
<td>-410</td>
<td>-76</td>
<td>-449</td>
</tr>
</tbody>
</table>

**Steel Stresses for Overload 3.81 Ft from Pier**

Bottom of Steel (Compression)

For \( DL₁ \):

\[
F_b = \frac{(410 - 0.095 \times 551)12}{300} = 14.3
\]

For \( DL₂ \):

\[
F_b = \frac{(76 - 0.095 \times 101)12}{356} = 2.2
\]

For \( L+I \):

\[
F_b = \frac{(449 - 0.095 \times 601)12 \times 5}{356} = 22.0
\]

\[
\frac{35.5}{35.5} < 40.0 \text{ ksi}
\]

Stresses at the theoretical cutoff of the cover plates are within the allowable. Next, fatigue is investigated at the actual ends of the plates adjacent to the fillet weld.

**Bending Moments 5 Ft from Pier**

<table>
<thead>
<tr>
<th></th>
<th>DL₁</th>
<th>DL₂</th>
<th>-(L+I)</th>
<th>+(L+I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M, kip-ft</td>
<td>-361</td>
<td>-69</td>
<td>-412</td>
<td>64</td>
</tr>
</tbody>
</table>

It is obvious from the above moments that no tension will exist in the bottom flange. Therefore, no restrictions are placed on the fatigue stress range there.
Tensile stress range in the top flange is computed to be

\[ f_{tr} = \frac{(412+64)(12)}{548} = 10.4 < 21 \text{ ksi} \]

Finally, the slab reinforcement is checked for Overload and for fatigue under Service Load at the theoretical cut-off location.

**Reinforcing Steel Stresses 3.81 Ft from Pier**

For \( DL_2 \): \( F_b = \frac{(76-0.095 \times 101)12}{394} = 2.0 \)

For \( L+I \): \( F_b = \frac{(449-0.095 \times 601)12 \times \frac{5}{3}}{394} = 19.9 \) \( \frac{21.9 < 32 \text{ ksi}}{21.9 < 32 \text{ ksi}} \)

**Reinforcing Steel Fatigue Stress Range 3.81 Ft from Pier**

\[ \text{Range} = \frac{(449+42)12}{394} = 15.0 < 20 \text{ ksi} \]

All stresses are within the allowable, and the stress range is satisfactory for fatigue. Therefore, the cover plates can be cut off 5 ft from the center support.

**COVER-PLATE WELDS**

Use the minimum weld size of \( \frac{3}{8} \) in. for the cover plates.

**FINAL ALTERNATE DESIGN**

An elevation view of the alternate design is shown below.

---

**BEAM ELEVATION—ALTERNATE DESIGN**

**COMPARISON OF ALTERNATE AND ORIGINAL DESIGN**

As a measure of the relative economy of the alternate design with a compact negative-
moment section and the original design with a noncompact negative-moment section, the difference in weights of steel required for the two designs is calculated.

**Weight Comparison for Beam Only**

**Noncompact Section**

<table>
<thead>
<tr>
<th>Material</th>
<th>Weight, lb per ft</th>
<th>Weight, lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>W30×140 ft</td>
<td>108</td>
</tr>
<tr>
<td>Top cover plate</td>
<td>⅜×8×8 ft</td>
<td>10.2</td>
</tr>
<tr>
<td>Bottom cover plate</td>
<td>1×12×22 ft</td>
<td>40.8</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Compact Section**

<table>
<thead>
<tr>
<th>Material</th>
<th>Weight, lb per ft</th>
<th>Weight, lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>W30×140 ft</td>
<td>108</td>
</tr>
<tr>
<td>Top cover plate</td>
<td>½×7×10 ft</td>
<td>7.4</td>
</tr>
<tr>
<td>Bottom cover plate</td>
<td>½×9½×10 ft</td>
<td>16.2</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Beam material savings = 16,100 − 15,356 = 744 lb per beam

**Weight of Two Lines of Diaphragms**

2 C10×15.3×25 ft

Weight = 2×25×15.3 = 765 lb

Weight per beam = \(\frac{765}{4}\) = 191 lb

Net savings = 744 − 191 = 553 lb per beam, or 2,212 lb per bridge

With a weight savings of only about 3%, it is difficult to say whether the alternate design is more economical than the original design when added fabrication and erection costs for the additional line of diaphragms is taken into account.

A detail drawing of the original design is shown on the following page.
Composite Design Example
2-Span Continuous Rolled Beams
Load Factor Design

Note:
1. All material ASTM A688, Grade A (USS COR-TEN B)
2. All bolts ASTM A325 Type 3
   - Total Wt=69,695 lb
   - Wt per sq ft (0.0, slab) = 14.9 lbs per sq ft
   - Weight does not include bearing shoes, railing or studs.
Composite Design Example
2-Span Continuous Rolled Beams
Load Factor Design
Note:

Designs presented in this chapter are in accordance with the Tenth Edition of the Standard Specifications for Highway Bridges of the American Association of State Highway and Transportation Officials (AASHTO) dated 1969. They should be reviewed for adequacy in conforming to the latest edition of the AASHTO Specifications and subsequent interims, especially with regard to fatigue provisions and welded stud shear connectors.
Composite:
Welded Plate Girder

Introduction
With few exceptions, the design of composite, welded plate girders is similar to the
design of composite wide-flange beams, discussed in the previous chapter. For plate
girders also, the section supporting the loads consists of the steel girders and a portion
of the concrete slab, bonded together by shear connectors.
Welded plate girders may be designed either with webs stiffened by intermediate
stiffeners or with webs unstiffened except at load points or bearings. A design with
stiffened webs generally results in a minimum weight of steel. However, a design with
unstiffened webs often yields a lower price per pound of fabricated structural steel
because less welding is required.
This chapter presents four examples to illustrate the design of composite, welded
plate girders:
I. An 80-ft simple-span, composite, welded plate girder.
II. A two-span, continuous, welded plate girder (100 ft—100 ft), composite for
positive moment only.
III. A two-span, continuous, welded plate girder (100 ft—100 ft), composite for
positive moment and negative moment.
IV. A four-span, continuous, welded plate girder (100 ft—128 ft—128 ft—100 ft),
composite for positive moment only.
These design examples are for interior girders with stiffened webs. For compu-
tation of stresses and deflections, the concrete slab is transformed into an equivalent
area of steel.
Girders with unstiffened webs have been designed for spans comparable to Designs
I and IV. Weight comparisons are given near the end of this chapter.
All designs are in accordance with the 1969 Edition of the Standard Specifications
for Highway Bridges of The American Association of State Highway Officials.
The design examples are for ASTM A36 steel. Other structural steels such as
ASTM A441, A572, and A588 have also proven economical for these types of struc-
tures. Design procedures for these high-strength low-alloy steels are similar to those
for A36 steel except that the allowable stresses are generally higher.

General Design Considerations
The principles of composite construction for welded plate girders are identical to
those for rolled beam stringers, and have been discussed in detail in Chapter 3,
Composite: Wide Flange Beam.
LATERAL DISTRIBUTION OF DEAD AND LIVE LOADS
Dead and live loads are distributed laterally in exactly the same manner as for wide-flange beams. The method is described in detail in Chapter 3.

MILITARY LOADING
Military loading may govern moments and end shears for very short spans, as discussed in Chapter 3. Spans of sufficient length to require welded girders will not usually be controlled by military loading.

DISTRIBUTION OF MOMENTS IN CONTINUOUS GIRDERS
For total moments, an analysis based upon constant moment of inertia will generally be accurate within 3 or 4 percent. The compensating variations in moment due to \( DL_1 \), \( DL_2 \) and \( LL+I \) are explained in Chapter 3. These effects apply to welded plate girders as well as to rolled beams.

CHANGES IN FLANGE-PLATE THICKNESS
Composite designs with welded girders offer more flexibility than designs with rolled sections. For example, flange plates of varying thicknesses, and sometimes varying widths, may be butt welded to provide a section strength that closely approximates the variation in bending moment.

Theoretical locations at which flange-plate thickness or width may be changed are determined in the same manner as the theoretical cutoff locations for cover plates on rolled beams. The actual changes in flange-plate thickness or width are made near theoretical locations.

An excessive number of changes in flange-plate thickness and width should be avoided. Although a minimum weight of steel results from such reductions in flange size, the cost of making and radiographing the necessary butt welds increases the over-all cost of the fabricated girder. For a simple span, the top flange is usually held at one size for the full length of the girder. The bottom flange generally should be made from three plates of two sizes—a center plate covering approximately the middle 60% of the span, and two end plates butt welded to the center plate.

For continuous spans, the sizes of top and bottom flanges normally are changed only once in the negative-moment areas. If both width and thickness of the flanges must be changed, however, some designers prefer two splices, the width changing at one location and the thickness at another location. In positive-moment areas, the bottom flange usually is made from three plates of two sizes, as for simple spans. The top flange generally should comprise one plate extending through most of the positive-moment area and a thicker plate extending from the negative-moment area to the first plate.

When flange plates of different thicknesses are butt welded, AASHO Specifications require a uniform transition slope between the offset surfaces not exceeding 1 in 2½. If plates of different widths are joined, the wider plate must taper into the narrower plate as shown below:

---

FATIGUE
Allowable fatigue stresses for base metal, for weld metal or base metal adjacent to butt welds, and for base metal adjacent to fasteners at field splices are given in the AASHO Specifications.

II/4.2
If butt welded splices conform to the following conditions, they may be designed in accordance with the allowable fatigue stress for base metal:

1. The parts joined are of equal thickness.
2. The parts joined are of equal width, or tapered as illustrated on p. II/4.2.
3. Weld soundness, established by radiographic inspection, meets specified requirements.
4. Welds are made smooth and flush by grinding in the direction of applied stress.

For bridges on freeways, expressways, and major highways and streets, allowable fatigue stresses are based upon 100,000, 500,000 or 2,000,000 cycles of loading, depending on the type of loading. For continuous spans of about 100 ft, truck loading controls positive moments, as well as negative moments in the vicinity of inflection points where fatigue is most critical. In such cases 500,000 cycles of loading should be used. For longer spans where lane loading governs, fatigue considerations should be based upon 100,000 cycles of loading.

The design examples of this chapter are for 500,000 loading cycles. Allowable fatigue stresses for A36 base metal at 500,000 cycles are given by the AASHO Specifications as follows:

\[
\text{Tension: } F_r = \frac{20,500}{1 - 0.55R} \\
\text{Compression: } F_r = \frac{0.55F_y}{1 - \left(\frac{0.55F_y}{13.3} - 1\right)R}
\]

where \(F_r\) = allowable fatigue stress, psi

\(R\) = minimum stress

\(\text{maximum stress}\)

\(F_y\) = minimum yield stress of material, psi

Butt-welded splices that do not meet the above conditions must be designed for allowable fatigue stresses for weld metal or base metal adjacent to butt welds. These stresses are given by the following formulas for 500,000 loading cycles:

\[
\text{Tension: } F_r = \frac{17,200}{1 - 0.62R} \\
\text{Compression: } F_r = \frac{0.55F_y}{1 - \left(\frac{0.55F_y}{10.6} - 1\right)R}
\]

Welding of intermediate transverse web stiffeners or other attachments to the tension flange is usually avoided. However, when such welds are required, the flange stress should not exceed the allowable fatigue stress for base metal adjacent to or connected by fillet welds. This stress is given by the following formula for 500,000 loading cycles:

\[
F_r = \frac{12,000}{1 - R}
\]

Allowable fatigue stresses for 100,000 cycles of loading and for other grades of steel are given in the AASHO Specifications.

**LATERAL BUCKLING**

In continuous welded girders, allowable stresses for lateral buckling must be used along the compression flange in negative-moment regions of the span. These allowable stresses are determined in the same manner as for rolled beams, described in Chapter 3.
FLANGE AND WEB DIMENSIONS

To insure against buckling, AASHO Specifications recommend certain limits on the proportions of flange and web material in welded plate girders. A wide but thin compression flange is subject to buckling. Hence, the ratio of flange-plate width to thickness should not exceed the value given by the following formula:

\[
\frac{\text{width}}{\text{thickness}} \leq \frac{3,250}{\sqrt{f_b}} \leq 24
\]

where \( f_b \) is calculated maximum compressive stress, psi

For A36 steel, where \( f_b = 20,000 \) psi, the formula limits the width-thickness ratio to 23.

To guard against buckling of the web, the minimum thickness, \( t \), for webs with transverse stiffeners, but without longitudinal stiffeners, should exceed the value determined by the following formula:

\[
\text{minimum web thickness} = \frac{D\sqrt{f_b}}{23,000} \geq \frac{D}{170}
\]

where \( D \) = clear unsupported depth of the web, in.

For A36 steel, where \( f_b = 20,000 \) psi, the formula requires a web thickness of at least \( D/166 \).

WEB SHEARING STRESS

For normal spans, shearing stress in the webs of welded plate girders usually is not critical when flexural strength is satisfied. In those cases, web thickness usually will be governed by the buckling considerations previously described. However, where heavy loads are carried by a short girder, shearing stress may govern web design.

BEARING AND INTERMEDIATE STIFFENERS

Bearing stiffeners are required over the end and intermediate bearings of welded plate girders. A bearing stiffener is designed as a column to transmit the entire reaction from the bearing to the web of the girder. The column consists of plates welded to each side of the web and a centrally loaded strip of the web equal to not more than 18 times the web thickness. The stiffener plates are milled to fit against the flange through which they receive their reaction or are attached to the flange by full penetration groove welds. Thickness, \( t \), of the plates, as prescribed by AASHO Specifications, must be at least

\[
t = \frac{1}{12} b_y \sqrt{\frac{F_y}{33,000}}
\]

where \( F_y \) = yield stress of stiffener, psi

\( b_y \) = stiffener plate width, in.

Between bearings, web stiffeners may be required to prevent web buckling. Spacing of intermediate stiffeners is calculated by the following formula:

\[
\text{Spacing, in.} = \frac{11,000t}{\sqrt{f_s}}
\]

where \( t \) = web-plate thickness, in.

\( f_s \) = shearing stress in gross section of web plate at point considered, psi

If the girder is simply supported, the first two spaces at the ends should be one-half that given by the above formula.

Transverse intermediate stiffeners may be omitted if the web-plate thickness is at least equal to that given by

\[
t = \frac{D\sqrt{f_s}}{7,500} \geq \frac{D}{150}
\]
For a given web-plate thickness, stiffeners are not needed in regions where the shearing stress does not exceed $f_s$ determined from the above formula.

Transverse intermediate stiffeners must provide a moment of inertia, in.\(^4\), of at least

$$I = \frac{d_o^2 J}{10.92}$$

where $J = 25\frac{D^2}{d^2} - 20$, but not less than 5.0.

$d = \text{required stiffener spacing, in.}$

$d_o = \text{actual stiffener spacing, in.}$

$D = \text{clear unsupported depth of web, in.}$

$t = \text{web-plate thickness, in.}$

Furthermore, the stiffener-plate width should be at least $1/30$ the depth of the girder plus 2 in., and preferably at least $1/4$ the girder-flange width. Thickness of the stiffener should not be less than $1/8$ its width.

Intermediate transverse stiffeners are sometimes welded to the compression flange. This does not serve any structural purpose in the finished bridge but does improve the rigidity of individual girders and therefore may be an aid in transporting and erecting the girders.

Where the tensile stress in a flange exceeds the allowable fatigue stress, stiffeners should not be welded to that flange.

SHEAR CONNECTORS, DEFLECTIONS

Design of shear connectors and computation of dead- and live-load deflections are based on the principles applicable to composite rolled-beam stringers, discussed in the previous chapter.

DESIGN EXAMPLES

Four design examples as listed at the beginning of this chapter are presented to illustrate the design of welded plate girders.

The following applies to all designs:

Roadway Section: See sketch in Design I.


Loading: HS20-44.

Structural Steel: A36

- Allowable bending stress = 20,000 psi
- Allowable shearing stress = 12,000 psi

Concrete: $f' = 4,000$ psi

$\sigma_s = 1,600$ psi

$n = 8$

Loading Conditions:

Case 1—Weight of girder and slab ($DL_1$) supported by the steel girder alone.

Case 2—Superimposed dead load ($DL_2$) (curbs and railings) supported by the composite section with the modular ratio $n = 8$.

Case 3—Superimposed dead load ($DL_2$) (curbs and railings) supported by the composite section with the increased modular ratio $3n = 3 \times 8 = 24$.

Case 4—Live load plus impact ($LL + I$) supported by the composite section with the modular ratio $n = 8$. 

II/4.
Loading Combinations:
Combination A = Case 1 + 3 + 4.
Combination B = Case 2 + 4.
Combination C = Case 1 + 2 + 4.

Fatigue Considerations:
500,000 cycles of maximum stress.

Design I–80-Foot Simple-Span Composite Girder

LOADS, SHEARS AND MOMENTS
The dead load to be carried by the steel alone consists of the weights of the concrete slab, the steel girder, haunch, and framing details. The weights of the curb and railing comprise the superimposed dead load, carried by the composite section. The HS20-44 truck loading is distributed to each girder according to AASHO Specifications, and the impact factor is calculated for an 80-ft span.

![Diagram of typical section]

**Typical Section**

Dead Load Carried by Steel
Slab = \( \frac{7}{12} \times 8.33 \times 0.150 = 0.730 \)
Steel girder, details and concrete haunch = 0.170
\( DL_1 \) per girder = 0.900 k/ft

Dead Load Carried by Composite Section
Curbs and railings, \( DL_2 = 0.660 \) k/ft
\( DL_2 \) per girder = 0.660/4 = 0.165 k/ft
**Live Load**

Live-load distribution \( \frac{S}{5.5} = \frac{8.33}{5.5} = 1.51 \) wheels = 0.755 axle

Impact = \( \frac{50}{80 + 125} = 0.244 \)

For a system of two or three moving concentrated loads on a simple span, the curve of maximum moments is closely approximated by two half-parabolas joined near midspan by a straight line of length 2a, where a = distance of maximum-moment section from midspan (see diagram below). 2a is equal to 4.67 ft for all spans greater than 33.8 ft, except where military loading governs. The curves of maximum moments for \( DL_1 \) and \( DL_2 \) are simply parabolic moment diagrams for uniform loads of 0.900 and 0.165 kips per foot, respectively.

**MAXIMUM MOMENT CURVES**

The curves of live-load shear are obtained by moving an HS20-44 truck across the span to produce maximum and minimum shear at each tenth point.
DESIGN OF GIRDER SECTION

A 4-ft-deep web satisfies the minimum depth-to-span ratio of 1/25 for the girder plus slab and 1/30 for the girder alone. The web thickness is required to be at least $D/165 = 0.291$ in. Thus, a $5/16$-in.-thick web can be used. This thickness is also the lower limit allowed by AASHO Specifications regardless of the value of D. Shearing stresses in the $48 \times 5/16$-in. web plates are well under the allowable value of 12,000 psi.

Maximum Shear Stress

$$DL_1 \text{; } V = 36.0$$
$$DL_2 \text{; } V = 6.6$$
$$LL+I \text{; } V = 59.7$$

$$f_v = \frac{102.3}{48 \times \frac{5}{16}} = 6.82 \text{ ksi} < 12$$

Minimum Thickness of Compression Flange for Maximum Stress

Assuming that the compression flange of the girder will be fully stressed, the maximum permissible $b/t$ ratio for the flange plate is 23. With a 12-in.-wide top-flange plate, the minimum allowable thickness is slightly over $\frac{3}{4}$ in. Hence, a trial top-flange plate $12 \times \frac{3}{16}$ in. is selected.
COMPOSITE SECTION

\[ \frac{b}{t} = 23 \text{ or } t = \frac{12}{23} = 0.522 \text{ in. Use } 12 \times \frac{9}{16} \]

Properties of Composite Section

The maximum-positive-moment section of the girder is investigated with a 12 × 1\% in. bottom-flange plate. Properties are computed for the steel section alone, the composite section with \( n \) equal to 8, and the composite section with 3\( n \) equal to 24.

COMPOSITE SECTION

Effective Flange Width

\( \frac{1}{4} \text{ span} = \frac{1}{4} \times 80 \times 12 = 240 \text{ in.} \)

Girder spacing, \( c \) to \( c = 100 \text{ in.} \)

12 × slab thickness = 12 × 7 = 84 in. (governs)

Steel Section for Maximum Positive Moment

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>( d )</th>
<th>( Ad )</th>
<th>( Ad^2 )</th>
<th>( I_o )</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Flg. 12 × % ( \frac{9}{16} )</td>
<td>6.75</td>
<td>24.28</td>
<td>+164</td>
<td>3,980</td>
<td>2,880</td>
<td>3,980</td>
</tr>
<tr>
<td>Web 48 × % ( \frac{9}{16} )</td>
<td>15.00</td>
<td>24.28</td>
<td>-522</td>
<td>13,000</td>
<td>2,880</td>
<td>13,000</td>
</tr>
<tr>
<td>Bot. Flg. 12 × 1% ( \frac{9}{16} )</td>
<td>21.00</td>
<td>-24.88</td>
<td>-522</td>
<td>13,000</td>
<td>2,880</td>
<td>13,000</td>
</tr>
</tbody>
</table>

\[ d_s = \frac{-358}{42.75} = -8.37 \text{ in.} \]

\[ -358 \text{ in.}^3 \]

\[ I_NA = \frac{-8.37 \times 358}{16,860 \text{ in.}^4} \]

\[ d_{\text{top of steel}} = 24.56 + 8.37 = 32.93 \text{ in.} \]

\[ d_{\text{Bot. of steel}} = 25.75 - 8.37 = 17.38 \text{ in.} \]

\[ Z_{\text{Top of steel}} = \frac{16,860}{32.93} = 512 \text{ in.}^3 \]

\[ Z_{\text{Bot. of steel}} = \frac{16,860}{17.38} = 970 \text{ in.}^3 \]
Composite Section, $3n = 24$, for Maximum Positive Moment

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel section</td>
<td>42.75</td>
<td>29.06</td>
<td>-358</td>
<td>20,690</td>
<td>19,860</td>
<td>20,790</td>
</tr>
<tr>
<td>Conc. 84×7/24</td>
<td>24.50</td>
<td>29.06</td>
<td>+712</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
d_{3n} = \frac{354}{67.25} = 5.26 \text{ in.} \quad +354 \text{ in.}^3
\]
\[d_{NA} = \frac{-5.26 \times 354}{38,790} = -1.860 \text{ in.}^4\]

\[d_{\text{Top of steel}} = 24.56 - 5.26 = 19.30 \text{ in.}\]
\[d_{\text{Bot of steel}} = 25.75 + 5.26 = 31.01 \text{ in.}\]

\[Z_{\text{Top of steel}} = \frac{38,790}{19.30} = 2,010 \text{ in.}^3\]
\[Z_{\text{Bot of steel}} = \frac{38,790}{31.01} = 1,251 \text{ in.}^3\]

Composite Section, $n = 8$, for Maximum Positive Moment

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel section</td>
<td>42.75</td>
<td>29.06</td>
<td>-358</td>
<td>62,070</td>
<td>19,860</td>
<td>62,370</td>
</tr>
<tr>
<td>Conc. 84×7/8</td>
<td>73.50</td>
<td>29.06</td>
<td>+2,136</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[d_{s} = \frac{1,778}{116.25} = 15.29 \text{ in.}\]
\[d_{s} = 1,778 \text{ in.}^3\]
\[d_{s} = -15.29 \times 1778 = -27,190 \text{ in.}^4\]
\[d_{s} = \frac{1778}{116.25} = 15.29 \text{ in.}\]
\[d_{s} = \frac{1778}{116.25} = 15.29 \text{ in.}\]
\[d_{s} = \frac{1778}{116.25} = 15.29 \text{ in.}\]

\[d_{\text{Top of steel}} = 24.56 - 15.29 = 9.27 \text{ in.}\]
\[d_{\text{Bot of steel}} = 25.75 + 15.29 = 41.04 \text{ in.}\]

\[Z_{\text{Top of steel}} = \frac{55,040}{9.27} = 5,937 \text{ in.}^3\]
\[Z_{\text{Bot of steel}} = \frac{55,040}{41.04} = 1,341 \text{ in.}^3\]

\[d_{\text{Top of conc.}} = 9.27 + 8.00 = 17.27 \text{ in.}\]

\[Z_{\text{Top of conc.}} = \frac{55,040}{17.27} = 3,187 \text{ in.}^3\]

Check of Compression-Flange Thickness

Stresses are computed at the top and bottom of steel and at the top of concrete for $DL_1$, $DL_2$ and $LL+I$. The $b/t$ ratio of the compression flange is now checked by the general formula applicable for any stress level: $b/t = 3,250/\sqrt{f_s}$.

The formula yields a minimum permissible flange thickness of slightly over 3/4 in. Hence, the original trial flange of 12×7/8 in. is satisfactory.
Bending Moments

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$LL+I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$, kip-ft</td>
<td>720</td>
<td>132</td>
<td>1,094</td>
</tr>
</tbody>
</table>

Steel Stresses—Combination A

Top of Steel (Compression)

$$DL_1: \sigma_b = \frac{720 \times 12}{512} = 16.88 \text{ ksi}$$

$$DL_2: \sigma_b = \frac{132 \times 12}{2,010} = 0.79 \text{ ksi}$$

$$LL+I: \sigma_b = \frac{1,094 \times 12}{5,937} = 2.21 \text{ ksi}$$

Bottom of Steel (Tension)

$$f_b = \frac{720 \times 12}{970} = 8.91 \text{ ksi}$$

$$f_b = \frac{132 \times 12}{1,251} = 1.27 \text{ ksi}$$

$$f_b = \frac{1,094 \times 12}{1,341} = 9.79 \text{ ksi}$$

Check: $$\frac{b}{t} = \frac{3.250}{\sqrt{19.880}} = 23.05 < 24$$

$$t = \frac{12}{23.05} = 0.520 \text{ in. Use } \frac{3}{16} \text{ in. top flg.}$$

Concrete Stresses—Combination B

Top of Concrete

$$DL_2: \sigma_c = \frac{132 \times 12}{3,187 \times 8} = 0.062 \text{ ksi}$$

$$LL+I: \sigma_c = \frac{1,094 \times 12}{3,187 \times 8} = 0.515 \text{ ksi}$$

LOCATION OF FLANGE-PLATE TRANSITIONS

At a point yet to be determined, the thickness of the bottom flange may be reduced. For savings in weight alone, two steps in flange thickness, possibly from 1 1/4 in. to 1 1/2 in., then from 1 1/2 in. to 3/8 in., could be used. However, higher fabrication costs due to the additional butt welds will usually offset any savings in steel costs. A single transition, from the 12 x 1 1/4-in. plate to a 12 x 1-in. plate, provides an economical flange design for the welded girder under consideration. Properties are computed for this reduced section.

Steel Section near Supports

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad^2</th>
<th>I_o</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Flg. 12 x 3/8</td>
<td>6.75</td>
<td>24.28</td>
<td>164</td>
<td>3,980</td>
<td>3,980</td>
<td></td>
</tr>
<tr>
<td>Web 48 x 3/8</td>
<td>15.00</td>
<td>-24.50</td>
<td>-294</td>
<td>7,200</td>
<td>7,200</td>
<td></td>
</tr>
<tr>
<td>Bot. Flg. 12 x 1</td>
<td>12.00</td>
<td>24.28</td>
<td>164</td>
<td>3,980</td>
<td>3,980</td>
<td></td>
</tr>
</tbody>
</table>

$$d_t = \frac{-130}{33.75} = -3.85 \text{ in.}$$

$$d_{top \text{ of steel}} = 24.56 + 3.85 = 28.41 \text{ in.}$$

$$d_{Bot. \text{ of steel}} = 25.00 - 3.85 = 21.15 \text{ in.}$$

$$I_{NA} = \frac{-3.85 \times 130}{-500} = 14,060 \text{ in.}^4$$

$$d_{Top \text{ of steel}} = \frac{13,560}{28.41} = 477 \text{ in.}^3$$

$$Z_{Bot. \text{ of steel}} = \frac{13,560}{21.15} = 641 \text{ in.}^3$$
Composite Section, $3n = 24$, near Supports

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad$^2$</th>
<th>I$_v$</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel section</td>
<td>33.75</td>
<td>24.50</td>
<td>29.06</td>
<td>-130</td>
<td>+712</td>
<td>20,690</td>
</tr>
</tbody>
</table>

$d_N = \frac{582}{56.25} = 9.99$ in. $\quad +582$ in.$^3$ $\quad -9.99 \times 582 = -5,810$ $\quad I_{NA} = 29,040$ in.$^4$

$d_{top of steel} = 24.56 - 9.99 = 14.57$ in. $\quad d_{bot of steel} = 25.00 + 9.99 = 34.99$ in.

$Z_{top of steel} = \frac{29,040}{14.57} = 1,993$ in.$^3$ $\quad Z_{bot of steel} = \frac{29,040}{34.99} = 830$ in.$^3$

Composite Section, $n = 8$, near Supports

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad$^2$</th>
<th>I$_v$</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel section</td>
<td>33.75</td>
<td>73.50</td>
<td>29.06</td>
<td>-130</td>
<td>+2,136</td>
<td>62,070</td>
</tr>
</tbody>
</table>

$d_s = \frac{2,006}{107.25} = 18.70$ in. $\quad +2,006$ in.$^3$ $\quad -18.70 \times 2006 = -37,510$ $\quad I_{NA} = 38,920$ in.$^4$

$d_{top of steel} = 24.56 - 18.70 = 5.86$ in. $\quad d_{bot of steel} = 25.00 + 18.70 = 43.70$ in.

$Z_{top of steel} = \frac{38,920}{5.86} = 6,642$ in.$^3$ $\quad Z_{bot of steel} = \frac{38,920}{43.70} = 891$ in.$^3$

Change in Bottom-Flange Thickness

Exclusive of fatigue considerations, the bottom-flange thickness may be reduced if the stress in the reduced section would not exceed the allowable stress. An approximate formula, discussed in Cover Plate Cutoffs in Chapter 3, gives the length required for the heavier section.

Approximate length $L_{cp}$ for 1¼-in. flange:

$$L_{cp} = (L - 2a) \sqrt{1 - \frac{Z_{ss}}{Z_{ss}^*} + 2a}$$

where $L =$ girder span $= 80$ ft

$a =$ distance of maximum-moment section from midspan $= 2.33$ ft

$Z_{ss} =$ section modulus of steel girder with reduced flange $= 641$ in.$^3$

$Z_{ss}^* =$ section modulus of steel girder with thicker flange $= 970$ in.$^3$

$$L_{cp} = [80 - (2 \times 2.33)] \sqrt{1 - \frac{641}{970} + (2 \times 2.33)} = 48.5$ ft

Approximate cutoff point for 1¼-in. flange from center of bearing:

$L/2 - L_{cp}/2 = 40 - 24.25 = 15.75$ ft, say 16 ft
A location 16.0 ft from the end bearing is investigated as the transition point from the 1¾-in. flange to the 1-in. flange. Stresses are calculated and compared with the allowable fatigue stress at the butt weld joining the flange plates. The comparison indicates the transition point is satisfactory.

Try cutoff 16.0 ft from end bearing.

### Bending Moments

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$LL+I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$, kip-ft</td>
<td>460</td>
<td>85</td>
<td>730</td>
</tr>
</tbody>
</table>

### Steel Stresses—Combination A

**Top of Steel (Compression)**

$$DL_1: \sigma_b = \frac{460 \times 12}{477} = 11.57$$

$$DL_2: \sigma_b = \frac{85 \times 12}{1,993} = 0.51$$

$$LL+I: \sigma_b = \frac{730 \times 12}{6,642} = 1.32$$

**Bottom of Steel (Tension)**

$$f_b = \frac{460 \times 12}{641} = 8.61$$

$$f_b = \frac{85 \times 12}{830} = 1.23$$

$$f_b = \frac{730 \times 12}{891} = 9.83$$

### Fatigue Check at Butt Weld

Ratio of minimum to maximum stress in bottom flange at weld:

$$R_{Bot} = \frac{(8.61+1.23)}{19.67} = 0.500$$

The allowable fatigue stress is

$$F_r = \frac{17.2}{1 - 0.62 \times 0.500} = 24.93 \text{ ksi} > 20$$

For simple spans, fatigue seldom governs at flange transition locations.

### DESIGN OF SHEAR CONNECTORS

The stepped shear-connector spacing diagram is contained within the theoretical spacing curve based on service behavior. The allowable loads on the 3⁄8-in.-dia., 4-in.-high, stud shear connectors are determined by the AASHO Specification formulas for an $H/d$ ratio greater than 4.0.

Concrete: $f'_c = 4,000$ psi; $n = 8$

Studs: 3⁄8-in.-dia., 4-in.-high; $H/d = 4.0/0.875 = 4.6 > 4.0$

For $H/d > 4$, AASHO Specifications give the ultimate strength of a shear connector as

$$Q_u = 0.93 d^2 \sqrt{f'_c} = 0.93 (0.875)^2 \sqrt{4,000} = 45.0 \text{ kips per stud}$$

With $a$ given as 10.6 for 500,000 cycles of load in AASHO Specifications, the load range per shear connector is

$$Z_r = ad^2 = 10.6 (0.875)^2 = 8.11 \text{ kips per stud}$$
DETAIL AT SHEAR CONNECTOR

Shear Connectors—Strength Requirements

At midspan, the maximum compressive stress in the concrete is

\[ H_1 = A_f F_v = 42.75 \times 36.0 = 1,539 \text{ kips (governs)} \]

\[ H_2 = 0.85 f_y b t = 0.85 \times 4.0 \times 84.0 \times 7.0 = 1,999 \text{ kips} \]

Number of studs required between midspan and each support is

\[ N = \frac{H_1}{\phi Q_s} = \frac{1,539}{0.85 \times 45.0} = 40.2 \]

At transition in girder section:

\[ H_1 = A_f F_v = 33.75 \times 36.0 = 1,215 \text{ kips (governs)} \]

\[ H_2 = 1,999 \text{ kips} \]

Number of studs required between transition in girder section and support is

\[ N = \frac{H_1}{\phi Q_s} = \frac{1,215}{0.85 \times 45.0} = 31.8 \]

Shear-Connector Spacing for Service Behavior (Fatigue)

At support, shear range = \( V_s = 59.7 - 0.0 = 59.7 \) kips \((LL \text{ only})\)

For \( n = 8 \), the horizontal shear per linear inch is

\[ S_r = \frac{V_s Q}{I} = \frac{59.7(73.5 \times 10.36)}{38,920} = 1.168 \text{ kips per in.} \]

Spacing required \((3 \text{ studs})\) = \( \frac{3Z}{S_r} = \frac{3(8.11)}{1.168} = 20.8 \text{ in.} \)
Shear-connector spacing required at each tenth point is computed and the theoretical spacing envelope is drawn. The actual stepped spacing diagram is constructed and provides 72 studs between each support and midspan. It also provides 33 studs between each girder transition and the adjacent support. Since these numbers of studs exceed those required for strength, the spacing diagram is acceptable.

**SHEAR-CONNECTOR SPACING**

**DESIGN OF FLANGE-TO-WEB WELD**

The fillet weld connecting the girder web to the flange must resist the horizontal shear between flange and web. The weld is designed for the maximum shear at the end of the span, and is governed by loading combination C. In many cases, weld size will be determined by the minimum allowable fillet welds for the material thicknesses. In the present design example, the thickness of the flange plates controls the weld size.

### End Shears

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$LL + I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$, kips</td>
<td>36.0</td>
<td>6.6</td>
<td>59.7</td>
</tr>
</tbody>
</table>

**Steel Section Only**

- $Q_{Top \ filk} = 6.75 \times 28.13 = 190 \text{ in.}^3$
- $Q_{Bot \ filk} = 12.00 \times 20.65 = 249 \text{ in.}^3$

**Composite Section, $n = 8$**

- $Q_{Top \ filk} = 6.75 \times 5.88 = 38$
- $Q_{Conc.} = 73.50 \times 10.36 = 761$
- $\frac{Q_{Bot \ filk}}{799} = 12.00 \times 43.20 = 513 \text{ in.}^3$
Shear, Kips per In. = \( S = \frac{VQ}{I} \)

**Top Weld**

\[
DL_1: \quad S = \frac{36.0 \times 190}{13,560} = 0.504
\]

\[
DL_2: \quad S = \frac{6.6 \times 799}{38,920} = 0.135
\]

\[
LL_1 + I: \quad S = \frac{59.7 \times 799}{38,920} = 1.226
\]

1.865 kips per in.

**Bottom Weld**

\[
DL_1: \quad S = \frac{36.0 \times 249}{13,560} = 0.661
\]

\[
DL_2: \quad S = \frac{6.6 \times 513}{38,920} = 0.087
\]

\[
LL_1 + I: \quad S = \frac{59.7 \times 513}{38,920} = 0.787
\]

1.535 kips per in.

Top weld governs

**Allowable stress on weld:**

Fatigue: 500,000 cycles

Ratio of minimum to maximum stress in the top weld is

\[ R = \frac{0.639}{1.865} = 0.343 \]

The allowable fatigue stress is

\[ f_a = \frac{10.8}{1 - 0.55R} = \frac{10.8}{1 - 0.55 \times 0.343} = 13.3 \text{ ksi} > 12.4 \text{ ksi}. \text{ Use } 12.4 \text{ ksi} \]

**Weld Size:**

With a weld on each side of the web, the shear per weld is

\[ S = \frac{1.865}{2} = 0.933 \text{ kips per in.} \]

Allowable load on weld = \( 12.4 \times 0.707 = 8.76 \) kips per in.

Weld size required = \( \frac{0.933}{8.76} = 0.106 \) in., say \( \frac{1}{8} \) in.

Use minimum weld for material thickness as required by AASHO Specifications.

For Flg. 12 x 3\%, use two \( \frac{3}{4} \)-in. fillets.

For Flg. 12 x 1, use two \( \frac{5}{16} \)-in. fillets.

For Flg. 12 x 1\%, use two \( \frac{3}{8} \)-in. fillets.

**BEARING STIFFENERS**

Bearing stiffeners are required at all bearing locations of the welded, composite plate girder. Section properties are computed for these stiffener plates acting in combination with a portion of the web and considered as a column. The bearing stress between stiffener and flange is checked and compared with the allowable value. The compressive column stress is compared with the allowable compressive stress specified for concentrically loaded columns, with the radius of gyration calculated about the axis through the centerline of the web plate.

In accordance with AASHO Specifications, the effective area in bearing is considered to be only the portions of the stiffeners outside the flange-to-web plate welds.
END BEARING STIFFENERS

End Reaction

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$LL+I$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$, kips</td>
<td>36.0</td>
<td>6.6</td>
<td>59.7</td>
<td>102.3</td>
</tr>
</tbody>
</table>

Length of web acting with stiffeners is

$L_w = 18 \times 0.313 = 5.63$ in.

For stiffeners, assume 2 plates $5 \times \frac{1}{4}$ in. with $I_v = \frac{0.5 \times 5^3}{12} = 5.2$ in.$^4$

Minimum thickness, in., required for stiffeners is

$t = \frac{1}{12} b \sqrt{\frac{F_v}{33,000}} = 0.436 < \frac{1}{2}$

For the total end reaction on the effective stiffener area:

Bearing stress $= \frac{102.3}{(5.00 - 0.25) \times \frac{1}{2} \times 2} = 21.54$ ksi $< 29$ ksi (allowable)

Equivalent Column at Bearing

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>$Ad^2$</th>
<th>$I_o$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Plates $5 \times \frac{1}{4}$ Web $5.63 \times 0.313$</td>
<td>5.00</td>
<td>2.65</td>
<td>13.25</td>
<td>35.11</td>
<td>10.4</td>
<td>45.5</td>
</tr>
</tbody>
</table>

$r = \sqrt{\frac{45.5}{6.76}} = 2.59$ in. \hspace{2cm} D = \frac{48}{2.59} = 18.53$

$F_{allow} = 16,000 - 0.30 (18.53)^2 = 15.90$ ksi

$f_a = \frac{102.3}{6.76} = 15.13$ ksi $< 15.90$ ksi

Use 2 plates $5 \times \frac{1}{4}$ in.

INTERMEDIATE TRANSVERSE STIFFENERS

Intermediate transverse stiffeners are required to resist buckling of the web. The minimum allowable web thickness without stiffeners is calculated and found to exceed the $\frac{3}{16}$-in. actual web thickness. Hence, intermediate stiffeners are required.

Stiffener spacing at the end of the span is governed by web shearing stress and is well below the maximum permissible value of 48 in., the clear distance between flanges.

A check is made to determine whether stiffeners may be omitted in regions of lower shearing stress. Stiffeners are unnecessary where the shear is below 35.9 kips, a zone extending 4 ft on either side of midspan. Since a diaphragm is located at midspan, however, no stiffeners are actually omitted.
Stiffener Spacing near Supports

At the end reaction, the average shearing stress in the web is

\[ f_v = \frac{102.3}{48 \times 5\%} = 6.82 \text{ ksi} \]

Min. web thickness without stiff. = \[ \frac{48 \sqrt{6,820}}{7,500} = 0.526 > \frac{5}{32} \text{ in.} \] Stiffeners are required.

Stiffener spacing required near the supports is

\[ d_{\text{end}} = \frac{11,000t}{\sqrt{f_v}} = \frac{11,000 \times 0.313}{\sqrt{6,820}} = 3.443 \times 82.58 = 41.69 \text{ in. Use 40 in.} \]

\[ d_{\text{max}} = \text{clear distance between flanges} = 48 \text{ in.} \]

Note: First two spaces from bearing stiffeners should be \( \frac{3}{4} \) the calculated spacing, or 20 in.

Location Where Transverse Stiffeners May Be Omitted

Stiffeners may be omitted where the web thickness is

\[ t \geq \frac{D \sqrt{f_v}}{7,500} \]

From this, it follows that stiffeners may be omitted where the shearing stress is

\[ f_v \leq \left( \frac{7,500t}{D} \right)^2 = \left( \frac{7,500 \times 0.313}{48} \right)^2 = 2.39 \text{ ksi} \]

\[ V = 2.39 \times (48 \times \frac{5}{32}) = 35.9 \text{ kips} \]

\[ \text{. . . Where shear} \leq 35.9 \text{ kips, a distance 4 ft on either side of midspan, stiffeners may be omitted.} \]

Actual Spacing of Intermediate Stiffeners

The required stiffener spacing is calculated at intervals along the span, and a curve of required spacing is plotted. Since the maximum permissible spacing equals the depth of the web, calculations need be carried only from the supports to locations where the required spacing exceeds the depth of the web. The actual spacing diagram is enclosed within the theoretical envelope. Stiffeners are placed at diaphragms to serve as connection plates.

Size of Intermediate Stiffeners

The transverse intermediate stiffeners are proportioned to provide a moment of inertia of at least that given by the AASHO formula previously discussed. Stiffeners are attached to only one side of the web, and the moment of inertia is taken about the face of the web plate. A \( 4 \times \frac{5}{32} \)-in. stiffener plate satisfies minimum width and minimum thickness limitations and provides considerably more moment of inertia than required.

The required moment of inertia of the stiffeners is determined for the required spacing \( d = 41.69 \text{ in.} \) and actual spacing \( d_s = 40 \text{ in.} \), with clear unsupported depth of web \( D = 48 \text{ in.} \).
STIFFENER SPACING

Required $I$ of Stiffener

\[ I = \frac{d^2 J}{10.92} \quad \text{with} \quad J = \frac{25D^2}{d^2} - 20 > 5 \]

\[ J = 25 \times \frac{(48)^2}{(41.69)^2} - 20 = 13 > 5 \]

\[ I = \frac{40 \times (0.313)^2 \times 13}{10.92} = 1.46 \text{ in}^4 \]

Stiffener Dimensions

For stiffener, assume one plate $4 \times \frac{5}{16} \text{ in.}$

\[ I = \frac{0.313 \times (4)^2}{3} = 6.68 \text{ in}^4 > 1.46 \]

Min. width = $2 + \frac{48}{30} = 3.6 \text{ in.} < 4 \text{ in.}$

Min. width = $\frac{12}{4} - 3 \text{ in.} < 4 \text{ in.}$

Min. $t_s = \frac{1}{16} \times 4 = \frac{1}{4} \text{ in.} < \frac{5}{16} \text{ in.}$

The $4 \times \frac{5}{16}$-in. plate is satisfactory.
DEFLECTIONS

The deflections due to dead load are computed at the quarter points and at the centerline of span, by the theoretical deflection formula for uniform load and constant moment of inertia. Appropriate moment of inertia values are used for $DL_1$ and $DL_2$.

$$w$$

$$aL$$

$$L$$

$$bL$$

Deflections Due to Dead Load

$DL_1$: $w = 0.900$ kips per ft

$DL_2$: $w = 0.165$ kips per ft

$$\Delta = \frac{72wL^4}{E_sI}(1 + ab)$$

where $\Delta =$ deflection, in., at distance $aL$ from support

$b = 1 - a$

$w =$ dead load, kips per ft

$L =$ span, ft

$E_s = 29 \times (10)^3$ ksi

$I =$ moment of inertia at midspan, in.$^4$

Midspan deflection, in. = $\frac{45wL^4}{2E_sI}$

Deflections Under $DL_1$

$I_s = 16,880$ in.$^4$

At midspan: $\Delta = \frac{45 \times 0.900 \times (80)^4}{2 \times 29 \times (10)^3 \times 16,880} = 1.70$ in.

At $a = 0.25$: $\Delta = \frac{72 \times 0.900 \times (80)^4}{29 \times (10)^3 \times 16,880} \times 0.25 \times 0.75[1 + 0.25(0.75)] = 1.21$ in.

Deflections Under $DL_2$

$I_{z1} = 38,790$ in.$^4$

At midspan: $\Delta = \frac{45 \times 0.165 \times (80)^4}{2 \times 29 \times (10)^3 \times 38,790} = 0.13$ in.

At $a = 0.25$: $\Delta = \frac{72 \times 0.165 \times (80)^4}{29 \times (10)^3 \times 38,790} \times 0.25 \times 0.75[1 + 0.25(0.75)] = 0.10$ in.
CAMBER DIAGRAM

Total Camber at Midspan
1.70 + 0.13 = 1.83 in. Use 1 3/4 in.

Total Camber at 1/4 Pt.
1.21 + 0.10 = 1.31 in. Use 1 1/4 in.

Deflection Due to Live Load + Impact

Live-load deflections are limited to 1/800 of the span. They are computed for two lanes of truck loading distributed equally to the four girders, since the exterior girders have the same section as the interior girders. Computations show that live-load deflections are well within the allowable limits.

\[ \Delta = \frac{324}{E' I} P_r (L^4 - 555L + 4,780) \]

where \( \Delta \) = midspan deflection, in.
\( P_r \) = concentrated load, kips, on four girders
\( = \) weight of front truck wheels \( \times \) distribution factor, plus impact, kips
\( I \) = moment of inertia at midspan, in.\(^4\)
\( E' \) = 29 \times 10^3 ksi
\( L \) = span, ft

Assume that two lanes of live-load (four wheels abreast) plus 24.4% impact are equally distributed over four girders. Then,

\[ P_r = 4 \times 4 \times 1.244 = 19.9 \text{ kips} \]
\[ I_s = 4 \times 55,040 = 220,160 \text{ in.}^4 \]

\[ \Delta = \frac{324 \times 19.9 [(80)^3 - 555(80) + 4,780]}{29 \times (10)^3 \times 220,160} = \frac{324 \times 19.9 \times 472,380}{29 \times (10)^3 \times 220,160} = 0.48 \text{ in.} \]

The ratio of live-load deflection to span is

\[ \frac{0.48}{80 \times 12} = \frac{1}{2,000} < \frac{1}{800} \]

FINAL DESIGN

An elevation of the 80-ft simple-span, composite, welded girder is shown on the next page.
Design II—Two-Span Continuous Girder (100-100 Ft), Composite For Positive Moment Only

LOADS, SHEARS AND MOMENTS

The design procedure for a two-span, continuous, composite, welded plate girder in the positive-moment portion of the span is similar to the procedure for simple spans. Negative moments are assumed to be carried solely by the welded girder in this example.

$DL_1$ is calculated as the weight of the 7-in.-thick concrete slab and an assumed weight of 0.170 kips per ft for the steel girder and framing details. The weight of the concrete curbs and steel railings makes up the superimposed dead load, $DL_2$. Live-load is the standard AASHO HS20-44 truck loading distributed to each girder, and impact is calculated for a 100-ft span. The typical cross section is the same as that on p. II/4.6.

Dead Load Carried by Steel

Slab = $7/12 \times 8.33 \times 0.150 = 0.730$

Steel girder, details and conc. haunch = $0.170$

$DL_1$ per girder = $0.900$ k/ft

Dead Load Carried by Composite Section

Curbs and railing, $DL_2 = 0.660$ k/ft

$DL_2$ per girder = $0.660/4 = 0.165$ k/ft

Live Load

Live-load distribution = $\frac{S}{5.5} - \frac{8.33}{5.5} = 1.51$ wheels = $0.755$ axle

Impact = $\frac{50}{100+125} = 0.222$

Curves of maximum moments and maximum shears are computed for the two-span continuous girders with constant moment of inertia.
MAXIMUM-SHEAR CURVES—CONSTANT I
DESIGN OF GIRDER SECTION

The negative-moment region of the girder is designed first, to set the over-all controlling proportions of the section. This step is followed by design of the section for maximum positive moment and the section at the dead-load inflection point.

A 4-ft-deep web satisfies the minimum allowable depth-to-span ratios prescribed by AASHO Specifications. For the maximum negative-moment region, a symmetrical section composed of $16 \times 1\frac{3}{4}$-in. flange plates and a $48 \times \frac{5}{16}$-in. web plate is investigated. Computations indicate that the flexural stress in the bottom flange and shearing stress in the web are within allowable limits.

**Maximum Shearing Stress at Interior Support**

\[ DL_1: V = 56.2 \]
\[ DL_2: V = 10.3 \]
\[ LL + I: V = 62.4 \]
\[ 128.9 \text{ kips} \]

\[ f_s = \frac{128.9}{48 \times \frac{5}{16}} = 8.58 \text{ ksi} < 12 \]

**MAXIMUM NEGATIVE MOMENT AT INTERIOR SUPPORT**

Steel Section

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flanges 2 - $16 \times 1\frac{3}{4}$</td>
<td>56.00</td>
<td>24.88</td>
<td>1,393</td>
<td>34,660</td>
<td>2,880</td>
<td>34,660</td>
</tr>
<tr>
<td>Web $48 \times \frac{5}{16}$</td>
<td>16.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ 71.00 \text{ in.}^2 \quad \quad \quad I_{NA} = 37,540 \text{ in.}^4 \]

\[ Z_{Top} = Z_{Bot} = \frac{37,540}{25.75} = 1,458 \text{ in.}^3 \]

From the maximum-moment curves, the total moment at the interior support is $-2,387 \text{ kip-ft.}$

Maximum stress \[ f_b = \frac{-2,387 \times 12}{1,458} = 19.65 \text{ ksi} \]

Resisting moment for 20-ksi allowable stress, \[ M_R = \frac{20 \times 1,458}{12} = 2,430 \text{ kip-ft} \]

**Allowable Compressive Stress for 20-Ft Diaphragm Spacing**

\[ F_b = 20,000 - 7.5 \left( \frac{20}{16} \right)^2 = 20,000 - 7.5 \left( \frac{20 \times 12}{16} \right)^2 = 18.31 \text{ ksi} \]

Because of continuity, AASHO Specifications permit a 20% increase in allowable stresses, up to 20 ksi, at the interior support.

\[ F_b' = 1.20 \times 18.31 = 21.97 \text{ ksi}. \text{ Use } 20 \text{ ksi.} \]

The resisting moment remains unchanged, 2,430 kip-ft.
MAXIMUM POSITIVE MOMENT

A steel section consisting of a 12 \times \frac{1}{2}\text{-in.} top flange, a 48 \times \frac{3}{16}\text{-in.} web and a 12 \times 1\frac{1}{8}\text{-in.} bottom flange is investigated for the region of maximum positive moment. Properties are computed for the steel section alone, the composite section with \( n \) equal to 8, and the composite section with \( 3n \) equal to 24.

POSITIVE-MOMENT SECTION

Steel Section

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>( d )</th>
<th>( Ad )</th>
<th>( Ad^2 )</th>
<th>( I_o )</th>
<th>( I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Flg. 12 \times \frac{1}{2}</td>
<td>6.00</td>
<td>24.25</td>
<td>+146</td>
<td>3,540</td>
<td>3,540</td>
<td></td>
</tr>
<tr>
<td>Web 48 \times \frac{3}{16}</td>
<td>15.00</td>
<td>24.81</td>
<td>-484</td>
<td>12,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bot. Flg. 12 \times 1\frac{1}{8}</td>
<td>19.50</td>
<td>40.50</td>
<td>-338</td>
<td>-33,800</td>
<td></td>
<td>15,600</td>
</tr>
</tbody>
</table>

\[ d = \frac{-338}{40.50} = -8.35 \text{ in.} \]

\[ d_{\text{Top of steel}} = 24.50 + 8.35 = 32.85 \text{ in.} \]

\[ d_{\text{Bot. of steel}} = 25.63 - 8.35 = 17.28 \text{ in.} \]

\[ Z_{\text{Top of steel}} = \frac{15,600}{32.85} = 475 \text{ in.}^3 \]

\[ Z_{\text{Bot. of steel}} = \frac{15,600}{17.28} = 903 \text{ in.}^3 \]
Composite Section, $3n = 24$

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>$Ad^2$</th>
<th>$I_o$</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>40.50</td>
<td>24.50</td>
<td>30.38</td>
<td>-338</td>
<td>+744</td>
<td>22,610</td>
</tr>
<tr>
<td>Conc. $84 \times 7/24$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$$d_s = \frac{406}{65.00} = 6.25 \text{ in.}$$

$$d_{Top \ of \ steel} = 24.50 - 6.25 = 18.25 \text{ in.}$$

$$d_{Bot \ of \ steel} = 25.63 + 6.25 = 31.88 \text{ in.}$$

$$Z_{Top \ of \ steel} = \frac{38,590}{18.25} = 2,115 \text{ in}^3$$

$$Z_{Bot \ of \ steel} = \frac{38,590}{31.88} = 1,210 \text{ in}^3$$

Composite Section, $n = 8$

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>$Ad^2$</th>
<th>$I_o$</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>40.50</td>
<td>73.50</td>
<td>30.38</td>
<td>-338</td>
<td>+2,233</td>
<td>67,840</td>
</tr>
<tr>
<td>Conc. $84 \times \frac{3}{8}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$$d_s = \frac{1,895}{114.00} = 16.62 \text{ in.}$$

$$d_{Top \ of \ steel} = 24.50 - 16.62 = 7.88 \text{ in.}$$

$$d_{Bot \ of \ steel} = 25.63 + 16.62 = 42.25 \text{ in.}$$

$$Z_{Top \ of \ steel} = \frac{55,070}{7.88} = 6,989 \text{ in}^3$$

$$Z_{Bot \ of \ steel} = \frac{55,070}{42.25} = 1,303 \text{ in}^3$$

$$d_{Top \ of \ conc.} = 7.88 + 9.37 = 17.25 \text{ in.}$$

$$Z_{Top \ of \ conc.} = \frac{55,070}{17.25} = 3,191 \text{ in}^3$$

Check of Compression-Flange Thickness

Stresses are checked at the top and bottom of steel, and at the top of concrete. The maximum allowable $b/t$ ratio for the flanges is calculated for the compressive stress. The calculations show that the $12 \times \frac{3}{4}$-in. top flange satisfies the requirements. The concrete stress is well within the allowable for compression.
Bending Moments 40 Ft from End Support

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$LL+I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$, kip-ft</td>
<td>630</td>
<td>116</td>
<td>1,136</td>
</tr>
</tbody>
</table>

**Steel Stresses—Combination A**

**Top of Steel (Compression)**

$DL_1$: $f_b = \frac{630 \times 12}{475} = 15.92$ ksi

$DL_2$: $f_b = \frac{116 \times 12}{2,115} = 0.66$ ksi

$LL+I$: $f_b = \frac{1,136 \times 12}{6,989} = 1.95 \frac{\text{ksi}}{18.53}$

Bottom of Steel (Tension)

$DL_1$: $f_b = \frac{630 \times 12}{903} = 8.37$ ksi

$DL_2$: $f_b = \frac{116 \times 12}{1,210} = 1.15$ ksi

$LL+I$: $f_b = \frac{1,136 \times 12}{1,303} = 10.46 \frac{\text{ksi}}{19.98}$

Check: $\frac{b}{t} = \frac{3,250}{\sqrt{18,530}} = 23.9 < 24$

$t = \frac{12}{23.9} = 0.502$ in.

Use $\frac{3}{8}$-in. top flange.

**Concrete Stresses—Combination B**

**Top of Concrete (Compression)**

$DL_1$: $f_b = \frac{116 \times 12}{3,191 \times 8} = 0.055$ ksi

$LL+I$: $f_b = \frac{1,136 \times 12}{3,191 \times 8} = 0.534 \frac{\text{ksi}}{0.589}$

**SECTION NEAR DEAD-LOAD INFLECTION POINT**

A section consisting of two $12 \times \frac{3}{4}$-in. flange plates and the $48 \times \frac{5}{16}$-in. web is investigated at the dead-load inflection point, about 25 ft from the interior support. Composite action is not considered to be developed at this location. Stresses are computed on the basis of the steel section alone, and compared with the allowable fatigue stress at the butt-welded field splice to be made at the inflection point. Since no change in plate thickness is anticipated, the allowable fatigue stress for base metal governs.

**Steel Section**

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>Iₜ</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flanges 2—$12 \times \frac{3}{4}$</td>
<td>18.00</td>
<td>24.38</td>
<td>439</td>
<td>10,700</td>
<td>10,700</td>
<td></td>
</tr>
<tr>
<td>Web $48 \times \frac{5}{16}$</td>
<td>15.00</td>
<td></td>
<td></td>
<td></td>
<td>2,880</td>
<td>2,880</td>
</tr>
</tbody>
</table>

$33.00 \text{ in.}^2 \quad I_{NA} = 13,580 \text{ in.}^4$

$Z_{\text{Top}} = Z_{\text{Bot}} = \frac{13,580}{24.75} = 549 \text{ in.}^3$
Fatigue Stress Near Inflection Point

Maximum Positive Moment

\[ DL_{1} + DL_{2}: M = 0 \text{ kip-ft} \]
\[ LL + I: M = +650 \text{ kip-ft} \]

Maximum Negative Moment

\[ DL_{1} + DL_{2}: M = 0 \text{ kip-ft} \]
\[ LL + I: M = -460 \text{ kip-ft} \]

Actual stress in top and bottom flanges is

\[ f_{b} = \frac{650 \times 12}{549} = 14.21 \text{ ksi} \]

Ratio of minimum to maximum stress in butt-welded flange splice is

\[ R = \frac{-460}{650} = -0.708 \]

The allowable fatigue stress in tension is

\[ F_{a} = \frac{20.5}{1 - 0.55(-0.708)} = 14.76 \text{ ksi} > 14.21 \text{ ksi} \]

The allowable fatigue stress in compression is

\[ F_{c} = \frac{19.8}{1 - \left( \frac{19.8}{13.3} \right)(-0.708)} = 14.71 \text{ ksi} > 14.21 \text{ ksi} \]

The 12 x 3/4-in. flanges are satisfactory. Good design practice, however, generally limits changes in flange-plate thickness to not more than 50 percent of the thickness of the heavier plate. For this reason, although the 3/4-in. thick flange satisfies all stress requirements, as indicated above, the flange thickness throughout the vicinity of the inflection point is increased to 7/8-in., half the thickness of the adjacent maximum-negative-moment flange. The section modulus then becomes \( Z = 620 \text{ in.}^{3} \).

LOCATION OF FLANGE-PLATE TRANSITIONS

In the negative-moment, noncomposite region extending from the interior pier to the inflection point, usually only one, but sometimes two, changes in section are made. It is desirable to maintain the flange thickness across the butt-welded field splice at the inflection point to take advantage of the higher allowable fatigue stress for base metal. If the flange widths at the maximum-positive-moment section and maximum-negative-moment section are the same, then a single change in section in the negative-moment region is made. The thickness is reduced at a location where the stress level permits. If the flange width at the maximum-negative-moment section is greater than at the maximum-positive-moment section, then in addition to the thickness transition, a width transition is made at the field splice. In the present example, both thickness and width transitions are necessary.

To determine the transition point from the 1 3/4-in.-thick flanges to the 7/8-in.-thick flanges in the negative-moment region, the resisting moment of the weaker section is calculated and plotted upon the curve of maximum negative moment. The transition point is the location at which the resisting moment is equal to the actual moment, a distance of about 13 ft from the interior pier.

The transition in width from the 16-in. flange of the negative-moment section to the 12-in. flange of the positive-moment section takes place at the butt-welded field splice, 25 ft from the interior pier. The plates are tapered on a 2-ft radius, in accordance with specifications as explained in the Introduction. If the butt weld is ground and radiographed, it may be treated as base metal in calculating the allowable fatigue stress for the splice.
NEGATIVE-MOMENT TRANSITION SECTION

Steel Section

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flanges 2 – 16 x ¾</td>
<td>28.00</td>
<td>15.00</td>
<td>24.44</td>
<td>684</td>
<td>16,720</td>
<td>2,880</td>
</tr>
<tr>
<td>Web 48 x 5/32</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

43.00 in.²

\[ I_{NA} = 19,600 \text{ in.}^4 \]

\[ Z_{Top} = Z_{Bot} = \frac{19,600}{24.88} = 788 \text{ in.}^3 \]

The allowable compressive stress in the bottom flange with diaphragms 20 ft apart is

\[ F_s = 20,000 - 7.5 \left( \frac{L}{b} \right)^2 = 20,000 - 7.5 \left( \frac{20 \times 12}{16} \right)^2 = 18.31 \text{ ksi} \]

Resisting moment,

\[ M_R = \frac{18.31 \times 788}{12} = 1,202 \text{ kip-ft} \]

The transition point is selected 13 ft from the interior support, where the total negative moment is about 1,150 kip-ft.

Stress in Negative-Moment Transition Section

Stresses are computed at the transition point. Also, the allowable fatigue stress is calculated adjacent to the butt weld joining the flange plates. The stress limitation for lateral buckling of the compression flange actually governs.

Maximum Moments 13 Ft from Interior Support

With Positive Live-Load Moment

\[ DL_1: M = -465 \]
\[ DL_2: M = -90 \]
\[ LL+I: M = +255 \]
\[ LL+I: M = +255 \text{ kip-ft} \]

With Negative Live-Load Moment

\[ DL_1: M = -465 \]
\[ DL_2: M = -90 \]
\[ LL+I: M = -570 \]
\[ LL+I: M = -570 \text{ kip-ft} \]

\[ LL+I: M = -1,125 \text{ kip-ft} \]
Actual stress in top and bottom flanges is

\[ f_b = \frac{1.125 \times 12}{788} = 17.13 \text{ ksi} \]

Ratio of minimum to maximum stress in butt-welded flange splice is

\[ R = \frac{-300}{-1,125} = 0.267 \]

The allowable fatigue stress in tension is

\[ F_r = \frac{17.2}{1 - 0.62(0.267)} = 20.61 \text{ ksi} > 17.13 \text{ ksi} \]

The allowable fatigue stress in compression is

\[ F_r = \frac{19.8}{1 - (19.8/10.6 - 1)(0.267)} = 25.77 \text{ ksi} > 17.13 \text{ ksi} \]

Lateral buckling governs, \( F_b = 18.31 \text{ ksi} \). Since the actual stress is smaller, the 16 x \( \frac{3}{8} \)-in. flanges are satisfactory.

**TRANSITION SECTION 70 FT FROM END SUPPORT**

The transition locations for the flange plates in the positive-moment region are determined by trial. For butt-welded transitions, static strength rather than fatigue limitations will normally control the locations at which flange thicknesses can be changed, except near inflection points, where fatigue limitations may control. A point 70 ft from the end bearing is investigated as a location for the transition from the 12 x \( \frac{3}{4} \)-in. top and bottom flange plates to the 12 x \( \frac{3}{4} \)-in. top flange and 12 x \( \frac{1}{8} \)-in. bottom flange of the positive-moment section. Since tensile stress in the bottom flange controls this transition, properties are calculated for the section with the thinner bottom flange and stresses are checked.

### Steel Section

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>( L_i )</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flanges 2 - 12 x ( \frac{3}{8} )</td>
<td>21.00</td>
<td>24.44</td>
<td>513</td>
<td>12,540</td>
<td>2,880</td>
<td>12,540</td>
</tr>
<tr>
<td>Web 48 x ( \frac{5}{16} )</td>
<td>15.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ 36.00 \text{ in.}^2 \]

\[ I_{NA} = 15,420 \text{ in.}^4 \]

\[ Z_{Top} = Z_{Bot} = \frac{15,420}{24.88} = 620 \text{ in.}^3 \]
Composite Section, $3n = 24$

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad$^2$</th>
<th>I&lt;sub&gt;s&lt;/sub&gt;</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>36.00</td>
<td>30.38</td>
<td>+744</td>
<td>22,610</td>
<td>100</td>
<td>15,420</td>
</tr>
<tr>
<td>Conc. 84 × 7/24</td>
<td>24.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>22,710</td>
</tr>
</tbody>
</table>

$d_{nt} = \frac{744}{60.50} = +12.30$ in.

$+744$ in.$^3$ 

$-12.30 \times 744 = -9,150$

$I_{NA} = 28,980$ in.$^4$

$d_{Top\ of\ steel} = 24.88 - 12.30 = 12.58$ in.

$d_{Bot\ of\ steel} = 24.88 + 12.30 = 37.18$ in.

$Z_{Top\ of\ steel} = \frac{28,980}{12.58} = 2,304$ in.$^3$

$Z_{Bot\ of\ steel} = \frac{28,980}{37.18} = 779$ in.$^3$

Composite Section, $n = 8$

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad$^2$</th>
<th>I&lt;sub&gt;s&lt;/sub&gt;</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>36.00</td>
<td>73.50</td>
<td>2,233</td>
<td>67,840</td>
<td>300</td>
<td>15,420</td>
</tr>
<tr>
<td>Conc. 84 × ¾</td>
<td></td>
<td>30.38</td>
<td></td>
<td></td>
<td></td>
<td>68,140</td>
</tr>
</tbody>
</table>

$d_{nt} = \frac{2,233}{109.50} = 20.39$ in.

$+2,233$ in.$^3$ 

$-20.39 \times 2,233 = -45,530$

$I_{NA} = 38,030$ in.$^4$

$d_{Top\ of\ steel} = 24.88 - 20.39 = 4.49$ in.

$d_{Bot\ of\ steel} = 24.88 + 20.39 = 45.27$ in.

$Z_{Top\ of\ steel} = \frac{38,030}{4.49} = 8,470$ in.$^3$

$Z_{Bot\ of\ steel} = \frac{38,030}{45.27} = 840$ in.$^3$

Steel Stresses Under Maximum Positive Moment—Combination A

Bending Moments 70 Ft from End Support

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$LL+I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$, kip-ft</td>
<td>158</td>
<td>29</td>
<td>786</td>
</tr>
</tbody>
</table>

Top of Steel (Compression)

$DL_1$: $f_b = \frac{158 \times 12}{620} = 3.06$

$DL_2$: $f_b = \frac{29 \times 12}{2,304} = 0.15$

$LL+I$: $f_b = \frac{786 \times 12}{8,470} = 1.11 \times 4.32$ ksi

Bottom of Steel (Tension)

$DL_1$: $f_b = \frac{158 \times 12}{620} = 3.06$

$DL_2$: $f_b = \frac{29 \times 12}{779} = 0.45$

$LL+I$: $f_b = \frac{786 \times 12}{840} = 11.23 \times 14.74$ ksi

II/4.3
Steel Stresses Under Maximum Negative Moment—Combination A

<table>
<thead>
<tr>
<th>Bending Moments 70 Ft from End Support</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>( M, \text{ kip-ft} ) &amp; ( DL_1 ) &amp; ( DL_2 ) &amp; ( LL + I ) &amp; ( \text{Total} )</td>
</tr>
<tr>
<td>---------------------------------------</td>
</tr>
<tr>
<td>158</td>
</tr>
</tbody>
</table>

Actual stress in steel section alone is

\[
f_s = \frac{239 \times 12}{620} = 4.63 \text{ ksi}
\]

Fatigue Stress at Butt Weld 70 Ft from End Support

Ratio of minimum to maximum stress in bottom flange is

\[
R = \frac{-4.63}{14.74} = -0.314
\]

The allowable fatigue stress in tension is

\[
F_r = \frac{17.2}{1 - 0.62(-0.314)} = 14.40 \text{ ksi} = 14.74 \text{ ksi}
\]

Compression is not critical.

TRANSITION SECTION 16 FT FROM END SUPPORT

Another positive-moment transition is located 16 ft from the end support by trial. The 1/4-in.-thick flange is reduced there to 3/4 in. Properties are calculated for the end section of the girder, and stresses computed. Again, bending stress not fatigue in the bottom flange governs.

### Steel Section

<table>
<thead>
<tr>
<th>Material</th>
<th>( A )</th>
<th>( d )</th>
<th>( Ad )</th>
<th>( Ad^2 )</th>
<th>( I_o )</th>
<th>( I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Flange 12×1/2</td>
<td>6.00</td>
<td>24.25</td>
<td>+146</td>
<td>3,540</td>
<td>2,880</td>
<td>3,540</td>
</tr>
<tr>
<td>Web 45×3/16</td>
<td>15.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bot. Flange 12×3/4</td>
<td>10.50</td>
<td>-24.44</td>
<td>-257</td>
<td>6,270</td>
<td>6,270</td>
<td></td>
</tr>
</tbody>
</table>

\( d_s = \frac{-111}{31.50} = -3.52 \text{ in.} \)

\( I_{NA} = 12,300 \text{ in.}^4 \)

\( d_{\text{Top of steel}} = 24.50 + 3.52 = 28.02 \text{ in.} \)

\( d_{\text{Bot. of steel}} = 24.88 - 3.52 = 21.36 \text{ in.} \)

\( Z_{\text{Top of steel}} = \frac{12,300}{28.02} = 439 \text{ in.}^2 \)

\( Z_{\text{Bot. of steel}} = \frac{12,300}{21.36} = 576 \text{ in.}^2 \)
### Composite Section, 3n = 24

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad^2</th>
<th>I_o</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>31.50</td>
<td>24.50</td>
<td>-111</td>
<td>744</td>
<td>22,610</td>
<td>100</td>
</tr>
<tr>
<td>Conc. 84 x 7/24</td>
<td>30.38</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ d_{2n} = \frac{633}{56.00} = 11.30 \text{ in.} \]

\[ d_{\text{Top of steel}} = 24.50 - 11.30 = 13.20 \text{ in.} \]

\[ d_{\text{Bot. of steel}} = 24.88 + 11.30 = 36.18 \text{ in.} \]

\[ Z_{\text{Top of steel}} = \frac{28,250}{13.20} = 2,140 \text{ in.}^3 \]

\[ Z_{\text{Bot. of steel}} = \frac{28,250}{36.18} = 781 \text{ in.}^3 \]

### Composite Section, n = 8

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad^2</th>
<th>I_o</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>31.50</td>
<td>73.50</td>
<td>-111</td>
<td>2,233</td>
<td>67,840</td>
<td>300</td>
</tr>
<tr>
<td>Conc. 84 x %</td>
<td>30.38</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ d_8 = \frac{2,122}{105.00} = 20.21 \text{ in.} \]

\[ d_{\text{Top of steel}} = 24.50 - 20.21 = 4.29 \text{ in.} \]

\[ d_{\text{Bot. of steel}} = 24.88 + 20.21 = 45.09 \text{ in.} \]

\[ Z_{\text{Top of steel}} = \frac{37,940}{4.29} = 8,844 \text{ in.}^3 \]

\[ Z_{\text{Bot. of steel}} = \frac{37,940}{45.09} = 841 \text{ in.}^3 \]

### Steel Stresses Under Maximum Positive Moment—Combination A

**Bending Moments 16 Ft from End Support**

<table>
<thead>
<tr>
<th>M, kip-ft</th>
<th>DL_1</th>
<th>DL_2</th>
<th>LL + I</th>
</tr>
</thead>
<tbody>
<tr>
<td>430</td>
<td>85</td>
<td>705</td>
<td></td>
</tr>
</tbody>
</table>

**Top of Steel (Compression)**

\[ DL_1: f_s = \frac{430 \times 12}{439} = 11.75 \]

\[ DL_2: f_s = \frac{85 \times 12}{2,140} = 0.48 \]

\[ LL + I: f_s = \frac{705 \times 12}{8,844} = 0.96 \]

### Bottom of Steel (Tension)

\[ f_b = \frac{430 \times 12}{576} = 8.96 \]

\[ f_b = \frac{85 \times 12}{781} = 1.31 \]

\[ f_b = \frac{705 \times 12}{841} = 10.06 \]

\[ f_b = \frac{705 \times 12}{20.33} = 13.19 \text{ ksi} \]
Steel Stresses Under Minimum Positive Moment—Combination A

Bending Moments 16 Ft from End Support

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$LL+I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$, kip-ft</td>
<td>430</td>
<td>85</td>
<td>-95</td>
</tr>
</tbody>
</table>

Top of Steel (Compression)

$DL_1$: $f_b = 11.75$

$DL_2$: $f_b = 0.48$

$LL+I$: $f_b = \frac{-95 \times 12}{8,844} = -0.13$

Bottom of Steel (Tension)

$f_b = 8.96$

$f_b = 1.31$

$f_b = \frac{-95 \times 12}{841} = -1.36$

Fatigue Stress at Butt Weld 16 Ft from End Support

Ratio of minimum to maximum stress in bottom flange is

$R = \frac{8.91}{20.33} = 0.438$

The allowable fatigue stress in tension is

$F_t = \frac{17.2}{1 - 0.62(0.438)} = 23.61 \text{ ksi} > 20 \text{ ksi}$

The 12 x 3/4-in. plate is satisfactory.

RE-ANALYSIS BASED ON VARIABLE SECTION

A re-analysis of the two-span continuous beam is made for moments based on the actual variation in moment of inertia along the span. The curves of maximum moments are shown for variable and constant moment of inertia. The solid line represents moments based on variable $I$; the dashed line represents moments based on constant $I$.

Stresses in Flanges for Maximum Negative Moment

Stresses are checked for the variable-$I$ analysis at the section of maximum negative moment. The calculations indicate an increase in stress of 4.5 percent from the values computed for constant moment of inertia.

$f_b = \frac{2,494 \times 12}{1,458} = 20.53 \text{ ksi}$

Stresses in Composite Section for Maximum Positive Moment

At the section of maximum positive moment, the analysis for variable moment of inertia results in a 4.2 percent decrease in bottom-flange stress from the value obtained under the assumption of uniform moment of inertia.
Bending Moments 40 Ft from End Support

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$LL+I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$, kip-ft</td>
<td>538</td>
<td>118</td>
<td>1,176</td>
</tr>
</tbody>
</table>

Steel Stresses—Combination A

Top of Steel (Compression)  
$DL_1$: $f_b = \frac{538 \times 12}{475} = 13.59$

$DL_2$: $f_b = \frac{118 \times 12}{2,115} = 0.67$

$LL+I$: $f_b = \frac{1,176 \times 12}{6,989} = \frac{2.02}{16.28} \text{ ksi}$

Bottom of Steel (Tension)

$DL_1$: $f_b = \frac{538 \times 12}{903} = 7.15$

$DL_2$: $f_b = \frac{118 \times 12}{1,210} = 1.17$

$LL+I$: $f_b = \frac{1,176 \times 12}{1,303} = \frac{10.83}{19.15} \text{ ksi}$

Stresses in Flanges Near Dead-Load Inflection Point

For the $12 \times \frac{3}{8}$-in. flange plates at the inflection point, stresses are lower for the variable-$I$ analysis than for constant $I$.

Maximum Moments

With Positive Live-Load Moment

$DL_1$: $M = -185$

$DL_2$: $M = +10$

$LL+I$: $M = +685$

$+510 \text{ kip-ft}$

$\frac{580 \times 12}{620} = 11.23 \text{ ksi}$

With Negative Live-Load Moment

$DL_1$: $-185$

$DL_2$: $+10$

$LL+I$: $-405$

$-580 \text{ kip-ft}$

Ratio of minimum to maximum stress in butt-welded flange splice is

$R = \frac{-510}{-580} = 0.880$

The allowable fatigue stress in tension is

$F_r = \frac{20.5}{1 - 0.55(-0.880)} = 13.81 \text{ ksi} > 11.23 \text{ ksi}$

The allowable fatigue stress in compression is

$F_r = \frac{19.8}{1 - \left(\frac{19.8}{13.3}\right)\left(-0.880\right)} = 13.85 \text{ ksi} > 11.23 \text{ ksi}$
Stresses in Flanges at Transition Points

Finally, stresses are checked at flange transition points 13 ft from the interior support, 70 ft from the end support and 16 ft from the end support. It is evident that the analysis of the two-span continuous girder based on constant moment of inertia is sufficiently accurate for proportioning the control sections of the girder and for determining flange-plate transition points. Although in this instance the section of maximum negative moment is slightly overstressed when variable $I$ is considered, the maximum positive-moment section is correspondingly understressed, and the total ultimate strength of the girder remains about the same.

Maximum Moments 13 ft from Interior Support

<table>
<thead>
<tr>
<th>With Negative Live-Load Moment</th>
<th>With Positive Live-Load Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DL_1$: $M = -660$</td>
<td>$DL_1$: $M = -660$</td>
</tr>
<tr>
<td>$DL_2$: $M = -80$</td>
<td>$DL_2$: $M = -80$</td>
</tr>
<tr>
<td>$LL + I$: $M = -495$</td>
<td>$LL + I$: $M = +260$</td>
</tr>
<tr>
<td>$\overline{-1,235 \text{ kip-ft}}$</td>
<td>$\overline{-480 \text{ kip-ft}}$</td>
</tr>
</tbody>
</table>

Actual stress in top and bottom flange 13 ft from interior support is

$$f_b = \frac{1,235 \times 12}{788} = 18.81 \text{ ksi}$$

Ratio of minimum to maximum stress in the bottom flange at this transition is

$$R = \frac{-480}{-1,235} = 0.389$$

The allowable fatigue stress in tension is

$$F_t = \frac{17.2}{1 - 0.62(0.389)} = 22.67 \text{ ksi} > 18.81 \text{ ksi}$$

The allowable fatigue stress in compression is

$$F_c = \frac{19.8}{1 - \left(\frac{19.8}{10.6} - 1\right)(0.389)} = 29.89 \text{ ksi} > 18.81 \text{ ksi}$$

Lateral buckling governs, $F_b = 18.31 \text{ ksi}$. This is sufficiently close to the actual stress.

Stresses Under Maximum Positive Moment—Combination A—
70 ft from End Support

Bending Moments

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$LL + I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$, kip-ft</td>
<td>-4 (neglect)</td>
<td>34</td>
<td>823</td>
</tr>
</tbody>
</table>
Top of Steel (Compression)  

\[ DL_2: f_b = \frac{34 \times 12}{2,304} = 0.18 \]

\[ LL+I: f_b = \frac{823 \times 12}{8,470} = 1.17 \] 

\[ \text{ksi} \]

Bottom of Steel (Tension)

\[ f_b = \frac{34 \times 12}{779} = 0.52 \]

\[ f_b = \frac{823 \times 12}{840} = 11.76 \]

\[ \text{12.28 ksi} \]

Stresses Under Maximum Negative Moment—Combination A—

70 ft from End Support

Bending Moments

<table>
<thead>
<tr>
<th></th>
<th>( DL_1 )</th>
<th>( DL_2 )</th>
<th>( LL+I )</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M, \text{kip-ft} )</td>
<td>-4</td>
<td>34</td>
<td>-384</td>
<td>-354</td>
</tr>
</tbody>
</table>

Actual stress in top and bottom flange 70 ft from end support is

\[ f_b = \frac{354 \times 12}{620} = 6.85 \text{ ksi} \]

Ratio of minimum to maximum stress in bottom flange is

\[ R = \frac{-6.85}{12.28} = -0.558 \]

The allowable fatigue stress in tension is

\[ F_t = \frac{17.2}{1-0.62(-0.558)} = 12.78 \text{ ksi} > 12.28 \text{ ksi} \]

Compression is not critical.

Stresses Under Maximum Positive Moment—Combination A—

16 ft from End Support

Bending Moments

<table>
<thead>
<tr>
<th></th>
<th>( DL_1 )</th>
<th>( DL_2 )</th>
<th>( LL+I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M, \text{kip-ft} )</td>
<td>390</td>
<td>80</td>
<td>750</td>
</tr>
</tbody>
</table>

Top of Steel (Compression)

\[ DL_1: f_b = \frac{390 \times 12}{439} = 10.66 \]

\[ DL_2: f_b = \frac{80 \times 12}{2,140} = 0.45 \]

\[ LL+I: f_b = \frac{750 \times 12}{8,844} = 1.02 \]

\[ \text{ksi} \]

Bottom of Steel (Tension)

\[ f_b = \frac{390 \times 12}{576} = 8.13 \]

\[ f_b = \frac{80 \times 12}{781} = 1.23 \]

\[ f_b = \frac{750 \times 12}{841} = 10.70 \]

\[ 20.06 \text{ ksi} \]
Stresses Under Minimum Positive Moment—Combination A
16 Ft from End Support

Bending Moments

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$LL+I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$, kip-ft</td>
<td>390</td>
<td>80</td>
<td>-85</td>
</tr>
</tbody>
</table>

Top of Steel (Compression)

- $DL_1$: $f_b = 10.66$
- $DL_2$: $f_b = 0.45$
- $LL+I$: $f_b = \frac{-85 \times 12}{8,844} = -0.12 \text{ ksi}$

Bottom of Steel (Tension)

- $f_b = 8.13$
- $f_b = 1.23$
- $f_b = \frac{-85 \times 12}{841} = -1.21 \text{ ksi}$

Ratio of minimum to maximum stress in the bottom flange is

$$R = \frac{8.15}{20.06} = 0.406$$

The allowable fatigue stress in tension is

$$F_r = \frac{17.2}{1 - 0.62(0.406)} = 22.99 \text{ ksi} > 20 \text{ ksi}$$

DESIGN OF SHEAR CONNECTORS

Welded stud shear connectors, ¾ in. in diameter and 4-in. high, are provided in the composite region of the span and in the negative-moment region adjacent to the dead-load point of inflection. The allowable stud loads are determined for an $H/d$ ratio greater than 4.0. The required shear-connector spacing for service behavior is calculated for points along the span, and the theoretical spacing curve is plotted. The actual, stepped shear-connector spacing diagram is constructed within this theoretical curve.

Concrete: $f' = 4,000$ psi; $n = 8$

Studs: ¾-in. dia., 4-in. high; $H/d = 4.0/0.875 = 4.6 > 4.0$

For $H/d > 4$, AASHO Specifications give the ultimate strength of a shear connector as

$$Q_s = 0.93d^2 \sqrt{f'_c} = 0.93(0.875)^2 \sqrt{4,000} = 45.0 \text{ kips per stud}$$

With $\alpha$ given in AASHO Specifications as 10.6 for 500,000 cycles of load, the load range per shear connector is

$$Z_r = \alpha d^2 = 10.6(0.875)^2 = 8.11 \text{ kips per stud}$$
Shear Connectors—Strength Requirements

At point of maximum moment \((0.4L)\), the maximum compressive force in the concrete is

\[
H_1 = A_s F_v = 40.5 \times 36.0 = 1,458 \text{ kips (governs)}
\]

\[
H_2 = 0.85 f_y t = 0.85 \times 4.0 \times 84.0 \times 7.0 = 1,999 \text{ kips}
\]

Number of studs required from point of maximum moment \((0.4L)\) to end support and to dead-load inflection point is

\[
N = \frac{H_1}{\phi Q_u} = \frac{1,458}{0.85 \times 45.0} = 38.1
\]

At 16 ft from end support:

\[
H_1 - A_s F_v = 31.5 \times 36.0 = 1,134 \text{ kips (governs)}
\]

\[
H_2 = 1,999 \text{ kips}
\]

Number of studs required between end support and first transition in girder section is

\[
N = \frac{H_1}{\phi Q_u} = \frac{1,134}{0.85 \times 45.0} = 29.6
\]

At 70 ft from end support:

Since this transition point was located by fatigue considerations, the number of studs required for strength is assumed proportional to the actual stress divided by the allowable tensile stress neglecting fatigue.

\[
H_1 = A_s F_v = 36.0 \times 36.0 = 1,296 \text{ kips (governs)}
\]

\[
H_2 = 1,999 \text{ kips}
\]

Number of studs required between dead-load inflection point and transition in girder section 70 ft from end support is

\[
N = \frac{H_1}{\phi Q_u} \times \frac{14.74}{20.00} = \frac{1,296}{0.85 \times 45.0 \times 20.00} \times 14.74 = 25.0
\]

\[
\frac{7}{8\text{'}} \times 4\text{'} \text{ granular flux-filled studs}
\]

\[
\text{2'' + 4'' + 4'' + 2''}
\]

\[
3.5''
\]

\[
7''
\]

\[
2.38''
\]

\[
4.29''
\]

\[
10.17''
\]

N.A. for \(n = 8\)

DETAIL AT SHEAR CONNECTORS
Shear-Connector Spacing for Service Behavior (Fatigue)

At the end support, shear range \( V_r = 58.7 - (-6.0) = 64.7 \text{ kips (LL only)} \). For \( n = 8 \),

\[
S_r = \frac{V_r Q}{I} = \frac{64.7 (73.5 \times 10.17)}{37,940} = 1,275 \text{ kips per in.}
\]

Spacing req'd. (3 studs) \( \frac{3(8.11)}{1,275} = 19.1 \text{ in.} \)

---

**SHEAR-CONNECTOR SPACING**

The required shear-connector spacing is computed at each tenth point along the span, and an envelope of required spacing is plotted. The actual, stepped shear-connector spacing diagram is then drawn within the theoretical curve. The actual connector spacing satisfies strength requirements by providing 69 studs between the end support and the 0.4\( L \) point and 84 studs from there to the dead-load inflection point. Thirty-three studs are placed between the end support and the first transition in girder section. Thirty-six studs at 6-in. pitch are placed adjacent to the dead-load inflection point to satisfy strength and insure full composite action at the butt-welded girder transition 70 ft from the end support.
Shear Connectors Required for Slab Reinforcement

For longitudinal reinforcement within the effective flange width in the negative-moment region, 14 No. 5 bars are specified. These provide an area of

\[ A_r = 14 \times 0.31 = 4.34 \text{ in.}^2 \]

and have a yield strength of

\[ H_3 = A_r F_y = 4.34 \times 40.0 = 173.6 \text{ kips} \]

This is equivalent to

\[ N = \frac{H_3}{\frac{\phi Q_u}{0.85 \times 45.0}} = 4.5 \text{ studs} \]

Live load plus impact, however, requires

\[ N = \frac{A_{f_r}}{Z_r} = \frac{4.34 \times 10.0}{8.11} = 5.3 \text{ studs} \]

Use 6 extra studs at 6-in. pitch adjacent to the dead-load inflection point.

DESIGN OF FLANGE-TO-WEB WELD

The flange-to-web weld must transmit the horizontal shear between the flange and web plates. At the interior support, the point of maximum shear, a \( \frac{3}{8} \)-in. fillet weld is required. It is sufficient for strength throughout the girder, but minimum weld requirements govern the weld size for the \( \frac{3}{8} \), 1\%, and 1\%-in.-thick flanges.

Shear at Interior Support

\[ V_{\text{max}} = 128.9 \text{ kips} \quad V_{\text{min}} = 66.5 \text{ kips} \]

\[ S = \frac{VQ}{I} = \frac{128.9 \times 28.00 \times 24.88}{37,540} = 2.391 \text{ kips per in.} \]

Allowable Weld Stress

Fatigue: 500,000 cycles

Ratio of minimum to maximum stress in the weld is

\[ R = \frac{66.5}{128.9} = 0.516 \]

The allowable fatigue stress in the weld is

\[ F_s = \frac{10.8}{1 - 0.55(0.516)} = 15.08 \text{ ksi} > 12.4 \text{ ksi} \]
Weld Size

With a weld on each side of the web, the shear per weld is
\[ \frac{S}{2} = \frac{2.391}{2} = 1.196 \text{ kips per in.} \]

Allowable load on weld = \(12.4 \times 0.707 = 8.76\) kips per in.

Weld size required = \(\frac{1.196}{8.76} = 0.137\) in., say \(\frac{3}{16}\) in.

Use minimum weld for material thickness as required by AASHO Specifications:

- For flange thickness \(\frac{3}{8}\) in., use two \(\frac{3}{16}\)-in. fillet welds.
- For flange thickness \(\frac{3}{8}\) in., use two \(\frac{3}{16}\)-in. fillet welds.
- For flange thickness \(1\frac{1}{4}\) in., use two \(\frac{3}{8}\)-in. fillet welds.
- For flange thickness \(1\frac{1}{4}\) in., use two \(\frac{3}{8}\)-in. fillet welds.

BEARING STIFFENERS

The bearing stiffeners are designed as columns to carry the reaction forces at points of support. A stiffener consisting of two \(5 \times \frac{1}{2}\)-in. plates welded to opposite sides of the web plate is investigated at the end support. The \(\frac{1}{2}\)-in. plate satisfies the minimum thickness requirement. Section properties are computed for the column about an axis through the centerline of the web. The compressive stress is calculated and compared to the allowable stress for concentrically loaded columns. Bearing stress is also checked.

<table>
<thead>
<tr>
<th>End Reaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>DL₁</td>
</tr>
<tr>
<td>R, kips</td>
</tr>
</tbody>
</table>

**END BEARING STIFFENERS**

Length of web acting with stiffeners is
\[ L_w = 18 \times 0.313 = 5.63 \text{ in.} \]

For end stiffeners, assume 2 plates \(5 \times \frac{1}{2}\) in. with
\[ I_s = \frac{0.5(5)^3}{12} = 5.2 \text{ in.}^4 \]
Minimum thickness, in., required for stiffeners is
\[
t = \frac{1}{12} b \sqrt{\frac{F_s}{33,000}} = 0.436 < \frac{3}{16}
\]
Bearing stress = \[
\frac{98.6}{4.75 \times \frac{3}{16} \times 2} = 20.76 \text{ ksi} < 29 \text{ ksi (allowable)}
\]
The equivalent column, stiffeners plus 5.63 in. of web, has \( J = 45.5 \text{ in.}^4, r = 2.59 \text{ in.}, D/r = 18.53, A = 6.76 \text{ in.}^2, \) and \( F_{\text{allow}} = 15.90 \text{ ksi} \) (see p. II/4.17). The stress in the column is
\[
f_s = \frac{98.6}{6.76} = 14.59 \text{ ksi} < 15.90 \text{ ksi}
\]
Use 2 plates 5 × \( \frac{3}{16} \) in. at end support.

**Bearing Stiffeners At Interior Support**

The bearing stiffeners at the interior support are designed in a similar manner. The stiffener plates should extend as nearly as practicable to the edge of the flange plates. For the 16-in.-wide flange at this location, a 7-in.-wide stiffener plate is used. Plates 1-in. thick are necessary to resist the 225-kip reaction.

**Interior Reaction**

<table>
<thead>
<tr>
<th></th>
<th>( DL_1 )</th>
<th>( DL_2 )</th>
<th>( LL+I )</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R, \text{ kips} )</td>
<td>112.4</td>
<td>20.6</td>
<td>92.0</td>
<td>225.0</td>
</tr>
</tbody>
</table>

Length of web acting with stiffeners again is 5.63 in. For stiffeners, assume two plates 7 × 1 in. with
\[
I_w = \frac{1.0(7)^3}{12} = 28.6 \text{ in.}^4
\]
Minimum thickness, in., required for stiffeners is
\[
t = \frac{1}{12} b \sqrt{\frac{F_s}{33,000}} = 0.609 \text{ in.} < 1 \text{ in.}
\]
Bearing stress = \[
\frac{225.0}{(7.00 - 0.438) \times 1.0 \times 2} = 17.14 \text{ ksi} < 29 \text{ ksi (allowable)}
\]

![Diagram of stiffeners at interior support](image)

**STIFFENERS AT INTERIOR SUPPORT**
Equivalent Column at Interior Support

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>Io</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Plates 7×1</td>
<td>14.00</td>
<td>3.65</td>
<td>51.10</td>
<td>186.5</td>
<td>57.2</td>
<td>243.7</td>
</tr>
<tr>
<td>Web 5.63×5/16</td>
<td>1.76</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>15.76</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>243.7 in.²</td>
</tr>
</tbody>
</table>

\[ r = \sqrt{\frac{243.7}{15.76}} = 3.93 \text{ in.} \]

\[ D = \frac{48}{3.93} = 12.21 \]

\[ F_{\text{allow.}} = 16,000 - 0.3(12.21)^2 = 15.96 \text{ ksi} \]

\[ f_s = \frac{225.0}{15.76} = 14.28 \text{ ksi} < 15.96 \text{ ksi} \]

Use 2 plates 7×1 in.

INTERMEDIATE STIFFENERS

Intermediate transverse stiffeners are required to prevent buckling of the girder web. Sample calculations for stiffener spacing adjacent to the end support and the interior support are shown.

Spacing at End Support

The average shearing stress in the web is

\[ f_v = \frac{98.6}{48 \times 5/16} = 6.57 \text{ ksi} \]

Stiffener spacing required at the end support is

\[ d_{\text{end}} = \frac{11,000 \times 0.313}{\sqrt{6,570}} = 42.48 \text{ in.} \text{ Use 40 in.} \]

\[ d_{\text{max}} = \text{clear distance between flanges} = 48 \text{ in.} \]

Note: First two spaces from bearing stiffeners should be 1/4 the calculated spacing = 20 in.
Spacing at Interior Support

Shear at Interior Support

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$LL+I$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$, kips</td>
<td>56.2</td>
<td>10.3</td>
<td>62.4</td>
<td>128.9</td>
</tr>
</tbody>
</table>

The average shearing stress in the web is

$$f_s = \frac{128.9}{48 \times \frac{3}{8}} = 8.59 \text{ ksi}$$

Stiffener spacing required near the interior support is

$$d_{int} = \frac{11,000 \times 0.313}{\sqrt{8590}} = 37.15 \text{ in. Use } 34\% \text{ in.}$$

Location Where Stiffeners May Be Omitted

In regions of low shearing stress, intermediate stiffeners may be omitted. The critical web shearing stress, below which stiffeners are not required, is calculated as 2.39 ksi. The corresponding shear force is 35.9 kips. Since there is only a short distance within which the shear is below 35.9 kips, stiffeners are used over the full span.

From the formula for minimum web thickness without stiffeners, $t = D \sqrt{f_s}/7500$,

$$f_s = \left( \frac{7500t}{D} \right)^2 = \left( \frac{7500 \times 0.313}{48} \right)^2 = 2.39 \text{ ksi}$$

$$V = 2.39 \times (48 \times \frac{3}{8}) = 35.9 \text{ kips}$$

Where shear is less than 35.9 kips, stiffeners may be omitted. This occurs over a distance of about 10 ft, from about 35 to 45 ft from the end support. Use stiffeners full length of span.

Actual Spacing of Intermediate Stiffeners

The required stiffener spacing is calculated at intervals along the span, and a curve of required spacing is plotted. Since the maximum permissible spacing equals the depth of the web, calculations need be carried only from the supports to locations where the required spacing exceeds the depth of the web. The actual spacing diagram is enclosed within the theoretical envelope. Stiffeners are placed at diaphragm locations to serve as connection plates.
STIFFENER SPACING

Size of Intermediate Stiffeners

Intermediate stiffeners consist of $4 \times \frac{5}{16}$-in. plates, welded to one side of the web. The moment of inertia of these plates taken about the face of the web is larger than the moment of inertia required by AASHO Specifications. Minimum width and minimum thickness limitations are also checked. These, rather than moment of inertia requirements, control the dimensions of the stiffeners.

Required $I$ of Stiffener

\[
I = \frac{d_s t^3 f}{10.92} \quad \text{with} \quad J = 25 \frac{D^2}{d^2} - 20 > 5
\]

\[
J = 25 \times \frac{(48)^2}{(37.15)^2} - 20 = 21.74 > 5
\]

\[
I = \frac{34.29 \times (0.313)^2 \times (21.74)}{10.92} = 2.09 \text{ in.}^4
\]
Stiffener Dimensions

For stiffener, assume one plate $4 \times \frac{3}{16}$ in.

$$I = \frac{0.313 \times (4)^2}{3} = 6.68 \text{ in.}^4 > 2.09$$

Min. width $= 2 + \frac{48}{30} = 3.6 \text{ in.} < 4 \text{ in.}$

Min. $t_* = \frac{16}{4} = 4 \text{ in.}$

Min. $t_* = \frac{3}{16} \times 4 = \frac{3}{4} \text{ in.} < \frac{5}{16} \text{ in.}$

The $4 \times \frac{3}{16}$-in. plate is satisfactory.

Fatigue Stresses Near Inflection Point

In accordance with AASHO Specifications, it is recommended that stiffeners attached to only one side of the web be welded to the compression flange. Near the dead-load inflection point, however, both flanges may be alternately in tension and compression. Since flange-to-stiffener welding imposes severe fatigue restrictions at the tension flange, it is undesirable to weld the stiffener to either flange in this region.

Flange stresses are checked at the three intermediate stiffener locations nearest the inflection point and compared with the allowable fatigue stresses. Allowable fatigue stress is exceeded only at the stiffener nearest the inflection point. This stiffener should not be welded to either flange.

Maximum Moments 23 Ft 4 In. from Interior Support

<table>
<thead>
<tr>
<th>With Positive Live-Load Moment</th>
<th>With Negative Live-Load Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DL_1: M = -240$</td>
<td>$DL_1: M = -240$</td>
</tr>
<tr>
<td>$DL_2: M = 0$</td>
<td>$DL_2: M = 0$</td>
</tr>
<tr>
<td>$LL + I: M = +630$</td>
<td>$LL + I: M = -420$</td>
</tr>
<tr>
<td>$\frac{+390 \text{ kip-ft}}{788}$</td>
<td>$\frac{-660 \text{ kip-ft}}{788}$</td>
</tr>
</tbody>
</table>

For these moments, stresses in top and bottom flanges 23 ft 4 in. from the interior support are

$$f_s = \frac{390 \times 12}{788} = 5.94 \text{ ksi}$$

$$f_s = \frac{660 \times 12}{788} = 10.05 \text{ ksi}$$

Ratio of minimum to maximum stress in the flanges is

$$R = \frac{390}{-660} = -0.591$$

The allowable fatigue stress in tension is

$$F_* = \frac{12.0}{1 - (-0.591)} = 7.54 \text{ ksi} > 5.94 \text{ ksi}$$

Weld stiffener to bottom flange. Maximum tension in bottom flange $= 5.94$ ksi.
Maximum Moments 26 Ft 8 In. from Interior Support

With Positive Live-Load Moment
\[ DL_1: M = -125 \]
\[ DL_2: M = + 15 \]
\[ LL + I: M = +730 \]
\[ +620 \text{ kip-ft} \]

With Negative Live-Load Moment
\[ DL_1: M = -125 \]
\[ DL_2: M = + 15 \]
\[ LL + I: M = -405 \]
\[ -515 \text{ kip-ft} \]

For these moments, stresses in top and bottom flanges 26 ft 8 in. from the interior support are

\[ f_b = \frac{620 \times 12}{620} = 12.00 \text{ ksi} \]
\[ f_b = \frac{515 \times 12}{620} = 9.97 \text{ ksi} \]

Ratio of minimum to maximum stresses in the flanges is

\[ R = \frac{-515}{620} = -0.831 \]

The allowable fatigue stress in base metal at fillet welds is

\[ F_f = \frac{12.0}{1 - (-0.831)} = 6.55 \text{ ksi} < 9.97 \text{ ksi} \]

Do not weld stiffener to top or bottom flange.

Maximum Moments 30 Ft from Interior Support

With Positive Live-Load Moment
\[ DL_1: M = 0 \]
\[ DL_2: M = 34 \]
\[ LL + I: M = 823 \]
\[ 857 \text{ kip-ft} \]

With Negative Live-Load Moment
\[ DL_1: M = 0 \]
\[ DL_2: M = + 34 \]
\[ LL + I: M = -384 \]
\[ -350 \text{ kip-ft} \]

For these moments, stresses in top and bottom flanges 30 ft from the interior support are

\[ f_b = \frac{857 \times 12}{620} = 16.59 \text{ ksi} \]
\[ f_b = \frac{350 \times 12}{620} = 6.77 \text{ ksi} \]

Ratio of minimum to maximum stresses in the flanges is

\[ R = \frac{-350}{857} = -0.408 \]

The allowable fatigue stress in base metal at fillet welds is

\[ F_f = \frac{12.0}{1 - (-0.408)} = 8.52 \text{ ksi} > 6.77 \text{ ksi} \]

Weld stiffener to top flange. Maximum tension in top flange = 6.77 ksi.
WELDED FIELD SPLICE

A full-penetration butt weld is used to make the field splice for the two-span girder at one inflection point, 25 ft from the interior support. There is no change in flange thickness at this location. But the flange width is changed, with a taper as discussed in the Introduction. If all welds are radiographed and ground smooth, the splice may be designed for stresses allowed for base metal. Stresses at the inflection point are well within these allowable values.

BOLTED FIELD SPLICE

A field splice, with ¼-in.-dia, high-strength bolts (ASTM A325), is designed as an alternate to the welded splice, 25 ft from the interior support.

For static strength, the splice material is proportioned to carry the greater of:

1. Seventy-five percent of the moment capacity of the net section.
2. The average of the actual maximum moment and the moment capacity of the net section.

The net section is the gross section on the weaker side of the splice less bolt holes.

Fatigue need not be considered when calculating bolt stresses, since no reduction in allowable bolt stress is required by the AASHO Specifications regardless of the stress range or number of stress repetitions. Fatigue should be taken into account, however, in the design of the splice plates. A fatigue design moment, defined as follows, assures that splice-plate stresses will be less than the allowable fatigue stress for base metal adjacent to friction-type fasteners.

\[
\text{Fatigue design moment} = \frac{\text{actual maximum moment} \times \text{allowable tensile stress}}{\text{allowable fatigue stress}}
\]

The fatigue design moment governs if it is greater than (1) or (2). In the example under consideration, the fatigue design moment does govern splice material, while (2) controls the bolt arrangement.

The shear to be used in designing a splice is not as well standardized as is the moment. Some designs are made for 75 percent of the shear capacity of the web or the average of the actual shear and the shear capacity of the web, whichever is greater. Such a design usually is quite conservative when the web thickness is governed by minimum thickness requirements rather than by shearing stress. Since both shear and moment are directly related to applied load, it would appear reasonable to use a design shear increased by the same proportion as the design moment. Accordingly, the field splice is designed for shear determined as follows:

\[
\text{Design shear} = \frac{\text{actual maximum shear} \times \text{design moment}}{\text{actual maximum moment}}
\]

Design calculations for the splice begin with tabulation of maximum shear and moment at the splice, and computation of net-section properties.
Maximum Shear

\[ DL_1: V = 33.8 \]
\[ DL_2: V = 6.5 \]
\[ LL + I: V = 49.4 \]
\[ 89.7 \text{ kips} \]

Maximum Moments

Positive \( M = +650 \) kip-ft
Negative \( M = -460 \) kip-ft

Net Section Properties (12 \( \times \frac{3}{8}\)-In. Flanges)

Use: \( \frac{3}{8}\)-in.-dia. H.S. Bolts (ASTM A325)
Flange Hole Area = \( 1 \times \frac{3}{8} = 0.875 \) in.\(^2\)
Web Hole Area = \( 1 \times \frac{3}{16} = 0.313 \) in.\(^2\)

\[ d^2 \text{ for Holes} \]
\[ (5)^2 = 25 \]
\[ (10)^2 = 100 \]
\[ (15)^2 = 225 \]
\[ (20)^2 = 400 \]
\[ \sum d^2 = 750 \text{ in.}^2 \]

\[ Ad^2 \text{ web holes} = 2 \times 0.313 \times 750 = 470 \]
\[ Ad^2 \text{ fig. holes} = 2 \times 2 \times 0.875 \times (24.44)^2 = 2,091 \]
\[ I_{holes} = 2,561 \text{ in.}^4 \]

The moment of inertia of the 48 \( \times \frac{3}{8}\)-in. web is 2,880 in.\(^4\) and of the two 12 \( \times \frac{3}{8}\)-in. flange plates, 12,540 in.\(^4\). Hence, the moment of inertia \( I_{net} \) of the net section is

\[ I_w = 2,880 - 470 = 2,410 \]
\[ I_f = 12,540 - 2,091 = 10,449 \]
\[ I_{net} = 12,859 \text{ in.}^4 \]

Required Moment and Shear Capacity of Splice

The moments and shears for design are determined by allowable fatigue stress. The fatigue design moment is computed. Calculations are then made for the net-section capacity, 75 percent of the net-section capacity, and the average of the net-section capacity and the actual moment. The shear capacity is determined on a comparable basis.

Ratio of minimum to maximum stresses in flanges is

\[ R = \frac{-460}{650} = -0.708 \]

The allowable fatigue stress in tension is

\[ F_r = \frac{20.5}{1 - 0.55(-0.708)} = 14.75 \text{ ksi} \]

The allowable fatigue stress in compression is

\[ F_r = \frac{19.8}{1 - \left( \frac{19.8}{13.3} - 1 \right) (-0.708)} = 14.71 \text{ ksi} \]
For the splice plates, then, the fatigue design moment is

\[ M_{\text{tot}} = \frac{650 \times 20.0}{14.71} = 884 \text{ kip-ft} \]

The calculated moment capacity of the net section is

\[ M_{\text{net}} = \frac{20.0 \times 12,859}{12 \times 24.88} = 861 \text{ kip-ft} \]

75\% \ M_{\text{net}} = 0.75 \times 861 = 646 \text{ kip-ft}

The average moment capacity is

\[ M_{av} = \frac{861 + 650}{2} = 756 \text{ kip-ft} \]

The fatigue design moment is larger than \( M_{av} \) and 75\% of \( M_{\text{net}} \) and therefore governs the splice-plate design.

The shear capacity corresponding to the average-moment capacity is

\[ V = \frac{756 \times 89.7}{650} = 104 \text{ kips} \]

**Web Splice Design**

The web splice carries a moment equal to the total design moment on the section multiplied by the ratio of the net moment of inertia of the web itself to the net moment of inertia of the entire section. In addition to this moment, the web splice also carries the design shear and the moment due to the eccentricity of this shear, 3.25 in.

From the shear and moment, the maximum shear on the extreme bolt is determined. Web splice plates, 43 x 5\% in., are then checked for extreme fiber stress.

**Web Moment**

\[ M_{w} = \frac{756 \times 2,410}{12,859} = 142 \]

**Moment of Inertia of Bolts**

\[ I_{xx} = 2 \times 2 \times 750 = 3,000 \]

\[ I_{yy} = 18 \times (1.5)^2 = \frac{41}{3,041 \text{ in.}^4} \]

### Shear on Bolts

Load per bolt due to shear is

\[ P_s = \frac{104}{18} = 5.78 \text{ kips} \]

Load on the outermost bolt due to moment is

\[ P_n = \frac{170 \times 12 \times 20.06}{3,041} = 13.46 \text{ kips} \]

The vertical component of this load is

\[ P_v = \frac{13.46 \times 1.5}{20.06} = 1.01 \text{ kips} \]

And the horizontal component is

\[ P_h = \frac{13.46 \times 20.0}{20.06} = 13.42 \text{ kips} \]

Hence, the total load on the outermost bolt is the resultant

\[ P = \sqrt{(5.78 + 1.01)^2 + (13.42)^2} = 15.04 \text{ kips} \]
Allowable double shear on \( \frac{3}{8} \)-in.-dia HS bolt = 16.2 kips > 15.04 kips. The bolts and bolt arrangement are satisfactory.

**Web Splice Plates—Design for Fatigue Moment**

For the web splice, assume 2 plates \( 12\frac{1}{2} \times \frac{3}{16} \) in. by 3 ft 7 in. long.

\[
I = 2 \left[ \frac{0.313(43)^3}{12} - 470 \right] = 3,208 \text{ in.}^4
\]

The portion of the fatigue design moment carried by these plates is

\[
M_{\text{fat}} = 844 \times \frac{2,410}{12,859} = 166 \text{ kip-ft}
\]

The maximum stress in the plates due to this moment is

\[
f = \frac{166 \times 12 \times 21.5}{3,208} = 13.35 \text{ ksi} < 14.71 \text{ ksi (allowable)}
\]

The assumed plates are satisfactory.

**Flange Splice Design**

The flange splice carries that portion of the total design moment not carried by the web splice. The splice plates transmit the moment couple across the splice in axial tension and compression, and into the girder flange by double shear on \( \frac{3}{8} \)-in. bolts. A staggered bolt pattern is used, with \( \frac{3}{16} \) and \( \frac{1}{2} \)-in. splice plates as shown.

**Flange Bolts Required**

The required average-moment capacity of the flange splice is

\[
M_f = 756 - 142 = 614 \text{ kip-ft}
\]

Compressive and tensile forces in the flanges form a couple that supply this capacity.

\[
P_f = \frac{614 \times 12}{48.88} = 151 \text{ kips}
\]

Bolts required = \( \frac{151}{16.2} \approx 9.32 \)

Use 12 bolts for sealing.
**Flange Splice Plates—Design For Fatigue Moment**

The fatigue-design-moment capacity required for the flange splice is

\[ M_{fat} = 884 - 166 = 718 \text{ kip-ft} \]

\[ P_{fat} = \frac{718 \times 12}{48.88} = 176 \text{ kips} \]

The area required for the flange splice plates then is

\[ A_{fat} = \frac{176 \text{ kips}}{20 \text{ ksi}} = 8.80 \text{ in.}^2 \]

For the flange splice, assume one \(12 \times \frac{3}{16}\)-in. plate and two \(5\frac{1}{2} \times \frac{1}{2}\)-in. plates. Splice plate area:

\[ (12 - 2)\frac{3}{16} = 4.38 \]

\[ (11 - 2)\frac{1}{2} = 4.50 \]

\[ 8.88 \text{ in.}^2 > 8.80 \text{ in.}^2 \]

The splice plates are adequate.

---

**DEFLECTIONS**

Dead-load deflections are computed for uniform dead load, assuming constant moment of inertia. The appropriate moments of inertia at the positive-moment region of the span are used for \(DL_1\) and \(DL_2\) deflections. Camber ordinates are determined at the quarter and midpoints of the positive-moment region, at the inflection point, and at the midpoint of the negative-moment region.

**Deflections Due to Dead Load**

- \(DL_1\): \(w = 0.900 \text{ kips per ft}\)
- \(DL_2\): \(w = 0.165 \text{ kips per ft}\)
- \(M_R = 1,359 \text{ kip-ft for } DL_1\)
- \(M_R = 199 \text{ kip-ft for } DL_2\)
\[ \Delta = \frac{72wL^4}{EI}a[1+ab-4C_R(1+a)] \]

where \( \Delta \) = deflection in., at distance \( aL \) from end support
\( w \) = dead load, kips per ft
\( b = 1 - a \)
\( L \) = span, ft.
\( E_s = 29 \times (10)^3 \) ksi
\( I \) = moment of inertia at midspan, in.\(^4\)
\( C_R = \frac{M_R}{wL^2} \)

**Deflections Under DL\(_1\)**

\[ I_1 = 15,600 \text{ in.}^4 \]
\[ C_R = \frac{1.359}{0.960(100)^2} = 0.151 \]
\[ \Delta = \frac{72 \times 0.900 \times (100)^4}{29 \times (10)^3 \times 15,600}a[1+ab-4(0.151)(1+a)] = 14.26ab[1+ab-0.604(1+a)] \]

At \( a = \frac{3}{6} \),
\[ \Delta = 14.26(0.1875)(0.8125)[1+0.1875(0.8125) - 0.604(1.1875)] = 0.95 \text{ in.} \]

At \( a = \frac{4}{6} \),
\[ \Delta = 14.26(0.375)(0.625)[1+0.375(0.625) - 0.604(1.375)] = 1.34 \text{ in.} \]

At \( a = \frac{5}{6} \),
\[ \Delta = 14.26(0.563)(0.437)[1+0.563(0.437) - 0.604(1.563)] = 1.06 \text{ in.} \]

At \( a = \frac{3}{4} \),
\[ \Delta = 14.26(0.75)(0.25)[1+0.75(0.25) - 0.604(1.75)] = 0.35 \text{ in.} \]

At \( a = \frac{7}{8} \),
\[ \Delta = 14.26(0.875)(0.125)[1+0.875(0.125) - 0.604(1.875)] = -0.03 \text{ in.} \]

**Deflections Under DL\(_2\)**

\[ I_2 = 38,590 \text{ in.}^4 \]
\[ C_R = \frac{199}{0.165(100)^2} = 0.121 \]
\[ \Delta = \frac{72 \times 0.165 \times (100)^4}{29 \times (10)^3 \times 38,590}a[1+ab-4(0.121)(1+a)] = 1.064ab[1+ab-0.484(1+a)] \]

At \( a = \frac{3}{6} \),
\[ \Delta = 1.064(0.1875)(0.8125)[1+0.1875(0.8125) - 0.484(1.1875)] = 0.10 \text{ in.} \]

At \( a = \frac{4}{6} \),
\[ \Delta = 1.064(0.375)(0.625)[1+0.375(0.625) - 0.484(1.375)] = 0.15 \text{ in.} \]

At \( a = \frac{5}{6} \),
\[ \Delta = 1.064(0.563)(0.437)[1+0.563(0.437) - 0.484(1.563)] = 0.13 \text{ in.} \]
At $a = \frac{3}{4}$,
\[ \Delta = 1.064(0.75)(0.25)[1 + 0.75(0.25) - 0.484(1.75)] = 0.08 \text{ in.} \]

At $a = \frac{7}{8}$,
\[ \Delta = 1.064(0.875)(0.125)[1 + 0.875(0.125) - 0.484(1.875)] = 0.02 \text{ in.} \]

**Total DL Deflections**

At $a = \frac{3}{6}$, $\Delta = 0.95 + 0.10 = 1.05$ in., say 1 in.
At $a = \frac{7}{8}$, $\Delta = 1.34 + 0.15 = 1.49$ in., say $1\frac{1}{2}$ in.
At $a = \frac{9}{16}$, $\Delta = 1.06 + 0.13 = 1.19$ in., say $1\frac{3}{4}$ in.
At $a = \frac{3}{4}$, $\Delta = 0.35 + 0.08 = 0.43$ in., say $\frac{3}{8}$ in.
At $a = \frac{7}{8}$, $\Delta = -0.03 + 0.02 = -0.01$ in., say 0

**Camber Diagram**

**Deflection Due to Live-Load + Impact**

The maximum live-load deflection occurs near the 0.4 point, with the live load placed to give maximum moment at that location. Again, the moment of inertia in the positive-moment region is assumed constant throughout the span.

The live-load deflection, in., 40 ft from the end support is given by

\[ \Delta = \frac{300}{E_s I} \left[ P_T (L^3 + 3.89L^2 - 680L + 5,910) - 0.32M_R L^2 \right] \]

where $P_T$ = weight of front truck wheel $\times$ distribution factor, plus impact, kips

$I$ = moment of inertia at midspan, in.$^4$

$L$ = span, ft

$E_s = 29 \times (10)^3$ ksi

$M_R$ = live-load + impact moment over interior support, kip-ft
GIRDER LOADED FOR MAXIMUM DEFLECTION

Assume that two lanes of live load (four wheels abreast) plus 22.2% impact are equally distributed over four girders.

\[ P_r = 4 \times 4 \times 1.22 = 19.6 \text{ kips} \quad \quad \quad I_s = 4 \times 55,070 = 220,280 \text{ in.}^4 \]

The moment at the interior support \( M_R \) can be computed in any of several ways; for example, by influence coefficients, as shown below. (See Reference 3 or 4 in the Bibliography of Chapter 3.)

\[
\begin{align*}
19.6 \times 100 \times 0.0602 &= 118 \\
78.4 \times 100 \times 0.0840 &= 658 \\
78.4 \times 100 \times 0.0947 &= 743 \\
M_R &= 1,519 \text{ kip-ft}
\end{align*}
\]

The maximum live-load deflection therefore is

\[
\Delta = \frac{300(19.6(100)^3 + 3.89(100)^2 - 680(100) + 5,910 - 0.32(1,519)(100)^2)}{29 \times (10)^3 \times 220,280}
\]

\[
\Delta = \frac{300 \times 14,284,676}{29 \times 10^3 \times 220,280} = 0.66 \text{ in.}
\]

The ratio of live-load deflection to span is

\[
\frac{0.66}{100 \times 12} = \frac{1}{1,818} < \frac{1}{800}
\]

FINAL DESIGN

An elevation of the two-span composite girder is shown on the next page.
Design III—Two-Span Continuous Girder (100-100 Ft), Composite For Positive And Negative Moment

The design procedure for this example is similar to that for Design II except that the longitudinal reinforcement in the concrete slab is assumed to assist the girder in the negative-moment region. The concrete is considered cracked under tensile stress and therefore ineffective. The reinforcement assumed effective is the steel embedded in the direction of the girder span in a width of slab over the girder not exceeding the following:

1. One-fourth the span of the girder.
2. Distance center to center of girders.
3. Twelve times the least thickness of the slab.

MAXIMUM NEGATIVE MOMENT

A steel section consisting of a $48 \times \frac{5}{6}$-in. web, a $16 \times 1\frac{1}{8}$-in. top flange and a $16 \times 1\frac{3}{4}$-in. bottom flange is investigated for the maximum negative-moment region. Properties are computed for the steel girder alone and for the composite section. Stresses over the interior support are checked at the top and bottom of the steel girder and in the reinforcing bars. The steel girder alone resists moments due to $DL_1$. The composite section composed of the steel girder and reinforcing bars resists moments due to $DL_2$ and $LL+I$.
### Steel Section

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>Io</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Flg. 16×1½&quot;</td>
<td>26.00</td>
<td>24.81</td>
<td>-645</td>
<td>16,000</td>
<td>2,880</td>
<td>16,000</td>
</tr>
<tr>
<td>Web 48×3½&quot;</td>
<td>15.00</td>
<td>24.88</td>
<td>-697</td>
<td>17,330</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bot. Flg. 16×1¾&quot;</td>
<td>28.00</td>
<td>24.88</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ d_s = \frac{-52}{69.00} = -0.75 \text{ in.} \]

\[ d_{\text{Top of steel}} = 25.63 \times 0.75 = 26.38 \text{ in.} \]

\[ d_{\text{Bot of steel}} = 25.75 - 0.75 = 25.00 \text{ in.} \]

\[ Z_{\text{Top of steel}} = \frac{36,170}{26.38} = 1,371 \text{ in.}^3 \]

\[ Z_{\text{Bot of steel}} = \frac{36,170}{25.00} = 1,447 \text{ in.}^3 \]

### Composite Section

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>Io</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel section</td>
<td>69.00</td>
<td>4.34</td>
<td>30.57</td>
<td>-52</td>
<td>4,060</td>
<td>36,210</td>
</tr>
<tr>
<td>Reinf. Steel 14 No. 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ d_e = \frac{81}{73.34} = 1.10 \text{ in.} \]

\[ d_{\text{Reinf.}} = 30.57 - 1.10 = 29.47 \text{ in.} \]

\[ Z_{\text{Reinf.}} = \frac{40,180}{29.47} = 1,363 \text{ in.}^3 \]

\[ d_{\text{Top of steel}} = 25.63 - 1.10 = 24.53 \text{ in.} \]

\[ d_{\text{Bot of steel}} = 25.75 + 1.10 = 26.85 \text{ in.} \]

\[ Z_{\text{Top of steel}} = \frac{40,180}{24.53} = 1,638 \text{ in.}^3 \]

\[ Z_{\text{Bot of steel}} = \frac{40,180}{26.85} = 1,496 \text{ in.}^3 \]

### Bending Moments (Constant I)

<table>
<thead>
<tr>
<th>M, kip-ft</th>
<th>DL₁</th>
<th>DL₂</th>
<th>LL + I</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1,125</td>
<td>-206</td>
<td>-1,056</td>
<td></td>
</tr>
</tbody>
</table>
Steel Stresses for Maximum Negative Moment

Top of Steel Girder (Tension)  
\[ DL_1: f_b = \frac{1.125 \times 12}{1.371} = 9.85 \]  
\[ DL_2: f_b = \frac{206 \times 12}{1.638} = 1.51 \]  
\[ LL+I: f_b = \frac{1.056 \times 12}{1.638} = 7.74 \]  
\[ 19.10 \text{ ksi} \]

Bottom of Steel Girder (Compression)  
\[ f_b = \frac{1.125 \times 12}{1.447} = 9.33 \]  
\[ f_b = \frac{206 \times 12}{1.496} = 1.65 \]  
\[ f_b = \frac{1.056 \times 12}{1.496} = 8.47 \]  
\[ 19.45 \text{ ksi} \]

The stress in the reinforcing bars is

\[ f_b = \frac{(206 + 1.056) \times 12}{1.363} = 11.11 \text{ ksi} \]

Allowable Compressive Stress for 20-ft Diaphragm Spacing

\[ F_b = 20,000 - 7.5 \left( \frac{L}{b} \right)^2 = 20,000 - 7.5 \left( \frac{20 \times 12}{16} \right)^2 = 18.31 \text{ ksi} \]

Because of continuity, the allowable stress at the interior support may be increased 20%, up to 20 ksi.

\[ F_b = 1.20 \times 18.31 = 21.97 \text{ ksi}. \] Use 20 ksi.

Since the bending stresses are less than 20 ksi, the assumed section is satisfactory.

Fatigue Stress in Top Flange

Since the shear connectors are welded to the tension flange of the stringer, fatigue must be considered. The allowable fatigue stress for flange metal adjacent to welded-stud shear connectors is calculated but found not to govern.

Minimum Flange Stress

\[ DL_1: f_b = 9.85 \]  
\[ DL_2: f_b = 1.51 \]  
\[ 11.36 \text{ ksi} \]

Ratio of minimum to maximum stress in the top flange is

\[ R = \frac{11.36}{19.10} = 0.595 \]

The allowable fatigue stress in tension is

\[ F_f = \frac{16.5}{1 - 0.65(0.595)} = 26.91 \text{ ksi} > 20 \text{ ksi} \]

Fatigue does not govern.

Shear-Connector Spacing

Shear-connector spacing with 3 studs per space is determined from the formula for the range of horizontal shear per inch:

\[ S_r = \frac{V \cdot Q}{I} \]

\( Q \) in this case is the statical moment of the area of the reinforcement about the neutral axis. The calculated spacing required for service behavior exceeds the maximum allowable pitch of 24 in permitted by AASHO Specifications.
The range of the shear for live load at the interior support is \( V_v = 62.4 \text{ kips}. \)

\[
Q = 4.34 \times 29.47 = 128 \text{ in.}^3
\]

\[
S_v = \frac{62.4 \times 128}{40,180} = 0.199 \text{ kips per in.}
\]

Spacing required = \( \frac{3 \times 8.11}{0.199} = 122.3 \text{ in.} \)

Use the maximum allowable spacing = 24 in.

The tensile force in the reinforcing bars is

\[
H_3 = A_r F_v = 4.34 \times 40.0 = 173.6 \text{ kips}
\]

The number of studs required is

\[
N = \frac{H_3}{\phi Q_v} = \frac{173.6}{0.85 \times 45} = 4.5
\]

This check for strength indicates that 4.5 studs are required between the interior pier and the dead-load inflection point. The 24-in. maximum allowable spacing provides considerably more than this number of connectors.

---

**Design IV—Four-Span Continuous Girder (100-128-128-100 Ft), Composite For Positive Moment Only**

This design is included to illustrate the advantages of multi-span continuous construction. The procedure followed in the design is identical to that for the two-span girder.

For the four-span girder, the end spans are held at 100 ft and the interior spans are increased to 128 ft. With this arrangement, positive moments are slightly less than in the two-span girder. A section consisting of a 12 × ½-in. top flange, 48 × ⅜-in. web, and 12 × 1½-in. bottom flange is sufficient for the positive-moment regions in both end and interior spans.

Negative moments over both the first interior support and the center support are somewhat greater than the negative moment for the two-span continuous girder of Design II. A section consisting of 18 × 1¼-in. top and bottom flanges at the first interior support and 18 × 1½-in. top and bottom flanges at the center support, with a 48 × ⅝-in. web at both locations, is sufficient for the negative-moment regions.

The four-span continuous welded girder is not governed by fatigue except near inflection points. Near these locations, fatigue controls allowable stresses for butt-welded splices of flanges having different thicknesses, and restricts welding of the intermediate stiffeners so that they are not welded to the flanges.

For the spans considered, the weight of girders and framing details per square foot of roadway are nearly the same for two-span and four-span construction. Thus, four-span construction permits a considerably longer interior span length without an increase in quantity of steel per linear foot. Four-span construction is also more economical because fewer bearing assemblies, expansion joints and piers are required for a given length of roadway. The reduced number of expansion joints provides the further benefit of a smoother riding roadway.
<table>
<thead>
<tr>
<th>Span 1: 100'</th>
<th>First Interior Bearing</th>
<th>Span 2: 128'</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Flange</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60' x 6'</td>
<td>12'-0&quot;</td>
<td>20'-0&quot;</td>
</tr>
<tr>
<td>48' x 6'</td>
<td>18'-0&quot;</td>
<td>20'-0&quot;</td>
</tr>
<tr>
<td>36' x 6'</td>
<td>12'-0&quot;</td>
<td>20'-0&quot;</td>
</tr>
<tr>
<td>24' x 6'</td>
<td>18'-0&quot;</td>
<td>20'-0&quot;</td>
</tr>
<tr>
<td>Diaphragm Spacing</td>
<td>20'-0&quot;</td>
<td>12'-0&quot;</td>
</tr>
<tr>
<td>16'-0&quot;</td>
<td>18'-0&quot;</td>
<td>12'-0&quot;</td>
</tr>
<tr>
<td>12'-0&quot;</td>
<td>18'-0&quot;</td>
<td>12'-0&quot;</td>
</tr>
<tr>
<td>8'-0&quot;</td>
<td>18'-0&quot;</td>
<td>12'-0&quot;</td>
</tr>
<tr>
<td>4'-0&quot;</td>
<td>18'-0&quot;</td>
<td>12'-0&quot;</td>
</tr>
<tr>
<td>Stiffener Spacing</td>
<td>10'-0&quot;</td>
<td>12'-0&quot;</td>
</tr>
<tr>
<td>8'-0&quot;</td>
<td>18'-0&quot;</td>
<td>12'-0&quot;</td>
</tr>
<tr>
<td>6'-0&quot;</td>
<td>18'-0&quot;</td>
<td>12'-0&quot;</td>
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<tr>
<td>4'-0&quot;</td>
<td>18'-0&quot;</td>
<td>12'-0&quot;</td>
</tr>
<tr>
<td>2'-0&quot;</td>
<td>18'-0&quot;</td>
<td>12'-0&quot;</td>
</tr>
<tr>
<td>Shear Connector Spacing</td>
<td>10'-0&quot;</td>
<td>12'-0&quot;</td>
</tr>
<tr>
<td>8'-0&quot;</td>
<td>18'-0&quot;</td>
<td>12'-0&quot;</td>
</tr>
<tr>
<td>6'-0&quot;</td>
<td>18'-0&quot;</td>
<td>12'-0&quot;</td>
</tr>
<tr>
<td>4'-0&quot;</td>
<td>18'-0&quot;</td>
<td>12'-0&quot;</td>
</tr>
<tr>
<td>2'-0&quot;</td>
<td>18'-0&quot;</td>
<td>12'-0&quot;</td>
</tr>
</tbody>
</table>

**End Bearing**
- 12'-0" x 1-1/2"

**Center Bearing**
- 18'-1-1/2"

**End Diaphragm**
- 12'-0"

**Bottom Flange**
- 18'-0"

**Diaphragm Connection Plate**
- 6" x 1-1/2" (Typ.)

**Stiffener Plate**
- (One Side Only)
  - 4" x 1-1/2" (Typ.)

**Web Plate**
- 48" x 1-1/2"

**Bearing Stiffener Plate**
- 7" x 1" Each Side

**Field Splice**
- 12'-0"

**Use:** Minimum size weld for material thickness.
UNSTIFFENED WEBS

As stated in the Introduction, composite welded girders may be designed without intermediate transverse web stiffeners. The additional thickness of web required in unstiffened designs will normally add more than the amount of weight saved by the elimination of stiffeners and a slight reduction in flange material. Thus, the quantity of steel will usually be greater than that for girders with stiffeners. On the other hand, girders without stiffeners require less fabrication, with a resulting lower unit cost of steel.

Girders with unstiffened webs have been designed for the same spans as Designs I and IV. Weight comparisons appear below:

<table>
<thead>
<tr>
<th>Types of Web</th>
<th>Total Weight, Kips 80-Ft</th>
<th>Total Weight, Kips 100-128-128-100-Ft</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Simple-Span Composite</td>
<td>4-Span Continuous Composite</td>
</tr>
<tr>
<td>Unstiffened</td>
<td>54.93</td>
<td>358.57</td>
</tr>
<tr>
<td>Stiffened</td>
<td>51.18</td>
<td>332.31</td>
</tr>
<tr>
<td>Difference:</td>
<td>3.75</td>
<td>26.26</td>
</tr>
</tbody>
</table>

The unstiffened web design is about 7.2 percent heavier than the stiffened design for the 80-ft simple span, and about 8.0 percent heavier for the four-span continuous girder.

CANTILEVER AND SUSPENDED SPANS

If foundation conditions are such that excessive settlement of piers would cause high stresses in continuous stringers, cantilever design can be employed to eliminate stresses from foundation settlement and still give many of the savings obtained in a continuous design. Sufficient hinges are added near points of dead-load inflection to make the girders statically determinate. The following arrangements of hinges are recommended:

- **2 Spans**
- **3 Spans**
- **4 Spans**
If a series of approximately equal spans is required, the following arrangement is recommended:

Cantilever and suspended-span construction generally will be more economical than a series of simple spans, but not as economical as continuous construction. The weight and cost of girders will be about the same for a continuous design and for a cantilever and suspended span design. However, hanger assemblies and expansion-joint assemblies generally required at each hinge will increase the total cost of cantilever and suspended-span construction.
Note:
All material ASTM-A36

*Total Wt. = 51,180 lb
Wt. per sq ft (o. to o. slab) = 19.19 lb per sq ft

*Wt. does not include bearing shoes, railing or studs.
Composite Design Example
2-Span Continuous Welded Girders
Design II
Composite: Welded Plate Girder Load Factor Design

Introduction
Chapter 4 illustrates the design of a composite, welded plate girder bridge by the working stress method. This chapter illustrates load factor design for the same type of construction.

The example presented in this chapter deals with the design of a two-span, continuous girder (100 ft.—100 ft.), composite for positive and negative moments similar to Design III of Chapter 4. The load factor design is in accordance with the 1973 Standard Specifications for Highway Bridges of the American Association of State Highway and Transportation Officials and their Interim Specifications dated 1974, 1975 and 1976. These specifications will be referred to for brevity as AASHTO followed by an article and section reference. USS COR-TEN B (ASTM A588, Grade A) is used for the steel portion of the composite beam. This is a high-strength low-alloy structural steel that is widely used in unpainted bridges where its enhanced atmospheric corrosion resistance is desired to help minimize maintenance costs.

The procedures for dead-load distribution, lateral distribution of live load, computation of reactions, shears, moments and deflections, determination of effective slab widths, section properties (except for plastic section modulus and related properties) and stresses in composite sections are the same for load factor and working stress designs. Descriptive text and illustrative calculations similar to those presented in Chapter 4 are not repeated but the similarity is pointed out.

General Design Considerations
Members designed by the Load Factor method are proportioned for multiples of the design loads. They are required to meet certain criteria for three theoretical load levels: 1) Maximum Design Load 2) Overload and 3) Service Load. The Maximum Design Load and Overload requirements are based on multiples of the service loads with certain coefficients necessary to insure the required capabilities of the structure. Service loads are defined as the same loads as used in working stress design.

The Maximum Design Load criteria insures the structure’s capability of withstanding a few passages of exceptionally heavy vehicles (simultaneously in more than one lane), in times of extreme emergency, that may induce significant permanent deformations.

The Overload criteria insures control of permanent deformations in a member, caused by occasional overweight vehicles equal to 5.3 the design live and impact loads (simultaneously in more than one lane), that would be objectionable to riding quality of the structure.

The Service Load criteria insures that the live load deflection and fatigue life (for assumed fatigue loading) of a member are controlled within acceptable limits.
Moments, shears and other forces are determined by assuming elastic behavior of the structure except for a continuous beam of compact section where negative moments over supports, determined by elastic analysis, may be reduced by a maximum of 10%. This reduction, however, must be accompanied by an increase in the maximum positive moment equal to the average decrease of the negative moments in the span.

DESIGN LOADS
The moments, shears or forces to be sustained by a stress-carrying steel member are computed from the following formulas for the three loading levels.

Service Load: \[ D + (L + I) \]
Overload: \[ D + \frac{5}{3}(L + I) \]

Maximum Design Load: \[ 1.30 \left[ D + \frac{5}{3}(L + I) \right] \]

where \( D \) = dead load
\( L \) = live load
\( I \) = impact load

The factor 1.30 is included to compensate for uncertainties in strength, theory, loading, analysis and material properties and dimensions. The factor 5/3 is incorporated to allow for overloads. Factors for other group loading combinations are given in AASHTO Art. 1.2.22.

DESIGN FOR MAXIMUM DESIGN LOADS
Welded plate girders of normal proportions are not likely to satisfy the requirements for compactness described in Chapter 3A. Therefore, the maximum strength or maximum moment capacity of a girder section is less than it would be if the fully plastic bending strength could be developed.

If a girder meets the requirements for a symmetrical, braced, noncompact section, the maximum strength may be computed from

\[ M_u = F_s S \]

where \( F_s \) = specified minimum yield point or yield strength, psi, of the type of steel being used
\( S \) = elastic section modulus

The section consequently must be proportioned so that

\[ F_s S \geq 1.30 \left[ D + \frac{5}{3}(L + I) \right] \]

Here, \( D \), \( L \), and \( I \) represent moments induced by the service loads.

For this relationship to be permitted, the following criteria must be satisfied:

1. Width-thickness ratio of the compression flange projection, when the bending moment \( M \) induced by the Maximum Design Load equals the maximum strength \( M_u \), should not exceed

\[ \frac{b'}{t} = \frac{2.200}{\sqrt{F_s}} \]

where \( b' \) = width of projecting flange element
\( t \) = flange thickness

When \( M < M_u \), \( b'/t \) may be increased in the ratio \( \sqrt{M_u/M} \).
The b/t requirement need not be satisfied for the compression flanges of composite girders in the positive bending regions.

2. Depth-thickness ratio of the web should not exceed

\[
\frac{D}{t_w} = 150
\]

where \( D \) = clear unsupported distance between flange components
\( t_w \) = web thickness

3. Spacing of lateral bracing of the compression flange should not exceed

\[
L_b = \frac{20,000,000A_f}{F_y d}
\]

where \( A_f \) = cross-sectional area of compression flange
\( d \) = depth of girder

The displacement or twisting of girders, called lateral buckling, may also be prevented by embedment of the top and sides of the compression flange in concrete.

4. Axial compression should not exceed

\[
P = 0.15F_y A
\]

where \( A \) = cross-sectional area of girder

5. Shear should not exceed either of the following values:

\[
V = \frac{3.5Et_w^3}{D}
\]

\[
V = 0.58F_y D t_w
\]

where \( E \) = steel modulus of elasticity

If a girder section acting together with the longitudinal slab reinforcing steel meets the preceding requirements, it may be designed as a braced, noncompact section, though it is unsymmetrical about its horizontal centroidal axis. Its maximum strength is the moment inducing yielding at an extreme surface under Maximum Design Loads composed of the initial dead load, superimposed dead load, and live load plus impact, taking into account whether the construction is shored or unshored when the concrete slab is cast.

When a member does not meet Criterion 3 for spacing of lateral bracing of braced, noncompact sections, it is considered an unbraced section. For symmetrical sections, the calculated maximum strength is reduced to

\[
M_u = F_y S \left[ 1 - \frac{3F_y}{4\pi^2E} \left( \frac{L_b}{b'} \right)^2 \right]
\]

When the ratio of the smaller moment to the larger moment at the ends of the braced length \( L_b \) is less than 0.7, this value of \( M_u \) may be increased 20% but may not exceed \( F_y S \).

For sections unsymmetrical about the horizontal axis but symmetrical about the vertical axis, along the web, maximum strength may be computed from the appropriate formula previously given, except that when the preceding formula for \( M_u \) is used, \( b' \) should be replaced by 0.9b'. Because the girder section with longitudinal slab steel is unsymmetrical about the horizontal axis, this modification applies to the calculation of maximum bending strength when the section does not qualify as braced, in accordance with Criterion 3.

The above AASHTO lateral buckling equation for maximum strength was developed for prismatic compression flanges. In the case where there is a transition in compression flange width or thickness within an unbraced length, the compression flange section throughout this length is no longer prismatic and the AASHTO lateral
buckling requirements are not directly applicable. However, it can be shown that by a modification of application the AASHTO lateral buckling formula can be applied conservatively to girders with stepped flanges.* This can be done by rearranging the AASHTO formula and computing the critical buckling stress of the braced panel, in which the transition occurs as that of the girder in the stepped-down region. This stress may then be increased by 20% providing the ratio of compression flange axial forces at the ends of the braced panel are equal to or less than 0.7. Although concurrent axial forces are theoretically correct, maximum axial forces, as obtained from the moment envelopes, may conservatively be used.

The critical buckling stress is determined from the following rearrangement of the AASHTO buckling formula:

\[ F_{cr} = \frac{M_u}{S} = F_y \left[1 - \frac{3E}{4\pi^2EI} \left( \frac{L_o}{b'} \right)^2 \right] \]

where \( b' \) = projecting compression flange width of the girder in the stepped-down region
\( S = \) section modulus of the steel section in the stepped-down region

The maximum strength at any point in the panel is expressed as:

\[ M_u = F_{cr} S_x \]

where \( S_x \) = section modulus at the point considered.

**BEARING AND INTERMEDIATE STIFFENERS**

In the section of the AASHTO Specifications dealing with load factor design, there are no provisions for bearing stiffeners, though intermediate transverse stiffeners and longitudinal stiffeners are covered. AASHTO Art. 1.7.73, however, requires stiffeners to be placed over bearings of welded plate girders. These stiffeners preferably should be made of plates and should satisfy the following requirements:

- They should extend as nearly as practicable to the outer edges of the flange plates.
- The plates should be placed on both sides of the web.
- The stiffeners should be designed as columns. For stiffeners composed of a pair of plates, the column section should be assumed to comprise those plates plus a centrally located strip of web with width not exceeding 18 times the web thickness.
- The connection of the stiffeners to the web should be capable of transmitting the entire end reaction to the bearings.
- The stiffeners should be ground to fit against the flange through which they receive their reaction or attached to the flange by full-penetration groove welds.
- Only the portion of the stiffeners outside the flange-to-web plate welds should be considered effective in bearing.
- Thickness of the stiffener plates should be at least

\[ t = \frac{b'}{12} \frac{F_y}{12\sqrt{33,000}} \]

AASHTO 1.7.134 contains load factor-design provisions for compression members. Presumably, these would apply to design of bearing stiffeners as columns, whereas the bearing pressure would be limited by the allowable stress in bearing. The total end reaction transmitted to the bearings and caused by the Maximum Design Loads, therefore, should not exceed the maximum strength of the bearing stiffeners as a column. By AASHTO 1.7.134, the maximum strength may be computed from

\[ P_u = 0.85A_s F_{cr} \]

*United States Steel Research reviewed the basis for the AASHTO requirements and analyzed the buckling loads of stepped columns with various geometries. Based on these results, a design procedure was developed which relates the strength of a stepped flange to that of a prismatic flange. For additional information on this procedure contact a U.S.S. Construction Representative through the nearest USS Sales Office.
where $A_e =$ gross effective area of the column cross section

$F_{cr} =$ critical stress, determined from whichever of the following formulas is appropriate

$$F_{cr} = F_y \left[ 1 - \frac{F_y}{4\pi^2E \left( KL/c \right)^2} \right] \quad \frac{KL}{c} \leq \sqrt{\frac{2\pi^2E}{F_y}}$$

$$F_{cr} = \frac{\pi^2E}{(KL/c)^2} \quad \frac{KL}{c} > \sqrt{\frac{2\pi^2E}{F_y}}$$

$K =$ effective length factor, which may be taken as unity for bearing stiffeners

$L_e =$ length of member between points of support $= D$ for bearing stiffeners

$r =$ radius of gyration of the column section in the plane of buckling

Intermediate stiffeners must be provided if a girder section does not satisfy Criterion 5, for shear. The depth-thickness ratio of the web with transverse stiffeners should not exceed

$$\frac{D}{t_w} = \frac{36,500}{F_y^{0.5}}$$

For composite girders and other girders unsymmetrical about the horizontal centroidal axis, if $D_o$, the clear distance between neutral axis and compression flange, exceeds $D/2$, the depth-thickness ratio should not exceed

$$\frac{D_o}{t_w} = \frac{18,250}{F_y^{0.5}}$$

The shear capacity of girder webs with transverse stiffeners is given by

$$V_w = V_p \left[ C + \frac{0.87(1-C)}{\sqrt{1+(d_o/D)^2}} \right]$$

where $V_p = 0.58F_yD t_w$

$d_o =$ distance between transverse stiffeners

$$C = 18,000 \frac{t_w}{D} \sqrt{\frac{1+(D/d_o)^2}{F_y}} - 0.3 \leq 1.0$$

The effect of shear on the bending strength of a girder can usually be ignored. If, however, the shear $V$ on any panel of a girder with transverse stiffeners exceeds $0.6V_w$, the moment $M$ at that section should be limited to

$$M = M_o \left( 1.375 - 0.625 \frac{V}{V_w} \right)$$

where $M_o$ is the bending strength of the section unreduced for shear.

Spacing of transverse stiffeners along a girder should not exceed the distance $d_o$ determined from the preceding formula for $V_w$ nor $1.5D$. At simply supported ends of girders, though, the first stiffener space may not be larger than $D$ nor

$$d_o = 14,500 \sqrt{\frac{Dt_w^2}{V}}$$

Transverse stiffeners should be proportioned so that the width-thickness ratio does not exceed

$$\frac{b'}{t} = \frac{2,600}{F_y^{0.5}}$$

Also, the gross cross-sectional area of each one-sided stiffener or pair of two-sided stiffeners should be at least

$$A = Y \left[ 0.15BDt_w(1-C) \frac{V}{V_w} - 18t_w^2 \right]$$

where $Y =$ ratio of web yield strength to stiffener yield strength

$B = 1.0$ for stiffener pairs

$= 1.8$ for single angles

$= 2.4$ for single plates

$C$ is the same as for the computation of $V_w$. 

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In addition, the required moment of inertia of each stiffener with respect to the mid-plane of the web is

\[ I = d_t \cdot \frac{D}{d_s}^2 - 2 \cdot \frac{D}{d_s} \]

where \( J = 2.5 \left( \frac{D}{d_s} \right)^2 - 2 \geq 0.5 \)

Transverse stiffeners need not bear on a tension flange, but the maximum distance between a face of that flange and the nearest web-to-stiffener weld should not exceed \( 6t_2 \). When stiffeners are provided on only one side of the web, they should bear on the compression flange, but need not be attached to it.

**DESIGN FOR OVERLOAD**

To guard against objectionable deformation under occasional overloads, the following moment relationship must be observed for noncomposite sections and negative bending of composite sections.

\[ 0.8 F_r S \geq D + \frac{5}{3}(L + I) \]

For the same purpose, composite sections in positive bending must satisfy the relationship

\[ 0.95 F_r S \geq D + \frac{5}{3}(L + I) \]

**DESIGN FOR SERVICE LOADS**

Fatigue is investigated in the same manner as in working stress design, using service loads and the provisions of AASHTO Art. 1.7.3. If the longitudinal reinforcing steel in tension over the negative moment region is considered in computing section properties, the stress range in the reinforcing steel is limited to 20,000 psi.

**SHEAR CONNECTORS**

Provisions for shear connectors in load factor design are identical to provisions for working-stress design. These are illustrated in Chapter 3A.

**Design Example—Two-Span Continuous Girder (100-100 Ft) Composite for Positive and Negative Moment**

To illustrate load factor design, an interior girder of a two-span bridge, similar to Design III of Chapter 4, will be designed. The section in the positive-moment region consists of the steel girder acting compositely with the concrete slab. In the negative-moment region, the section consists of the steel girder and the longitudinal slab reinforcing steel. The following data apply to this design:
**Roadway Section:** The same as that shown for Design I in Chapter 4 (see Typical Section).


**Loading:** HS20-44

**Structural Steel:** ASTM A588, Grade A, with $F_v = 50,000$ psi

**Concrete:** $f'_c = 4,000$ psi, modular ratio $n = 8$

**Slab Reinforcing Steel:** ASTM A615, Grade 40 with $F_y = 40,000$ psi

**Loading Conditions:**

Case 1—Weight of girder and slab ($DL_1$) supported by the steel girder alone.

Case 2—Superimposed dead load ($DL_1$) (curbs and railings) supported by the composite section with the modular ratio $n = 8$.

Case 3—Superimposed dead load ($DL_1$) (curbs and railings) supported by the composite section with the increased modular ratio $3n = 3 \times 8 = 24$.

Case 4—Live load plus impact ($L+I$) supported by the composite section with the modular ratio $n = 8$.

**Loading Combinations:**

- Combination A = Case 1 + 3 + 4.
- Combination B = Case 1 + 2 + 4.

**Stress Cycles for Fatigue:**

- 500,000 cycles of truck loading.
- 100,000 cycles of lane loading.

**LOADS, SHEARS AND MOMENTS**

Analysis is based on the assumption of constant moment of inertia throughout the length of the girder.

The initial dead load $DL_1$ consists of an estimated weight of 0.170 kips per ft for the girder and framing details, plus the weight of the 7-in.-thick concrete deck slab. The dead load $DL_1$ carried by the composite section is made up of the weight of the curbs and railings. The live load is AASHTO HS20-44 truck loading with impact for a 100-ft span.

**Dead Load Carried by Steel**

- Slab = $7/12 \times 8.33 \times 0.150 = 0.730$
- Steel girder, details and conc. haunch = 0.170
- $DL_1$ per girder = 0.900 k/ft

**Dead Load Carried by Composite Section**

- Curbs and railings, $DL_2 = 0.660$ k/ft
- $DL_1$ per girder = $0.660/4 = 0.165$ k/ft

**Live Load**

- Live-load distribution = $S = \frac{8.33}{5.5} = 1.51$ wheels = 0.755 axles
- Impact = $\frac{50}{100 + 125} = 0.222$

*No future wearing surface is anticipated for this bridge. If a future wearing surface will be required, its weight must be included in the dead load carried by the composite section and distributed equally to all stringers.
The curves shown for maximum moment and maximum shear may be calculated by any convenient method.
DESIGN OF GIRDER SECTION

In recognition of the fact that normal welded plate girders never satisfy the requirements for compactness, the section is assumed to be either a braced or unbraced, noncompact section. Minimum material requirements for either classification of section are calculated for a trial section with anticipated flange widths of 10 and 14 in. and a web depth of 42 in.

**Thicknesses for Noncompact Section**

For $F_u = 50,000$ psi, the criteria for width-thickness ratio of the projecting compression flanges becomes $b'/t \leq 9.8$ assuming $M = M_u$. Hence, the minimum thickness for a 14-in. compression flange in negative bending, with $b' = 7$ in., is

$$t = \frac{7}{9.8} = 0.714: \text{use } \frac{3}{4} \text{ in.}$$

For a 10-in. compression flange in negative bending, with $b' = 5$ in., the minimum thickness is

$$t = \frac{5}{9.8} = 0.510: \text{use } \frac{5}{8} \text{ in.}$$

To satisfy the criterion for unstiffened web depth-thickness ratio $D/t_w \leq 150$, the web thickness for $D = 42$ in. must be at least

$$t_w = \frac{42}{150} = 0.280: \text{use } \frac{5}{8} \text{ in.}$$

If in subsequent computations it is found that $M < M_u$, these thicknesses may be reduced by the ratio $\frac{M}{\sqrt{M_u}}$

**STIFFENED WEB**

Webs may be designed as fully stiffened, partly stiffened, or unstiffened. The best current design practice seeks to achieve maximum economy by minimizing fabrication. Thus, an optimum design would likely incorporate either a partly stiffened or unstiffened web. Initially, a $\frac{3}{8}$-in. stiffened web is investigated.

The maximum shear permitted on a $\frac{3}{8}$-in. web without stiffeners is the smaller of the following:

$$V = \frac{3.5Et_w^3}{D} = \frac{3.5 \times 29,000(\frac{3}{8})^3}{42} = 127 \text{ kips}$$

$$V = 0.58F_uDt_w = 0.58 \times 50 \times 42 \times \frac{3}{8} = 457 > 127 \text{ kips}$$

**Maximum Shearing Stress at Supports, Kips**

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$L+I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>At Interior Support</td>
<td>56.2</td>
<td>10.3</td>
<td>62.4</td>
</tr>
<tr>
<td>At End Bearing</td>
<td>33.7</td>
<td>6.2</td>
<td>58.7</td>
</tr>
</tbody>
</table>

The shear for Maximum Design Loads at the interior support is

$$V = 1.30 \left(56.2 + 10.3 + \frac{5}{3} \times 62.4\right) = 222 > 127 \text{ kips}$$

Similarly, the shear at the end bearing is

$$V = 1.30 \left(33.7 + 6.2 + \frac{5}{3} \times 58.7\right) = 179 > 127 \text{ kips}$$
Because the shear exceeds that permitted at both the interior support and the end bearing, stiffeners are required at both locations.

If, however, the shears for Maximum Design Loads are plotted along the girder span, there is a region within the span for which the maximum shear at any section is less than the 127 kip capacity of the section. As indicated in the Maximum Design Shear Diagram, web stiffeners may be omitted over a 46-ft-long length in the positive-moment region.

\[ V_p = 0.58F_p D_t = 0.58 \times 50 \times 42 \times \frac{3}{8} = 457 \text{ kips} \]

Also, for use in the formula for \( V_u \),

\[ C = 18,000 \frac{f_y}{D} \sqrt{\frac{1 + (D/d_w)^2}{F_y}} - 0.3 = 18,000 \times \frac{3}{8} \sqrt{\frac{1 + (42/54)^2}{50,000}} - 0.3 = 0.610 \]

Substitution of these values gives the shear capacity as

\[ V_u = V_p \left[ C + \frac{0.87(1 - C)}{\sqrt{1 + (d_w/D)^2}} \right] = 457 \left[ 0.610 + \frac{0.87(1 - 0.610)}{\sqrt{1 + (54/42)^2}} \right] = 374 \text{ kips} \]

\[ 0.6V_u = 0.6 \times 374 = 224 > 222 \text{ kips} \]
No reduction in moment capacity is required for stiffener spacings up to 54 in. Use five spaces at 48 in. in the diaphragm panel adjacent to the pier.

**Stiffener Spacing at Field Splice**
A field splice, 22.5 ft from the pier, lies in the second diaphragm panel from that support. With the decrease in shear with distance from the pier, stiffeners can be placed farther apart than in the first diaphragm panel. The maximum permissible stiffener spacing is \(1.5D = 1.5 \times 42 = 63\) in. Calculations indicate that at 60 in. the shear capacity at the field splice is more than adequate. Shear due to Maximum Design Loads at the splice is 168 kips.

For use in the formula for \(V_s\), \(V_p = 457\) kips and

\[
C = 18,000 \times \frac{\frac{36}{42} \sqrt{\frac{1 + (42/60)^2}{50,000}}} - 0.3 = 0.577
\]

Substitution of these values gives the shear capacity at the splice as

\[
V_s = 457 \left[ 0.577 + \frac{0.87(1 - 0.577)}{\sqrt{1 + (60/42)^2}} \right] = 360\text{ kips}
\]

\(0.6V_s = 0.6 \times 360 = 216 > 168\) kips

**Stiffener Spacing at End Bearing**
AASHTO Specifications require that the first stiffener space at the end bearing be limited for a \(\frac{3}{4}\)-in. web to

\[
d_s = 14,500 \sqrt{\frac{D}{V}} = 14,500 \sqrt{\frac{42(\frac{3}{4})^2}{179,100}} = 51.0 \text{ in.}
\]

Since this spacing exceeds the web depth, the first stiffener spacing is set equal to 42 in. The shear capacity of the web with this stiffener spacing is then investigated. For use in the formula for \(V_s\), \(V_p = 457\) kips and

\[
C = 18,000 \times \frac{\frac{36}{42} \sqrt{\frac{1 + (42/42)^2}{50,000}}} - 0.3 = 0.716
\]

Substitution of these values gives the shear capacity at the end bearing as

\[
V_s = 457 \left[ 0.716 + \frac{0.87(1 - 0.716)}{\sqrt{1 + (42/42)^2}} \right] = 407 > 179\text{ kips}
\]

**Stiffener Spacing 42 In. from End Bearing**
The remaining stiffeners within the first diaphragm panel are placed at 49\(\frac{1}{2}\) in. intervals, and the shear in the web is investigated for this spacing at a distance of 42 in. from the end bearing. Shear due to the factored loads is 168 kips. For use in the formula for \(V_s\), \(V_p = 457\) kips and

\[
C = 18,000 \times \frac{\frac{36}{42} \sqrt{\frac{1 + (42/49.5)^2}{50,000}}} - 0.3 = 0.643
\]

Substitution of these values gives the shear capacity as

\[
V_s = 457 \left[ 0.643 + \frac{0.87(1 - 0.643)}{\sqrt{1 + (49.5/42)^2}} \right] = 386
\]

\(0.6V_s = 0.6 \times 386 = 231 > 168\) kips

**UNSTIFFENED WEB**
As an alternate design, an unstiffened web is investigated. Calculations show that without stiffeners a thickness of \(\frac{3}{16}\) in. is required for the positive-moment region and a thickness of \(\frac{3}{8}\) in. for the negative-moment region.
Required minimum thickness of web is determined from the criterion for shear capacity of an unstiffened web:

\[
t = 3\sqrt{\frac{VD}{3.5E}}
\]

At the end bearing, the shear has previously been calculated to be 179 kips. Hence the web thickness there must be at least

\[
t = 3\sqrt{\frac{179 \times 42}{3.5 \times 29,000}} = 0.420: \text{ use } \frac{5}{8} \text{ in.}
\]

At the interior support, the shear has previously been computed to be 222 kips, for which the web thickness must be at least

\[
t = 3\sqrt{\frac{222 \times 42}{3.5 \times 29,000}} = 0.451: \text{ use } \frac{3}{8} \text{ in.}
\]

**FATIGUE REQUIREMENTS**

Before the design of positive and negative moment sections is begun, it may be helpful to determine in general what fatigue checks should be made. For this welded plate girder with butt welded flange transitions, fillet welded web stiffeners or diaphragm connection plates and stud shear connectors, fatigue should be checked under tension or reversal at the following locations:

**AASHTO Category C**
1. Base metal adjacent to stud-type shear connectors.
2. Base metal in the girder web at the toe of transverse stiffener (or diaphragm connection plate) fillet welds.

**AASHTO Category B**
3. Base metal adjacent to full penetration groove welded flange transitions.
4. Base metal adjacent to continuous fillet welds parallel to applied stress in bottom flanges.

Groove welded flange splices at transitions in width or thickness can be assigned AASHTO fatigue category B providing that transition slopes not exceeding 1 to 2½ are used and that the welds are finished smooth and flush.

At maximum negative moment locations or at transitions in negative moment regions, the top flange is in tension. If shear studs are connected to this flange, AASHTO fatigue category C may govern the maximum stress range there, if less than that permitted by rebar fatigue.

At maximum positive moment locations or at transitions in positive moment regions, the bottom flange is in tension. If a transverse stiffener or diaphragm connection plate is near, the stress range in the web is governed by fatigue category C. Also, if the section is at a groove welded flange splice, the maximum bottom flange stress range cannot exceed that of fatigue category B.

Girder sections near points of contraflexure where stress reversals are likely to occur must be checked for category C allowables at the top flange where shear connectors are likely to be attached and in the bottom of the web if a transverse stiffener or diaphragm connection plate is located there. Also, the bottom flange must be checked for fatigue category B if the section is adjacent to a groove welded splice.

The AASHTO Specifications assign the following allowable ranges of stress to categories B and C:

<table>
<thead>
<tr>
<th></th>
<th>500,000 cycles (Truck Loading)</th>
<th>100,000 cycles (Lane Loading)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Category B</td>
<td>27.5 ksi</td>
<td>45.0 ksi</td>
</tr>
<tr>
<td>Category C</td>
<td>19.0 ksi</td>
<td>32.0 ksi</td>
</tr>
</tbody>
</table>
MAXIMUM NEGATIVE MOMENT AT INTERIOR SUPPORT

The girder will be designed with a stiffened web. The negative-moment section at the interior support is designed first. The section will be noncompact and either braced or unbraced. The design relationship for Maximum Design Load is

\[ F_v S \geq 1.2 \left( D + \frac{5}{3} (L + I) \right) \]

and that for Overload is

\[ 0.8 F_v S \geq \left[ D + \frac{5}{3} (L + I) \right] \text{ or } F_v S \geq 1.25 \left[ D + \frac{5}{3} (L + I) \right] \]

By inspection, the Maximum Design Load relationship governs.

A section made up of a 1\(\frac{1}{4}\) x 14-in. bottom flange, a 1\(\frac{1}{4}\) x 14-in. top flange and a 3\(\frac{3}{4}\) x 42-in. web plate is tried. The slab contains 14 No. 6 longitudinal bars at 6 in. spacing.

![Diagram of girder section](image)

MAXIMUM-NEGATIVE-MOMENT SECTION

With diaphragms spaced at 20 ft, the unbraced length of compression flange exceeds the maximum unbraced length for a braced section.

\[ L_b = \frac{20,000,000 A_f}{F_v d} = \frac{20,000,000 \times 21}{50,000 \times 44.75} = 188 \text{ in.} = 15.7 \text{ ft} < 20 \text{ ft} \]

The negative-moment section, therefore, is an unbraced, noncompact section. As a result, the calculated maximum bending strength is defined by

\[ M_u = F_v S \left[ 1 - \frac{3 F_v}{4 \pi^2 E} \left( \frac{L_b}{0.95 b} \right)^2 \right] \]

or, dividing through by \( S \), the critical lateral buckling stress for the compression flange is expressed as

\[ F_{cr} = \frac{M_u}{S} = F_v \left[ 1 - \frac{3 F_v}{4 \pi^2 E} \left( \frac{L_b}{0.95 b} \right)^2 \right] \]
Assuming that the compression flange does not change in width within the unbraced length, the critical allowable compression stress becomes

$$F_v\left[1 - \frac{3 \times 50,000}{4 \pi^2 \times 29,000,000 \left(\frac{20 \times 12}{0.9 \times 6.81}\right)^2}\right] = 0.799F_v$$

If the ratio of moments at the two ends of the unbraced length is less than 0.7, however, this stress may be increased by 20% but not to more than $F_v$.

### Maximum Negative Moments, Kip-Ft

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$L+I$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>At Interior Support</td>
<td>-1,125</td>
<td>-206</td>
<td>-1,056</td>
<td>-2,387</td>
</tr>
<tr>
<td>At Diaphragm</td>
<td>-180</td>
<td>-33</td>
<td>497</td>
<td>710</td>
</tr>
</tbody>
</table>

The ratio of the total factored moments is

$$R = \frac{1.30 \left[-180 - 33 + \frac{5}{3}(-497)\right]}{1.30 \left[-1,125 - 206 + \frac{5}{3}(-1,056)\right]} = 0.3 < 0.7$$

Hence, the allowable stress may be increased up to 20%.

$$1.20 \times 0.799F_v = 0.959F_v$$

From the preceding, the design relationship for Maximum Design Load at the maximum-negative-moment location is given by

$$1.30 \left[D + \frac{5}{3}(L+I)\right] \leq 0.959F_vS$$

and the design relationship for Overload is given by

$$D + \frac{5}{3}(L+I) \leq 0.80F_vS$$

By inspection, the Maximum Design Load relationship governs.

Section properties are calculated for the girder section alone and for the girder section plus the longitudinal reinforcing bars in the concrete slab. The allowable stresses in the girder section are

- $F_c = 0.959F_v = 0.959(50) = 48.0$ ksi for compression flange
- $F_s = F_v = 50.0$ ksi for tension flange
- $F_v = 40.0$ ksi for rebars

### Steel Section at Interior Support

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_v$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T. Fig. 1(\frac{3}{14}) × 14</td>
<td>17.50</td>
<td>21.62</td>
<td>378</td>
<td>8,184</td>
<td>8,184</td>
<td></td>
</tr>
<tr>
<td>B. Fig. 1(\frac{1}{16}) × 14</td>
<td>21.00</td>
<td>-21.75</td>
<td>-457</td>
<td>9,934</td>
<td>9,934</td>
<td></td>
</tr>
<tr>
<td>Web 3(\frac{1}{4}) × 42</td>
<td>15.75</td>
<td>-</td>
<td></td>
<td>2,315</td>
<td>2,315</td>
<td></td>
</tr>
</tbody>
</table>

$$d_v = \frac{-79}{54.25} = -1.46 \text{ in.}$$

$$d_{Top} \text{ of steel} = 22.25 + 1.46 = 23.71 \text{ in.}$$

$$d_{Bot} \text{ of steel} = 22.50 - 1.46 = 21.04 \text{ in.}$$

$$I_{NA} = \frac{-1.46(79)}{20,318 \text{ in.}^4} = \frac{-115}{20,318 \text{ in.}^4}$$

$$S_{Top} \text{ of steel} = \frac{20.318}{23.71} = 857 \text{ in.}^2$$

$$S_{Bot} \text{ of steel} = \frac{20.318}{21.04} = 966 \text{ in.}^2$$
Steel Section with Reinforcing Steel at Interior Support

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_o$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Girder</td>
<td>54.25</td>
<td>26.55</td>
<td>164</td>
<td>4,342</td>
<td>20,433</td>
<td>4,342</td>
</tr>
<tr>
<td>Reinf. 14-#6</td>
<td>6.16</td>
<td>26.55</td>
<td>164</td>
<td>4,342</td>
<td>20,433</td>
<td>4,342</td>
</tr>
</tbody>
</table>

$60.41 \text{ in.}^2 \quad 85 \text{ in.}^3 \quad 24,775$

$d_r = \frac{85}{60.41} = 1.41 \text{ in.}$

$1.41(85) = -120$

$I_{NA} = \frac{-120}{24,655 \text{ in.}^4}$

$d_{\text{Top of steel}} = 22.25 - 1.41 = 20.84 \text{ in.}$

$d_{\text{Bot. of steel}} = 22.50 + 1.41 = 23.91 \text{ in.}$

$S_{\text{Top of steel}} = \frac{24,655}{20.84} = 1.183 \text{ in.}^2$

$S_{\text{Bot. of steel}} = \frac{24,655}{23.91} = 1.031 \text{ in.}^2$

$d_{\text{Reinf.}} = 20.84 + 1 + 3.30 = 25.14 \text{ in.}$

$S_{\text{Reinf.}} = \frac{24,655}{25.14} = 981 \text{ in.}^3$

Steel Stresses for Maximum Negative Moment Due to Maximum Design Loads

**Top of Steel (Tension)**

For $DL_1$: $F_t = \frac{1.125(12)}{857} \times 1.3 = 20.5$

$F_t = \frac{1,125(12)}{966} \times 1.3 = 18.2$

For $DL_2$: $F_t = \frac{206(12)}{1,183} = 2.7$

$F_t = \frac{206(12)}{1,031} \times 1.3 = 3.1$

For $LL+I$: $F_t = \frac{1,056(12)}{1,183} \times \frac{1.3 \times 5}{3} = 23.2$

$F_t = \frac{1,056(12)}{1,031} \times \frac{1.3 \times 5}{3} = 26.6$

46.4 < 50 ksi

47.9 < 48.0 ksi

**Bottom of Steel (Compression)**

$F_b = \frac{1,125(12)}{966} \times 1.3 = 18.2$

$F_b = \frac{206(12)}{1,031} \times 1.3 = 3.1$

$F_b = \frac{1,056(12)}{1,031} \times \frac{1.3 \times 5}{3} = 26.6$

46.4 < 50 ksi

47.9 < 48.0 ksi

**Reinforcing Steel Stress (Tension)**

$F_t = \frac{1.3 \times 12 \left[ 206 + \frac{5}{3} \times 1,056 \right]}{981} = 31.3 < 40 \text{ ksi}$

Fatigue stress range in the reinforcing steel is limited to 20 ksi. The Service Load stress range is computed to be

$f_{sr} = \frac{1,056(12)}{981} = 12.9 < 20 \text{ ksi}$

In addition to Maximum Design Load, the maximum-negative-moment section should be checked for fatigue at the stud shear connector weld. Assuming that a row of connectors will be placed on the top flange near the interior support, the live load stress range at this location is determined to be

$f_{sr} = \frac{1,056(12)}{1,183} = 10.7 < 32 \text{ ksi (Lane Load Controls)}$

The section is satisfactory in fatigue near the interior support. By inspection, the connection of the stud shear connector is more critical than the top flange connection of the bearing stiffener.

**MAXIMUM POSITIVE MOMENT**

The maximum-positive-moment section qualifies as a braced, noncompact section, because the compression flange is braced throughout by the concrete slab. A trial section comprising a $\frac{3}{8} \times 10\text{-in.}$ top flange (minimum material), $\frac{3}{8} \times 42\text{-in.}$ web, and
1½ × 10-in. bottom flange is selected to meet strength requirements. Properties are calculated for the steel section alone, the composite section with modular ratio 3n = 24, and the composite section with modular ratio n = 8.

### Steel Section

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Flg. 9/10 × 10</td>
<td>5.63</td>
<td>21.28</td>
<td>120</td>
<td>2,549</td>
<td>2,549</td>
<td></td>
</tr>
<tr>
<td>Web ⅜ × 42</td>
<td>15.75</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bot. Flg. 1⅞ × 10</td>
<td>15.00</td>
<td>−21.75</td>
<td>−326</td>
<td>7,096</td>
<td>7096</td>
<td></td>
</tr>
</tbody>
</table>

36.38 in.² −206 in.³

\[ d_s = \frac{−206}{36.38} = −5.66 \text{ in.} \]

\[ d_{Top \text{ of steel}} = 21.56 + 5.66 = 27.22 \text{ in.} \]

\[ S_{Top \text{ of steel}} = \frac{10.79}{27.22} = 397 \text{ in.}³ \]

\[ S_{Bot. \text{ of steel}} = \frac{10.79}{16.84} = 641 \text{ in.}³ \]

### Composite Section, 3n = 24

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>36.38</td>
<td></td>
<td></td>
<td></td>
<td>11,960</td>
<td></td>
</tr>
<tr>
<td>Conc. 84 × 7/24</td>
<td>24.50</td>
<td>26.75</td>
<td>655</td>
<td>17,531</td>
<td>100</td>
<td>17,631</td>
</tr>
</tbody>
</table>

60.88 in.² 449 in.³

\[ d_s = \frac{449}{60.88} = 7.38 \text{ in.} \]

\[ d_{Top \text{ of steel}} = 21.56 − 7.38 = 14.18 \text{ in.} \]

\[ S_{Top \text{ of steel}} = \frac{26,277}{14.18} = 1,853 \text{ in.}³ \]

\[ S_{Bot. \text{ of steel}} = \frac{26,277}{29.88} = 879 \text{ in.}³ \]

### Composite Section, n = 8

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>36.38</td>
<td></td>
<td></td>
<td></td>
<td>11,960</td>
<td></td>
</tr>
<tr>
<td>Conc. 84 × 7/8</td>
<td>73.50</td>
<td>26.75</td>
<td>1,966</td>
<td>52,594</td>
<td>300</td>
<td>52,894</td>
</tr>
</tbody>
</table>

109.88 in.² 1,760 in.³

\[ d_s = \frac{1,760}{109.88} = 16.02 \text{ in.} \]

\[ d_{Top \text{ of steel}} = 21.56 − 16.02 = 5.54 \text{ in.} \]

\[ S_{Top \text{ of steel}} = \frac{36,659}{5.54} = 6,617 \text{ in.}³ \]

\[ S_{Bot. \text{ of steel}} = \frac{36,659}{38.52} = 952 \text{ in.}³ \]

The design relationship for Maximum Design Loads is

\[ P_r S \geq 1.30 \left[ D + \frac{5}{3} (L + I) \right] \]

*When the design does not include a wearing surface on the bridge deck, one-half inch is sometimes subtracted from the slab depth to account for wear from traffic. This was not done in this example design.
The design relationship for Overload, however, is

$$0.95F_p S \geq \left[ D + \frac{5}{3}(L + I) \right] \text{ or } F_p S \geq 1.053 \left[ D + \frac{5}{3}(L + I) \right]$$

By inspection, the Maximum Design Load governs, and the allowable stress is $F_p = 50$ ksi. Maximum positive moment occurs 40 ft from the end support.

**Bending Moments 40 Ft from End Support**

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$L + I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$, kip-ft</td>
<td>630</td>
<td>116</td>
<td>1,136</td>
</tr>
</tbody>
</table>

**Steel Stresses—Combination A**

- **Top of Steel (Compression)**
  - For $DL_1$: $F_p = \frac{630 \times 12}{397} \times 1.30 = 24.8$ kips
  - For $DL_2$: $F_p = \frac{116 \times 12}{1,853} \times 1.30 = 1.0$ kips
  - For $L + I$: $F_p = \frac{1,136 \times 12}{6,617} \times 1.30 \times \frac{5}{3} = 4.5$ kips

- **Bottom of Steel (Tension)**
  - For $DL_1$: $F_p = \frac{630 \times 12}{641} \times 1.30 = 15.3$ kips
  - For $DL_2$: $F_p = \frac{116 \times 12}{879} \times 1.30 = 2.1$ kips
  - For $L + I$: $F_p = \frac{1,136 \times 12}{952} \times 1.30 \times \frac{5}{3} = 31.0$ kips

\[30.3 < 50 \text{ ksi}\]
\[48.4 < 50 \text{ ksi}\]

In addition to Maximum Design Loads, the section used for maximum positive moment must also be investigated for fatigue at the toe of the transverse stiffener fillet weld. To accommodate painting and drainage of moisture, the transverse stiffeners are normally cut back at least 1 in. from the inside face of the tension flange. Furthermore, to lessen fatigue stresses during transportation, it is recommended practice to terminate the stiffener-to-web weld a distance of four to six times the web thickness from the inside face of the tension flange. Using four times the web thickness, the maximum bending stress at the toe of the stiffener fillet weld is given by

$$f' = \frac{M(y - (4t_w + t_f))}{I}$$

The maximum range of live load moments in the positive-moment region occurs at the diaphragm connection stiffener at the 0.4 point of span 1.

$$M_{l.l. \text{ Range}} = 1,136 - (-243) = 1,379 \text{ kip-ft.}$$

The range of tensile stress at the connection plate fillet weld can then be calculated as

$$f_b = \frac{1,379 \times 12}{36,659} \times [38.52 -\left\{(4)(\frac{3}{8}) + 1.5\right]\} = 16.0 < 19 \text{ ksi (Truck Load Controls)}$$

The trial section is satisfactory for maximum positive moment.

**FLANGE-PLATE TRANSITION 17 FT FROM END SUPPORT**

With the two main sections designed, attention is next directed to the transition sections. At 17 ft from the end bearing, the thickness of the bottom flange of the maximum-positive-moment section is stepped down from $1\frac{1}{2}$ to 1 in. Properties are calculated for the steel and composite sections.
Steel Section

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Flg. 3/8 x 10</td>
<td>5.63</td>
<td>21.28</td>
<td>120</td>
<td>2,549</td>
<td>2,315</td>
<td>2,315</td>
</tr>
<tr>
<td>Web 3/8 x 42</td>
<td>15.75</td>
<td>-21.50</td>
<td>-215</td>
<td>4,623</td>
<td>4,623</td>
<td></td>
</tr>
<tr>
<td>Bot. Flg. 1 x 10</td>
<td>10.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

31.38 in.²  - 95 in.²  9,487

\[ d_s = \frac{-95}{31.38} = -3.03 \text{ in.} \]
\[ I_{NA} = \frac{-3.03 \times 95}{16} = 288 \]

\[ d_{\text{Top of steel}} = 21.56 + 3.03 = 24.59 \text{ in.} \]
\[ d_{\text{Bot. of steel}} = 22.0 - 3.03 = 18.97 \text{ in.} \]

\[ S_{\text{Top of steel}} = \frac{9.199}{24.59} = 374 \text{ in.}³ \]
\[ S_{\text{Bot. of steel}} = \frac{9.199}{18.97} = 485 \text{ in.}³ \]

Composite Section, 3n = 24

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>31.38</td>
<td></td>
<td>-95</td>
<td></td>
<td>9,487</td>
<td></td>
</tr>
<tr>
<td>Conc. 84 x 7/24</td>
<td>24.50</td>
<td>26.75</td>
<td>655</td>
<td>17,531</td>
<td>100</td>
<td>17,631</td>
</tr>
</tbody>
</table>

55.88 in.²  560 in.²  27,118

\[ d_{s1} = \frac{560}{55.88} = 10.02 \text{ in.} \]
\[ I_{NA} = \frac{-10.02 \times 560}{16} = 5,611 \]

\[ d_{\text{Top of steel}} = 21.56 - 10.02 = 11.54 \text{ in.} \]
\[ d_{\text{Bot. of steel}} = 22.00 + 10.02 = 32.02 \text{ in.} \]

\[ S_{\text{Top of steel}} = \frac{21.507}{11.54} = 1,864 \text{ in.}³ \]
\[ S_{\text{Bot. of steel}} = \frac{21.507}{32.02} = 672 \text{ in.}³ \]

Composite Section, n = 8

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>31.38</td>
<td></td>
<td>-95</td>
<td></td>
<td>9,487</td>
<td></td>
</tr>
<tr>
<td>Conc. 84 x 3/8</td>
<td>73.50</td>
<td>26.75</td>
<td>1,966</td>
<td>52,594</td>
<td>300</td>
<td>52,894</td>
</tr>
</tbody>
</table>

104.88 in.²  1,871 in.³  62,381

\[ d_s = \frac{1.871}{104.88} = 17.84 \text{ in.} \]
\[ I_{NA} = \frac{-17.84 \times 1.871}{16} = 33.379 \]

\[ d_{\text{Top of steel}} = 21.56 - 17.84 = 3.72 \text{ in.} \]
\[ d_{\text{Bot. of steel}} = 22.00 + 17.84 = 39.84 \text{ in.} \]

\[ S_{\text{Top of steel}} = \frac{29,002}{3.72} = 7,852 \text{ in.}³ \]
\[ S_{\text{Bot. of steel}} = \frac{29,002}{39.84} = 728 \text{ in.}³ \]

As with previous sections, Maximum Strength is more critical than Overload and is investigated. Also, fatigue in base metal adjacent to the butt-welded flange transition and fatigue in the web at the toe of transverse stiffener fillet welds must be checked.

Bending Moments 17 Ft from End Support

<table>
<thead>
<tr>
<th>( M_i ), kip-ft</th>
<th>( DL_1 )</th>
<th>( DL_2 )</th>
<th>(- (L + I))</th>
<th>( + (L + I) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>440</td>
<td>80</td>
<td>-104</td>
<td>739</td>
<td></td>
</tr>
</tbody>
</table>
Steel Stresses 17 Ft from End Support Due to Maximum Design Loads

Top of Steel (Compression)  | Bottom of Steel (Tension)
---|---
For $DL_1$: $F_b = \frac{440 \times 12}{374} \times 1.30 = 18.3$  | $F_b = \frac{440 \times 12}{485} \times 1.30 = 14.1$
For $DL_2$: $F_b = \frac{80 \times 12}{1,864} \times 1.30 = 0.7$  | $F_b = \frac{80 \times 12}{672} = 1.9$
For $L+I$: $F_b = \frac{739 \times 12}{7,796} \times 1.30 \times \frac{5}{3} = 2.5$  | $F_b = \frac{739 \times 12}{728} \times 1.30 \times \frac{5}{3} = 26.4$

$21.5 < 50 \text{ ksi}$  |  $42.4 < 50 \text{ ksi}$

Strength of the section is satisfactory. Fatigue is investigated next.

The range of stress in the bottom flange at the transition is

$$ f_{tv} = \frac{(739 + 104)(12)}{728} = 13.9 < 27.5 \text{ ksi (Truck Load Controls)} $$

A transverse web stiffener is located 1'-1/2" away from the flange transition where the bottom flange has been reduced to a 1 in. thickness. By observation of the stress range at the transition, it can be concluded that the stress range in the girder web at the toe of the stiffener fillet weld does not exceed the 15 ksi allowable for such a detail.

Resistance to fatigue is therefore satisfactory at the transition 17 ft from the end support.

**FLANGE TRANSITIONS 6 FT FROM INTERIOR SUPPORT**

A transition in section in the negative-moment region is located 6 ft from the interior support. The top 14-in.-wide flange plate is reduced in thickness from 1\(\frac{1}{4}\) to \(\frac{3}{4}\) in. and the bottom flange plate is reduced in thickness from \(\frac{1}{2}\) to 1 in. Properties are calculated for the steel section alone and for the steel section plus the longitudinal reinforcing in the slab.

### Steel Section 6 Ft from Interior Support

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>$Ad^2$</th>
<th>$I_s$</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Flg. (\frac{3}{4}) x 14</td>
<td>10.50</td>
<td>21.37</td>
<td>224</td>
<td>4,795</td>
<td>2,315</td>
<td>4,795</td>
</tr>
<tr>
<td>Web (\frac{3}{4}) x 42</td>
<td>15.75</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bot. Flg. 1 x 14</td>
<td>14.00</td>
<td>-21.50</td>
<td>-301</td>
<td>6,472</td>
<td></td>
<td>6,472</td>
</tr>
</tbody>
</table>

\(\text{d} = \frac{77}{40.25} = 1.91 \text{ in.}\)

\(d_{\text{Top of steel}} = 21.75 + 1.91 = 23.66 \text{ in.}\)

\(d_{\text{Bot. of steel}} = 22.00 - 1.91 = 20.09 \text{ in.}\)

\(S_{\text{Top of steel}} = \frac{13,435}{23.66} = 568 \text{ in.}^3\)

\(S_{\text{Bot. of steel}} = \frac{13,435}{20.09} = 669 \text{ in.}^3\)

### Steel Section with Reinforcing Steel 6 Ft from Interior Support

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>$Ad^2$</th>
<th>$I_s$</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>40.25</td>
<td></td>
<td>-77</td>
<td>164</td>
<td>4,342</td>
<td>13,582</td>
</tr>
<tr>
<td>Reinf. 14-#6</td>
<td>6.16</td>
<td>26.55</td>
<td></td>
<td></td>
<td></td>
<td>4,342</td>
</tr>
</tbody>
</table>

\(\text{d} = \frac{1.87}{46.41} = 0.87 \text{ in.}\)

\(d_{\text{Top of steel}} = 21.75 - 1.87 = 19.88 \text{ in.}\)

\(d_{\text{Bot. of steel}} = 22.00 + 1.87 = 23.87 \text{ in.}\)

\(S_{\text{Top of steel}} = \frac{17,761}{19.88} = 893 \text{ in.}^3\)

\(S_{\text{Bot. of steel}} = \frac{17,761}{23.87} = 744 \text{ in.}^3\)

\(d_{\text{Reinf.}} = 22.25 - 1.87 + 3.30 = 24.68 \text{ in.}\)

\(S_{\text{Reinf.}} = 17,761 \div 24.68 = 720 \text{ in.}^3\)
Bending Moments 6 Ft from Interior Support

<table>
<thead>
<tr>
<th></th>
<th>DL_1</th>
<th>DL_2</th>
<th>-(L+I)</th>
<th>+(L+I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M, kip-ft</td>
<td>-805</td>
<td>-150</td>
<td>-721</td>
<td>+63</td>
</tr>
</tbody>
</table>

The allowable compressive stress for the 14-in.-wide girder flange which is not prismatic within the unbraced length was previously computed to be \(0.972F_v=0.972\) (50) = 48.6 ksi.

Steel Stresses 6 Ft from Interior Support Due to Maximum Design Loads

**Top of Steel (Tension)**

For \(DL_1\): \(F_t = \frac{805(12)}{568} \times 1.3 = 22.1\)

For \(DL_2\): \(F_t = \frac{150(12)}{893} \times 1.3 = 2.6\)

For \(L+I\): \(F_t = \frac{721(12)}{893} \times 1.3 \times \frac{5}{3} = 21.0\)

**Bottom of Steel (Compression)**

\(F_v = \frac{805(12)}{669} \times 1.3 = 18.8\)

\(F_v = \frac{150(12)}{744} \times 1.3 = 3.1\)

\(F_v = \frac{721(12)}{744} \times 1.3 \times \frac{5}{3} = 25.2\)

\(45.7 < 50\) ksi

\(47.1 < 48.6\) ksi

Since shear connectors are attached to the top flange, a fatigue check should be made at the transition to insure that the tensile stress range in the top flange is within the allowable. The range of live load tensile stress in the top flange is determined to be

\(f_{tu} = \frac{(721+63)(12)}{893} = 10.5 < 19\) ksi

Reinforcing Steel Stress (Tension) 6 Ft from Interior Support

\(f_t = \frac{1.3 \times 12 \left(150 + \frac{5}{3} \times 721\right)}{720} = 29.3 < 40\) ksi

The fatigue stress range in the reinforcing steel is

\(f_{tu} = \frac{12(721+63)}{720} = 13.1 < 20\) ksi

Thus the reinforcing steel is satisfactory for fatigue.

**WELDED FIELD SPLICE**

A field splice is located 22.5 ft from the pier. This location places the splice midway between a diaphragm and web stiffener and close to the dead-load inflection point.

For the positive-moment side of the splice, a section with a \(\frac{3}{4}\times10\)-in. top flange and a \(1\times10\)-in. bottom flange is tried. These flange plates then have the same thickness as the 14-in.-wide flanges on the negative-moment side of the splice. For a welded field splice, this offers the advantage that transition slopes are not required and, if the splice were to be bolted, filler plates would not be required.

A welded field splice is investigated 22.5 ft from the pier. Section properties are calculated for the steel, steel with slab reinforcing steel and composite sections.

*Connection of studs to the top flange governs over connection of stiffeners to the top flange.
### Steel Section on Positive-Moment Side of Splice

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_o$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Web ¾ × 42</td>
<td>15.75</td>
<td>21.50</td>
<td>215</td>
<td>4,622</td>
<td></td>
<td>2,315</td>
</tr>
<tr>
<td>B. Flg. 1 × 10</td>
<td>10.00</td>
<td>21.50</td>
<td>215</td>
<td>4,622</td>
<td></td>
<td>4,622</td>
</tr>
</tbody>
</table>

\[d = \frac{-55}{33.25} = -1.65 \text{ in.}\]

\[d_{\text{Top of steel}} = 21.75 + 1.65 = 23.40 \text{ in.}\]

\[d_{\text{Bot of steel}} = 22.00 - 1.65 = 20.35 \text{ in.}\]

\[S_{\text{Top of steel}} = \frac{10,271}{23.40} = 439 \text{ in.}^3\]

\[S_{\text{Bot of steel}} = \frac{10,271}{20.35} = 505 \text{ in.}^3\]

### Steel Section with Slab Reinforcing Steel on Positive-Moment Side of Splice

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_o$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>33.25</td>
<td>26.55</td>
<td>164</td>
<td>4,342</td>
<td></td>
<td>10,362</td>
</tr>
<tr>
<td>Reinf. 14-# 6</td>
<td>6.16</td>
<td>26.55</td>
<td>164</td>
<td>4,342</td>
<td></td>
<td>4,342</td>
</tr>
</tbody>
</table>

\[d = \frac{109}{39.41} = 2.77 \text{ in.}\]

\[d_{\text{Top of steel}} = 21.75 - 2.77 = 18.98 \text{ in.}\]

\[d_{\text{Bot of steel}} = 22.00 + 2.77 = 24.77 \text{ in.}\]

\[S_{\text{Top of steel}} = \frac{14,402}{18.98} = 759 \text{ in.}^3\]

\[S_{\text{Bot of steel}} = \frac{14,402}{24.77} = 581 \text{ in.}^3\]

\[d_{\text{Reinf.}} = 22.25 - 2.77 + 1.00 + 3.30 = 23.78 \text{ in.}\]

\[S_{\text{Reinf.}} = \frac{14,402}{23.78} = 606 \text{ in.}^3\]

### Composite Section, 3n = 24

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_o$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>33.25</td>
<td>24.50</td>
<td>655</td>
<td>17,531</td>
<td>100</td>
<td>17,631</td>
</tr>
<tr>
<td>Conc. 84×7/24</td>
<td>24.50</td>
<td>26.75</td>
<td>655</td>
<td>17,531</td>
<td>100</td>
<td>17,631</td>
</tr>
</tbody>
</table>

\[d_s = \frac{600}{57.75} = 10.39 \text{ in.}\]

\[d_{\text{Top of steel}} = 21.75 - 10.39 = 11.36 \text{ in.}\]

\[d_{\text{Bot of steel}} = 22.00 + 10.39 = 32.39 \text{ in.}\]

\[S_{\text{Top of steel}} = \frac{21,759}{11.36} = 1,915 \text{ in.}^3\]

\[S_{\text{Bot of steel}} = \frac{21,759}{32.39} = 672 \text{ in.}^3\]
Composite Section, \( n = 8 \)

<table>
<thead>
<tr>
<th>Material</th>
<th>( A )</th>
<th>( d )</th>
<th>( Ad )</th>
<th>( Ad^2 )</th>
<th>( I_a )</th>
<th>( I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>33.25</td>
<td>26.75</td>
<td>-55</td>
<td>52,594</td>
<td>300</td>
<td>52,894</td>
</tr>
<tr>
<td>Conc. 84 x ( \frac{3}{8} )</td>
<td>73.50</td>
<td>1,966</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ d_a = \frac{1,911}{106.75} = 17.90 \text{ in.} \]
\[ d_{\text{Bot of steel}} = 22.00 + 17.90 = 39.90 \text{ in.} \]
\[ d_{\text{Top of steel}} = \frac{29,046}{3.85} = 7,544 \text{ in}^2 \]

\[ S_{\text{Bot of steel}} = \frac{29,046}{39.90} = 728 \text{ in}^3 \]

**Maximum Moments at Field Splice**

<table>
<thead>
<tr>
<th>With Positive Live-Load Moment</th>
<th>With Negative Live-Load Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>( DL_1; M = -95 )</td>
<td>( DL_1; M = -95 )</td>
</tr>
<tr>
<td>( DL_2; M = -20 )</td>
<td>( DL_2; M = -20 )</td>
</tr>
<tr>
<td>( L+I; M = +575 )</td>
<td>( L+I; M = -445 )</td>
</tr>
</tbody>
</table>

Because of the location of the section close to the dead-load inflection point, bending strength is not critical. But fatigue must be investigated at the butt welds in the bottom flanges and at the shear-connector welds to the top flange.

The maximum range of stress in the bottom flange at the field splice is determined to be

\[ f' = \frac{480(12)}{581} + \frac{575(12)}{728} = 9.9 + 9.5 = 19.4 < 27.5 \text{ ksi} \]

Assuming that shear connectors are welded to the top flange of the girder near the splice, the stress range at that point cannot exceed 19 ksi. The actual stress range is

\[ f' = \frac{480(12)}{759} + \frac{575(12)}{7,544} = 7.59 + 0.91 = 8.50 < 19 \text{ ksi} \]

**BOLTED FIELD SPLICE**

A bolted field splice is designed as an alternative to the welded splice. The bolted splice is a friction-type connection made with \( \frac{3}{8} \)-in.-dia A325 bolts. Design calculations and details of this splice are given later, to permit continuation of the investigation of flange-plate transitions.

**FLANGE TRANSITION 28 FT FROM INTERIOR SUPPORT**

Next, the location must be determined at which the transition from the section with \( \frac{3}{4} \times 10 \)-in. top flange and \( 1 	imes 10 \)-in. bottom flange to the maximum-positive-moment section, with \( \frac{3}{8} \times 10 \)-in. top flange and \( 1 \frac{1}{2} \times 10 \)-in. bottom flange can be made. For this purpose, the transition is assumed and stresses are checked at 28 ft from the pier. Fatigue is the governing condition. Section properties are the same as those calculated for the transition at the field splice.

**Maximum Moments 28 Ft from Interior Support**

<table>
<thead>
<tr>
<th>With Positive Live-Load Moment</th>
<th>With Negative Live-Load Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>( DL_1; M = 95 )</td>
<td>( DL_1; M = 95 )</td>
</tr>
<tr>
<td>( DL_2; M = 20 )</td>
<td>( DL_2; M = 20 )</td>
</tr>
<tr>
<td>( L+I; M = 735 )</td>
<td>( L+I; M = -445 )</td>
</tr>
</tbody>
</table>
Steel Stresses 28 Ft from Interior Support Due To Maximum Design Loads

Top of Steel (Compression)  
For $DL_1$: $F_b = \frac{95 \times 12}{439} \times 1.30 = 3.4$  
For $DL_2$: $F_b = \frac{20 \times 12}{1,915} \times 1.30 = 0.2$  
For $L+I$: $F_b = \frac{735 \times 12}{7,544} \times 1.30 \times \frac{5}{3} = 2.5$  

Bottom of Steel (Tension)  
$F_b = \frac{95 \times 12}{505} \times 1.30 = 2.9$  
$F_b = \frac{20 \times 12}{672} \times 1.30 = 0.5$  
$F_b = \frac{735 \times 12}{728} \times 1.30 \times \frac{5}{3} = 26.2$  

Strength of the composite section is much greater than needed. Fatigue is investigated in the bottom flange butt weld and in shear connector welds to the top flange. There is no transverse stiffener at the transition but fatigue should be investigated at the toe of the stiffener-to-web weld 25 ft from the interior support.

The maximum range of stress in the bottom flange at the transition is determined to be

$$f_{nr} = \frac{445(12)}{581} + \frac{735(12)}{728} = 9.2 + 12.1 = 21.3 < 27.5 \text{ ksi}$$

The stress range in the top flange near stud shear connector welds is

$$f_{nr} = \frac{445(12)}{759} + \frac{735(12)}{7,544} = 7.0 + 1.2 = 8.2 < 19 \text{ ksi}$$

Earlier calculations for stiffener spacing place a stiffener 3.0 ft away from the flange transition or 25 ft from the interior support. Live-load moments at this position are determined to be

$$(L+I) = 645 \text{ kip-ft} \quad -(L+I) = -462 \text{ kip-ft}$$

and the live load stress range is

$$f_{nr} = \frac{462(12)(20+2.77)}{14,402} + \frac{645(12)(20+17.90)}{29,046} = 8.8 + 10.1 = 18.9 < 19 \text{ ksi}$$

**Bolted-Splice Alternative**

With a bolted field splice, it may, in some cases, be more economical to extend the heavier maximum-positive-moment section all the way to the splice and eliminate the butt-welded transition. Filler plates would then be required at the splice.

**FLANGE-TO-WEB WELDS**

The flange-to-web welds must have sufficient strength to transfer the horizontal shear between the girder flange and web plus adequate resistance to fatigue. AASHTO Art. 1.7.135 limits the maximum strength of the welds to 0.45 of the specified minimum tensile strength of the welding-rod metal. (The ultimate strength of the weld metal in fillet welds need not match the strength of the base metal.) Fatigue limitations are the same as those for weld metal in working-stress design.

With $F_u = 70$ ksi for A588 steel, the design relationship for strength of a fillet weld is

$$1.30 \left[ D + \frac{5}{3}(L+I) \right] \leq (0.45 \times 70 \times 0.707 = 22.3 \text{ ksi})$$

Here, $D$, $L$ and $I$ are the shear stresses due to dead, live and impact loads, respectively.

**FLANGE-TO-WEB WELDS AT END SUPPORT**

The flange-to-web welds are checked initially at the end bearing. Section properties needed for shear calculations are determined first. The moments of inertia of the steel
section alone and the composite section with \( n = 8 \) are the same as those computed for the flange-plate transition 17 ft from the end support. Calculations indicate that the weld size is governed by material thickness rather than maximum-strength requirements.

### Section Properties at End Support

<table>
<thead>
<tr>
<th>Steel Section Only</th>
<th>Composite Section, ( n = 8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I = 9,199 \text{ in.}^4 )</td>
<td>( I = 29,002 \text{ in.}^4 )</td>
</tr>
<tr>
<td>Top Flg.: ( Q = \frac{9}{16} \times 10 \times 24.31 = 137 \text{ in.}^3 )</td>
<td>Top Flg.: ( Q = \frac{9}{16} \times 10 \times 3.44 = 19 )</td>
</tr>
<tr>
<td>Bot. Flg.: ( Q = 1 \times 10 \times 18.47 = 185 \text{ in.}^3 )</td>
<td>Conc.: ( Q = \frac{7}{8} \times 84 \times 8.91 = 655 \frac{674}{674} \text{ in.}^3 )</td>
</tr>
<tr>
<td>Bot. Flg.: ( Q = 1 \times 10 \times 39.34 = 393 \text{ in.}^3 )</td>
<td>Bot. Flg.: ( Q = 1 \times 10 \times 39.34 = 393 \text{ in.}^3 )</td>
</tr>
</tbody>
</table>

### Maximum Shears at End Support

- **With Positive Live-Load Shear**
  - \( DL_1: V = 33.7 \)
  - \( DL_2: V = 6.2 \)
  - \( L + I: V = 58.7 \)
- **With Negative Live-Load Shear**
  - \( DL_1: V = -33.7 \)
  - \( DL_2: V = -6.2 \)
  - \( L + I: V = -6.0 \)

### Shear Flow \( S = VQ/I \) Due to Maximum Design Loads

<table>
<thead>
<tr>
<th>Top Weld</th>
<th>Bottom Weld</th>
</tr>
</thead>
<tbody>
<tr>
<td>For ( DL_1 ): ( S = \frac{33.7 \times 137}{9,199} \times 1.30 = 0.65 ) ( S = \frac{33.7 \times 185}{9,199} \times 1.30 = 0.88 )</td>
<td>( S = \frac{33.7 \times 137}{9,199} \times 1.30 = 0.65 ) ( S = \frac{33.7 \times 185}{9,199} \times 1.30 = 0.88 )</td>
</tr>
<tr>
<td>For ( DL_2 ): ( S = \frac{6.2 \times 674}{29,002} \times 1.30 = 0.19 ) ( S = \frac{6.2 \times 393}{29,002} \times 1.30 = 0.11 )</td>
<td>( S = \frac{6.2 \times 674}{29,002} \times 1.30 = 0.19 ) ( S = \frac{6.2 \times 393}{29,002} \times 1.30 = 0.11 )</td>
</tr>
<tr>
<td>For ( L + I ): ( S = \frac{58.7 \times 674}{29,002} \times 1.30 \times \frac{5}{3} = 2.96 ) ( S = \frac{58.7 \times 393}{29,002} \times 1.30 \times \frac{5}{3} = 1.72 )</td>
<td>( S = \frac{58.7 \times 674}{29,002} \times 1.30 \times \frac{5}{3} = 2.96 ) ( S = \frac{58.7 \times 393}{29,002} \times 1.30 \times \frac{5}{3} = 1.72 )</td>
</tr>
</tbody>
</table>

Shear in the top weld governs. For two welds the shear flow in each weld is 3.80/2 = 1.90 kips per in.

Weld size required = \( \frac{1.90}{2.23} = 0.85 \text{ in.} \)

This, however, is less than the minimum size of weld required by AASHTO Specifications for the thickness of flange plate. Therefore, use the following minimum-size welds:

- For \( \frac{9}{16} \)-in. top flange, \( \frac{3}{4} \)-in. fillet welds.
- For 1-in. bottom flange, \( \frac{5}{16} \)-in. fillet welds.

### Shear Flow Range in Top Weld Due to Service Loads

Maximum shear range occurs at the end-support of the girder and is equal to

\[
s_{max} = \left( \frac{58.7 + 6.0}{29,002} \right) = 1.50 \text{ kips per in.}
\]

Shear stress on the throat of fillet welds falls into AASHTO fatigue category F. For 500,000 cycles of truck loading, the associated allowable shear stress range is 12 ksi. The actual stress range in the top weld at the end-support section equals

\[
f_a = \frac{1.50}{2 \times 0.707 \times \frac{3}{4}} = 4.24 < 12 \text{ ksi}
\]

The web-to-flange welds have adequate resistance to fatigue.
FLANGE-TO-WEB WELDS AT INTERIOR SUPPORT

Calculations similar to those for the welds at the end bearing are made to determine flange-to-web weld size at the pier. They also show that material thickness rather than strength governs. Additional computations indicate that this also is the case throughout the length of the girder.

Section properties are computed first at the interior support. The moments of inertia for the girder and the girder plus reinforcing steel were calculated previously for determination of bending strength.

**Section Properties at Interior Support**

<table>
<thead>
<tr>
<th>Steel Section Only</th>
<th>Steel plus Reinforcing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I = 20,318 \text{ in.}^4$</td>
<td>$I = 24,655 \text{ in.}^4$</td>
</tr>
<tr>
<td>Top Flg.: $Q = 1.25(14)(23.09) = 404 \text{ in.}^3$</td>
<td>Top Flg.: $Q = 1.25(14)(20.22) = 354$</td>
</tr>
<tr>
<td>Reinf.: $Q = 6.16(25.14) = 155$</td>
<td>$= 509 \text{ in.}^3$</td>
</tr>
<tr>
<td>Bot. Flg.: $Q = 1.5(14)(20.29) = 426 \text{ in.}^3$</td>
<td>Bot. Flg.: $Q = 1.5(14)(23.16) = 486 \text{ in.}^3$</td>
</tr>
</tbody>
</table>

**Maximum Shears at Interior Support**

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$L + I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$, kips</td>
<td>56.2</td>
<td>0.3</td>
<td>62.4</td>
</tr>
</tbody>
</table>

Shear Flow $S = VQ/I$ Due to Maximum Design Loads

**Top Weld**

For $DL_1$: $S = \frac{56.2 \times 404}{20,318} \times 1.3 = 1.45$

For $DL_2$: $S = \frac{10.3 \times 509}{24,655} \times 1.3 = 0.28$

For $L + I$: $S = \frac{62.4 \times 509}{24,655} \times 1.3 \times \frac{5}{3} = 2.79$

4.52 kips per in.

Shear in the top weld governs. As before, the allowable stress on the fillet weld is 22.3 ksi. For two welds,

weld size required $= \frac{4.52}{2(22.3)} = 0.101 \text{ in.}$

Minimum size of fillet weld permitted for the 1¼ and 1½-in. thick flanges, however, is $\frac{5}{16}$ in. Next, this size of weld is investigated for fatigue.

The range of shear flow in the top weld due to Service Loads equals

$s_r = \frac{62.4 \times 509}{24,655} = 1.29 \text{ kips per in.}$

The corresponding stress range on a $\frac{5}{16}$-in. web-to-flange weld is

$f_{cr} = \frac{1.29}{2 \times 0.707 \times \frac{5}{16}} = 2.92 < 12 \text{ ksi}$

The web-to-flange welds at the interior support have adequate resistance to fatigue.

**FLANGE-TO-WEB WELDS FOR VARIOUS FLANGE THICKNESSES**

Minimum weld size for material thickness governs throughout the length of the girder.
Weld sizes are as follows: ¼ in. where the top flange is ⅜ in., thick and ⅜ in. for the top and bottom flanges elsewhere.

**SHEAR CONNECTORS**

Welded studs, ⅝ in. in diameter by 5 in. long, are selected for use as shear connectors. Because the calculations for this example are similar to the calculations made in Chapter 3A, only the shear-connector spacing diagram is shown here. Fatigue governs the spacing in the positive-moment region while the 24-in. maximum spacing controls in the negative-moment region.

**SHEAR CONNECTOR SPACING**

**TRANSVERSE INTERMEDIATE STIFFENERS**

Each transverse intermediate stiffener consists of a plate welded to one side of the web. As is the rest of the girder, the stiffener is made of A588 steel. A 4-in.-wide stiffener is tried.

Thickness required is determined from the requirement for maximum width-thickness ratio of stiffeners. Minimum thickness thus is

\[ t = \frac{b' \sqrt{F_y}}{2,600} = \frac{4 \sqrt{50,000}}{2,600} = 0.344 \text{ in.} \]

Use a ⅞-in.-thick stiffener plate. Cross-sectional area of the plate is \( A = 4 \times \frac{3}{8} = 1.50 \text{ in.}^2 \)

and the moment of inertia is

\[ I = \frac{(\frac{3}{8})(4)^3}{3} = 8.0 \text{ in.}^4 \]

Area and moment-of-inertia requirements are checked for the stiffener near the interior support, where shear is largest and equals 222 kips for the Maximum Design Loads. Distance of the stiffener from the support is \( d_s = 48 \text{ in.} \). For calculation of the minimum stiffener area required by AASHTO Specifications, \( t_s = \frac{3}{8} \text{ in.}, D = 42 \text{ in.}, Y = 1 \text{ and } B = 2.4 \). Also needed are \( C \) and \( V_u \), the shear capacity of the web. For computation of \( V_u \), \( V_s \) previously has been found to be 457 kips.

\[ C = 18,000 \frac{t_s}{D} \sqrt{\frac{1 + (D/d_s)^2}{F_y}} - 0.3 = 18,000 \times \frac{3/8}{42} \sqrt{\frac{1 + (42/48)^2}{50,000}} - 0.3 = 0.655 \]

The shear capacity of the web then is

\[ V_u = V_s \left[ C + \frac{0.87(1-C)}{\sqrt{1+(d_s/D)^2}} \right] = 457 \left[ 0.655 + \frac{0.87(1-0.655)}{\sqrt{1+(48/42)^2}} \right] = 390 \text{ kips} \]
Substitution in the formula for minimum cross-sectional area of stiffener required gives

\[ A = Y \left[ 0.15BDt_w(1-C) \frac{V}{V_u} - 18t_u^2 \right] \]

\[ = 0.15 \times 2.4 \times 42 \times \left( \frac{3}{8} \right)^2 \frac{222}{390} \times 18 \left( \frac{3}{8} \right)^2 = -1.42 \text{ in.}^2 \]

The negative result indicates that the area requirement can be ignored. The area formula is based on the assumption that a portion of the web assists the stiffener. The quantity $18t_u^2$ in the area formula represents the contribution of the web. When this quantity predominates; that is, when the formula yields a negative number, the web itself contributes more than the required area.

For computation of the required moment of inertia of the stiffener,

\[ J = 2.5 \left( \frac{D}{d_o} \right)^2 - 2 = 2.5 \left( \frac{42}{48} \right)^2 - 2 = 0.086 < 0.5 \]

The minimum value permitted for $J$ is 0.5. The required moment of inertia then is

\[ I = d_o t_w^3 J = 48 \left( \frac{3}{8} \right)^3 0.5 = 1.3 < 8.0 \text{ in.}^4 \]

Use $\frac{3}{8} \times 4$-in. stiffeners.

**BEARING STIFFENERS AT END SUPPORT**

The bearing stiffeners are designed as columns to carry the reaction forces at points of support. A stiffener consisting of two $4\frac{1}{2}$-in.-wide plates welded to opposite sides of the girder web is investigated at the end support. Minimum thickness of stiffener required is

\[ t = \frac{b'}{12} \sqrt{\frac{F_v}{33,000}} = \frac{4.5}{12} \sqrt{\frac{50,000}{33,000}} = 0.46 \text{ in.} \]

Use a $\frac{1}{2}$-in.-thick stiffener plate.

### End Reaction

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$L+I$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$, kips</td>
<td>33.7</td>
<td>6.2</td>
<td>58.7</td>
<td>98.6</td>
</tr>
</tbody>
</table>

Bearing stress = $\frac{98.6}{4.0 \times \frac{3}{8} \times 2} = 24.7 < 40 \text{ ksi (allowable)}$

The stiffener column consists of the two $\frac{1}{2} \times 4\frac{1}{2}$-in. plates and a length of web equal to

\[ L_w = 18 \times \frac{3}{8} = 6.75 \text{ in.} \]

Area of the equivalent column is

\[ A_e = 2 \times \frac{1}{2} \times 4\frac{1}{2} + \frac{3}{8} \times 6.75 = 7.03 \text{ in.}^2 \]

Moment of inertia of the equivalent column is

\[ I_e = \frac{(\frac{3}{8})(4.5 + 0.375 + 4.5)^3}{12} = 34.3 \text{ in.}^4 \]

and the radius of gyration is

\[ r = \sqrt[3]{\frac{34.3}{7.03}} = 2.21 \text{ in.} \]
Consequently, the slenderness ratio of the stiffener equals
\[
\frac{KL_c}{r} = \frac{D}{2.21} = 19.00
\]
\[
\sqrt{\frac{2\pi^2E}{60}} = \sqrt{\frac{(2)(\pi^2)(29,000)}{50}} = 107.0 > 19.0
\]
The allowable stress then is
\[
F_{cr} = F_v \left[ 1 - \frac{F_v}{4\pi^2E} \left( \frac{D}{r} \right)^2 \right] = 50 \left[ 1 - \frac{50}{4\pi^2 \times 29,000 \times (19.00)^2} \right] = 49.2 \text{ ksi}
\]
The Maximum Design Load on the columns was previously computed for the investigation of the stiffened web at the end bearing to be 179 kips. The capacity of the equivalent column is
\[
P_v = 0.85A_{cr}F_{cr} = 0.85 \times 7.03 \times 49.2 = 294 > 179 \text{ kips}
\]
Therefore, the two \( \frac{3}{4} \times 4\frac{1}{2} \)-in. plates are satisfactory as bearing stiffeners.

**BEARING STIFFENERS AT INTERIOR SUPPORT**

The bearing stiffeners at the interior support are designed in the same way as those at the end support. A stiffener consisting of two 6-in.-wide plates welded to opposite sides of the girder web is investigated at the interior support. Minimum thickness of stiffener required is
\[
t = \frac{6}{12} \sqrt{\frac{50,000}{33,000}} = 0.615 \text{ in.}
\]
Use \( \frac{5}{8} \times 6 \)-in. stiffeners.

**Interior Reaction**

<table>
<thead>
<tr>
<th></th>
<th>( DL_1 )</th>
<th>( DL_2 )</th>
<th>( L+I )</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R ), kips</td>
<td>112.4</td>
<td>20.6</td>
<td>92.0</td>
<td>225.0</td>
</tr>
</tbody>
</table>

Bearing stress = \[
\frac{225.0}{5.5 \times \frac{5}{8} \times 2} = 32.7 < 40 \text{ ksi (allowable)}
\]

The reaction due to the Maximum Design Loads is
\[
R = 1.30 \left( 112.4 + 20.6 + \frac{5}{3} \times 92.0 \right) = 372 \text{ kips}
\]

The stiffener column consists of the two \( \frac{3}{4} \times 6 \)-in. plates and a portion of the web 6.75 in. long, as at the end support. The area of the equivalent column is
\[
A_s = 2 \times \frac{3}{4} \times 6 + \frac{3}{4} \times 6.75 = 10.0 \text{ in.}^2
\]

Moment of inertia of the equivalent column is
\[
I = \frac{(\frac{3}{4})(6+0.375+6)^3}{12} = 98.7 \text{ in.}^4
\]
and the radius of gyration is
\[
r = \sqrt{\frac{98.7}{10.0}} = 3.14 \text{ in.}
\]
Therefore, the slenderness ratio is
\[
\frac{D}{r} = \frac{42}{3.14} = 13.4
\]

The allowable stress then is
\[
F_{cr} = 50 \left[ 1 - \frac{50}{4\pi^2 \times 29,000 \times (13.4)^2} \right] = 49.6 \text{ ksi}
\]
The capacity of the equivalent column consequently is

\[ P_e = 0.85 \times 10.0 \times 49.6 = 422 > 372 \text{ kips} \]

Therefore, the two \( \frac{3}{8} \times 6 \)-in. plates are satisfactory as bearing stiffeners.

DEFLECTIONS

Dead-load and live-load deflections are determined in the same manner as for working-stress design. The dead-load camber diagram is shown below.

CAMBER DIAGRAM

The maximum live-load deflection is 0.995 in. and the deflection-span ratio is \( 1/1,204 < 1/800 \).

DESIGN OF BOLTED FIELD SPLIC

The bolted alternate to the welded field splice, 22.5 ft from the pier, is to be a friction-type connection made with \( \frac{3}{4} \)-in.-dia A325 bolts. The web on both sides of the splice is \( \frac{3}{4} \times 42 \) in. Flange plates on the positive-moment side of the splice are \( \frac{3}{4} \times 10 \)-in. top flange and \( 1 \times 10 \)-in. bottom flange, whereas those on the negative-moment side are \( \frac{3}{4} \times 14 \)-in. top flange and \( 1 \times 14 \)-in. bottom flange.

Shears 22.5 Ft from Pier, Kips

<table>
<thead>
<tr>
<th></th>
<th>For Service Loads</th>
<th>Factor</th>
<th>For Overload</th>
<th>Factor</th>
<th>Max. Design Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>( DL_1 ):</td>
<td>36.1</td>
<td>1</td>
<td>36.1</td>
<td>1.30</td>
<td>46.9</td>
</tr>
<tr>
<td>( DL_2 ):</td>
<td>6.6</td>
<td>1</td>
<td>6.6</td>
<td>1.30</td>
<td>8.6</td>
</tr>
<tr>
<td>( L+I ):</td>
<td>52.0</td>
<td>5</td>
<td>86.7</td>
<td>1.30</td>
<td>112.7</td>
</tr>
<tr>
<td></td>
<td>94.7</td>
<td>3</td>
<td>129.4</td>
<td></td>
<td>168.2</td>
</tr>
</tbody>
</table>

Moments 22.5 Ft from Pier, Kip-Ft

<table>
<thead>
<tr>
<th></th>
<th>For Service Loads</th>
<th>Factor</th>
<th>For Overload</th>
<th>Factor</th>
<th>Max. Design Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>( DL_1 ):</td>
<td>-95</td>
<td>1</td>
<td>-95</td>
<td>1.30</td>
<td>-124</td>
</tr>
<tr>
<td>( DL_2 ):</td>
<td>-20</td>
<td>1</td>
<td>-20</td>
<td>1.30</td>
<td>-26</td>
</tr>
<tr>
<td>(- (L+I) ):</td>
<td>-480</td>
<td>5</td>
<td>-800</td>
<td>1.30</td>
<td>-1,040</td>
</tr>
<tr>
<td>(+ (L+I) ):</td>
<td>575</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum:</td>
<td>-595</td>
<td></td>
<td>-915</td>
<td></td>
<td>-1,190</td>
</tr>
<tr>
<td>Minimum:</td>
<td>460</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
For load factor design of a bolted field splice, AASHTO Specifications require that the splice material be designed for Maximum Design Loads and resistance to fatigue under service loads. Friction connections must resist slip deformation under Overload, and therefore the fasteners must be proportioned for an allowable stress $F_v = 21$ ksi for an Overload of $D + (5/3)(L + I)$. The allowable bolt load in double shear is

$$P = 2 \times 0.6013 \times 21 = 25.3 \text{ kips per bolt}$$

For design of the splice material for Maximum Design Loads, the design moment is computed as the greater of:

- Average of the calculated moment on the section and maximum capacity of the section
- 75% of the maximum capacity of the section

The calculated moment is that induced by the Maximum Design Load $1.30[D + (5/3)(L + I)]$. Splice material should have a capacity equal at least to the design moment. The section capacity is based on the gross section minus any flange-area loss due to bolt holes in excess of 15% of each flange area.

The section at the splice is subjected to negative moment which acts on the girder section only, negative moment which act on the girder section plus slab reinforcing steel and positive moment which acts on the girder plus concrete section. Because the effects of negative moment predominate at the splice, splice material will be designed for a negative-moment section. Also, because the $DL_i$ moment which acts on the girder section only is small in relation to the moment which acts on the girder section plus slab reinforcing steel, the calculation for net section at the splice will include the slab reinforcing steel.

The bolt holes remove from the $1\frac{3}{8} \times 10$-in. flange

$$\% \text{ of flange} = \frac{2 \times 1}{10} \times 100 = 20\%$$

Therefore, $20\% - 15\% = 5\%$ of the flange area must be deducted for determination of the net section. With this deduction, the net moment of inertia including the slab reinforcing steel is determined below.

**Net Section Properties at Splice**

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_s$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T. Flg. .95(3/4 x 10)</td>
<td>7.13</td>
<td>21.37</td>
<td>152</td>
<td>3,256</td>
<td>3,256</td>
<td></td>
</tr>
<tr>
<td>Web 3/4 x 42</td>
<td>15.75</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B. Flg. .95(1 x 10)</td>
<td>9.50</td>
<td>-21.50</td>
<td>-204</td>
<td>4,391</td>
<td>4,391</td>
<td></td>
</tr>
<tr>
<td>Reinf. 14-#6</td>
<td>6.16</td>
<td>26.55</td>
<td>164</td>
<td>4,342</td>
<td>4,342</td>
<td></td>
</tr>
</tbody>
</table>

38.54 in.$^2$ 112 in.$^3$ 14,304

\[
d_s = \frac{112}{38.54} = 2.91 \text{ in.}
\]

\[
d_{Top \text{ of steel}} = 21.75 - 2.91 = 18.84 \text{ in.}
\]

\[
d_{Bot \text{ of steel}} = 22.00 + 2.91 = 24.91 \text{ in.}
\]

\[
S_{Top \text{ of steel}} = \frac{13,978}{18.84} = 742 \text{ in.$^2$}
\]

\[
S_{Bot \text{ of steel}} = \frac{13,978}{24.91} = 561 \text{ in.$^2$}
\]

Base metal fatigue should be investigated at the gross girder section near friction type fasteners.

**Design Moments and Shears**

For $F_v = 50$ ksi, the net section capacity is

$$M_{net} = \frac{50 \times 561}{12} = 2,337 \text{ kip-ft}$$

75% $M_{net} = 0.75 \times 2,337 = 1,753 \text{ kip-ft}$
The average of the calculated moment for the design loads and the net capacity of the section is

\[ M_{av} = \frac{1,190 + 2,337}{2} = 1,764 > 1,753 \text{ kip-ft} \]

The design moment therefore is 1,764 kip-ft.

The design shear is determined by multiplying the calculated shear for the design loads by the ratio of the design moment to the calculated moment on the section.

\[ \text{Design shear} = 168.2 \times \frac{1,764}{1,190} = 249 \text{ kips} \]

**Web Splice**

The web splice plates must carry the design shear, a moment \( M_s \), due to the eccentricity of this shear, and a portion \( M_w \) of the design moment on the section. The portion of the design moment to be resisted by the web is obtained by multiplying the design moment by the ratio of the moment of inertia of the web to the net moment of inertia of the entire section.

\[ I_{web} = 2,315 + 15.75(2.91)^2 = 2,448 \text{ in.}^4 \]

**Web Moments for Design Loads**

\[ M_s = \frac{249 \times 3.5}{12} = 73 \]

\[ M_w = 1,764 \times \frac{2,448}{13,978} = 309 \text{ kip-ft} \]

Try two \( \frac{5}{16} \times 38\)-in. web splice plates and two rows of bolts with eight bolts per row on each side of the joint. The area of one hole is \( 0.312 \times 1 = 0.312 \text{ sq in.} \). The holes remove

\[ \% \text{ of plate} = \frac{8 \times 0.312}{0.312 \times 38} \times 100 = 21.05\% \]

Consequently, the fraction of the hole area that must be deducted in determination of the net section is

\[ \frac{21.05 - 15}{21.05} = 0.2874 \]

**\( d^2 \) for Holes**

\[(2.5)^2 = 6.25\]
\[(7.5)^2 = 56.25\]
\[(12.5)^2 = 156.25\]
\[(17.5)^2 = 306.25\]

\[ 2d^2 = 525.00 \text{ in.}^2 \]

\[ \Sigma Ad^2 \text{ web holes} = 0.2874 \times 4 \times 0.312 \times 525 = 188 \text{ in.}^4 \]
The assumption is made that the neutral axis of the splice material is in the same position as it is on the net section. The bending properties of the web splice plates are then computed as follows.

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>l*</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Splice Pl. (\frac{5}{16}&quot; \times 38&quot;)</td>
<td>23.75</td>
<td>2.91</td>
<td>201</td>
<td>2,853</td>
<td>3,054</td>
</tr>
<tr>
<td>Holes 2(8) (0.312) (0.287)</td>
<td>- 1.43</td>
<td>2.91</td>
<td>-12</td>
<td>-188</td>
<td>-200</td>
</tr>
</tbody>
</table>

\(d_{top\ of\ splice} = 19.0 - 2.91 = 16.09\ in.\)

\(d_{tot.\ of\ splice} = 19.0 + 2.91 = 21.91\ in.\)

\(S_{top\ of\ splice} = \frac{2.854}{15.09} = 177\ in.^3\)

\(S_{tot.\ of\ splice} = \frac{2.854}{21.91} = 130\ in.^3\)

Hence, the maximum bending stress in the plates for design loads is

\[ f_b = \frac{382 \times 12}{130} = 35.3 < 50\ ksi \]

The plates are satisfactory for bending. The allowable shear stress is

\[ F_s = 0.55 F_v = 0.55 \times 50 = 27.5\ ksi \]

The shear stress for the maximum design shear is

\[ f_s = \frac{249}{23.75} = 10.5 < 27.5\ ksi \]

The \(\frac{5}{16}\) \times 38-in. web splice plates are satisfactory for strength requirements. The plates are next checked for fatigue under service loads.

**Web Bending Stress Range for Service Loads**

The range of moment carried by the web equals

\[ M_w = (575 + 480) \times \frac{2,448}{13,978} = 185\ kip-ft \]

The maximum bending stress range in the gross section of the web splice plates then is

\[ f_b = \frac{185 \times 12 \times 21.91}{3,054} = 15.9\ ksi \]

**Allowable Fatigue Stresses for Splice Material**

Fatigue in base metal adjacent to friction-type fasteners is classified by AASHTO as Category B. For 500,000 cycles, the associated allowable stress range is 27.5 ksi. The plates are satisfactory for fatigue.

Use two \(\frac{5}{16}\) \times 38-in. web splice plates

**Web Bolts**

The 16 bolts in the web splice must carry the vertical shear, the moment due to the eccentricity of this shear about the centroid of the bolt group, and the portion of the beam moment taken by the web. These forces are induced by the Overload \(D + (5.3)(L + I)\). The allowable load in double shear was previously computed to be \(P = 25.3\) kips per bolt.

The polar moment of inertia of the bolt group about the assumed location of the neutral axis is

\[ I = 2 \times 2 \times 525 + 16(1.5)^2 + 16(2.91)^2 = 2,271\ in.^4 \]
The distance from the centroid to the outermost bolt is

\[ d = \sqrt{(17.5+2.91)^2 + (1.5)^2} = 20.5 \text{ in.} \]

**SPLICE DETAILS**

**Web Moments for Overload**

\[ M_w = \frac{129.4 \times 3.5}{12} = 38 \]

\[ M_w = 915 \times \frac{2448}{13978} = 160 \approx 198 \text{ kip-ft} \]

Load per bolt due to shear is

\[ P_s = \frac{129.4}{16} = 8.1 \text{ kips} \]

Load on the outermost bolt due to moment is

\[ P_m = \frac{198 \times 12 \times 20.5}{2271} = 21.4 \text{ kips} \]

The vertical component of this load is

\[ P_v = \frac{21.4 \times 1.5}{20.5} = 1.6 \text{ kips} \]
The horizontal component is

\[ P_h = \frac{21.4 \times (17.5 + 2.91)}{20.5} = 21.3 \text{ kips} \]

Therefore, the total load on the outermost bolt is the resultant

\[ P = \sqrt{(8.1 + 1.6)^2 + (21.3)^2} = 23.4 < 25.3 \text{ kips} \]

Use 16 \( \frac{3}{4} \)-in.-dia A325 bolts in two rows.

Flange-Splice Design

The flange splice plates are proportioned for Maximum Design Loads and checked for fatigue. The average stress in the top flange under the Maximum Design Load is

\[ F_{b, \text{Top}} = \frac{1,764(12)(21.37 - 2.91)}{13,978} = 28.0 \text{ ksi} \]

The total flange force is determined by multiplying the average stress by the net flange area

\[ P_{\text{Top}} = 28.0 \times 10 \times 0.75 \times 0.95 = 199 \text{ kips} \]

The required net area of top flange splice plate then becomes

\[ A_{\text{Top}} = \frac{199}{50} = 3.98 \text{ in.}^2 \]

Since this value is less than 75% of the net area of the top flange, the minimum required area will be

\[ A_{\text{Top}} = 0.75(7.13) = 5.35 \text{ in.}^2 \]

Try a \( \frac{3}{8} \times 10 \)-in. outer plate and two \( \frac{3}{8} \times 4 \)-in. inner plates. The net area after deduction of bolt holes in excess of 15% of the plate area is

\[ A_f = (\frac{3}{8} \times 10) + (2 \times \frac{3}{8} \times 4) - [(2 \times 1 \times \frac{3}{8}) + (2 \times 1 \times \frac{3}{8}) - 0.15 \left( (\frac{3}{8} \times 10) + (2 \times \frac{3}{8} \times 4) \right)] \]

\[ = 5.67 \text{ in.}^2 > 5.35 \text{ in.}^2 \]

The average stress in the bottom flange under the Maximum Design Load is

\[ F_{b, \text{Bot.}} = \frac{1,764(12)(21.5 + 2.91)}{13,978} = 37.0 \text{ ksi} \]

The total flange force is

\[ P_{\text{Bot.}} = 37.0 \times 10 \times 1 \times .95 = 351 \text{ kips} \]

The required net area of the bottom plate becomes

\[ A_{\text{Bot.}} = \frac{351}{50} = 7.02 \text{ in.}^2 \]

which is less than 75% of the net area of the bottom flange, or

\[ A_{\text{Bot.}} = 0.75(9.50) = 7.13 \text{ in.}^2 \]

Try a \( \frac{3}{4} \times 10 \)-in. outer plate and two \( \frac{1}{2} \times 4 \)-in. inner plates. The net area after deduction of bolt holes in excess of 15% of the plate area is

\[ A_f = (\frac{3}{4} \times 10) + (2 \times \frac{1}{2} \times 4) - [(2 \times 1 \times \frac{3}{4}) + (2 \times 1 \times \frac{1}{2}) - 0.15 \left( (\frac{3}{4} \times 10) + (2 \times \frac{1}{2} \times 4) \right)] \]

\[ = 7.16 \text{ in.}^2 > 7.13 \text{ in.}^2 \]

The flange splice plates are then checked for fatigue under Service Load. The range of live-load moment at the splice equals

\[ 460 + 595 = 1,055 \text{ kip-ft} \]
And the range of average stress in the flanges is calculated to be

\[
\text{Top flange: } f_r = \frac{1.055(12)(21.37 - 2.91)}{13,978} = 16.7 \text{ ksi}
\]

\[
\text{Bot. flange: } f_r = \frac{1.055(12)(21.50 + 2.91)}{13,978} = 22.1 \text{ ksi}
\]

The corresponding range of stress in the gross section of the flange splice plates is

\[
\text{Top flange: } f_r = \frac{16.7(10)(0.75)(0.95)}{(\frac{3}{8} \times 10) + (2 \times \frac{3}{4} \times 4)} = 19.4 < 27.5 \text{ ksi}
\]

\[
\text{Bot. flange: } f_r = \frac{22.1(10)(1)(0.95)}{(\frac{3}{8} \times 10) + (2 \times \frac{3}{4} \times 4)} = 27.1 < 27.5 \text{ ksi}
\]

**Flange Bolts**

The number of bolts required in the flange splice is determined by the capacity needed for transmitting the flange force under the Overload \(D + 5/3(L+I)\). The total moment on the section is 915 kip-ft.

The average stress in the top flange is

\[
F_r = \frac{915(12)(21.37 - 2.91)}{13,978} = 14.5 \text{ ksi}
\]

And the flange force becomes

\[
P_{\text{Top}} = 14.5(10 \times 0.75 \times 0.95) = 103 \text{ kips}
\]

For this flange force

\[
\frac{103}{25.3} = 4.1 \text{ bolts are required}
\]

Use 6 bolts in two rows.

The average stress in the bottom flange is

\[
F_r = \frac{915(12)(21.50 + 2.91)}{13,978} = 19.2 \text{ ksi}
\]

And the bottom flange force is

\[
P_{\text{Bot}} = 19.2(10 \times 1 \times 0.95) = 182 \text{ kips}
\]

For this flange force

\[
\frac{182}{25.3} = 7.2 \text{ bolts are required}
\]

Use 8 bolts in two rows. Details of the splice are shown on page 33.
FINAL DESIGN
An elevation of the two-span, continuous girder with a stiffened web is shown below. An elevation of the girder with an unstiffened web follows. A detail drawing of the complete design is shown on the following page.

ELEVATION OF GIRDER WITH STIFFENED WEB

ELEVATION OF GIRDER WITH UNSTIFFENED WEB
Composite Design Example
2-Span Continuous Welded Girders Load Factor Design

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Intermediate Stiffener Detail

Camber Diagram

Welded Field Splice

Note:
- All materials ASTM A508, Grade A (U80 Con-Ten R)
- All fasteners designated as H.S. Bolts shall be ASTM A325 Type 3

- Total Wt. = 112,500 lb
- Wt. per sq ft (6.0 to 9.5 slab) = 16.9 lb per sq ft

- Weight does not include bearing shoes, railing or studs.
Composite: Medium Span Welded Plate Girder Load Factor Design

Introduction

Chapter 5 illustrates the use of load factor design for a welded plate girder bridge with medium spans and composite construction. Design procedures are similar to those described in Chapter 4A for two-span, continuous, welded plate girders with 100-ft. spans.

A three-span, continuous, plate girder bridge with spans of 273, 350 and 273 ft. is used as a design example. The width of roadway and framing arrangement are as shown in a cross section.

The framing consists of three continuous girders, cross frames and stringers. This arrangement was compared with three other framing arrangements for economy of material. The studies indicated that:

A two-girder, floorbeam-stringer system with the same roadway slab requires
about 3.6% more structural steel.

A four-girder system requires a thicker slab and about 7.7% more structural steel.

A three-girder system, without stringers, requires a thicker slab and about 10% more steel, including more slab reinforcement.

Preliminary design studies for the example framing with constant web depth versus webs haunched over interior supports indicated that about 1.2% more structural steel would be required for constant-depth girders. But fabricating costs would be greater for the haunched girders. Overall construction costs would be about equal. The example framing with constant-depth girders is an economical system for a medium-span bridge with the selected spans.

As in Chapter 4A, girders are considered composite in the positive-moment and negative-moment regions. Load factor design is executed in accordance with the 1977 Standard Specifications for Highway Bridges of the American Association of State Highway and Transportation Officials and with the Interim Specification dated 1978. These specifications will be referred to for brevity as AASHTO followed by an article and section reference.

USS COR-TEN B (ASTM A588, Grade A) is used for the steel portion of the composite girder. This is a high-strength, low-alloy structural steel that is widely used in unpainted bridges where its enhanced atmospheric corrosion resistance is desired to help minimize maintenance costs.

The procedures for dead-load distribution, lateral distribution of live load, computation of reactions, shears, moments and deflections, determination of effective slab widths, section properties (except for plastic section modulus and related properties) and stresses in composite sections are the same for load factor and working stress designs. Descriptive text and illustrative calculations similar to those presented in Chapters 4 and 4A are not repeated but the similarity is pointed out.

General Design Considerations

Members designed by the load factor method are proportioned for multiples of the design loads. They are required to meet certain criteria for three theoretical load levels: (1) Maximum Design Load; (2) Overload, and (3) Service Load. The Maximum Design Load and Overload requirements are based on multiples of the service loads with certain other coefficients necessary to insure the required capabilities of the structure. Service Loads are defined as the same loads as those used in working stress design.

The Maximum Design Load criteria insures the structure's capability of withstanding a few passages of exceptionally heavy vehicles (simultaneously in more than one lane), in times of extreme emergency, that may induce significant permanent deformations.

The Overload criteria insures control of permanent deformations in a member, caused by occasional overweight vehicles equal to 5/3 the design live and impact loads (simultaneously in more than one lane), that would be objectionable to riding quality of the structure.

The Service Load criteria insures that the live-load deflection and fatigue load (for assumed fatigue loading) of a member are controlled within acceptable limits.

Service Load moments, shears and other forces are generally determined by assuming elastic behavior of the structure. In analysis of continuous beams with compact sections (AASHTO Art. 1.7.124A), however, negative moments over interior supports may be reduced up to a maximum of 10%. This reduction, however, must be accompanied by an increase in the maximum positive moment equal to the average decrease of the negative moments in the span.

DESIGN LOADS

The moments, shears or forces to be sustained by a stress-carrying steel member are
computed from the following formulas for the three loading levels.

Service Load: \[ D + (L + I) \]

Overload: \[ D + \frac{5}{3} (L + I) \]

Maximum Design Load: \[ 1.30 \left[ D + \frac{5}{3} (L + I) \right] \]

where \( D \) = dead load
\( L \) = live load
\( I \) = impact load

Uncertainties in strength, theory, loading, analysis, material properties and dimensions are included in the factor 1.30. The factor 5/3 is incorporated to allow for overloads. Factors for other group loading combinations are given in AASHTO Art. 1.2.22.

DESIGN FOR MAXIMUM DESIGN LOADS*

Welded plate girders of normal proportions are not likely to satisfy the requirements for compactness described in Chapter 3A. Therefore, the maximum-moment capacity of a girder section is less than the computed plastic bending strength.

If a girder meets the requirements for a symmetrical, braced, noncompact section, the maximum strength may be computed from

\[ M_u = F_y S \]

where \( F_y \) = specified minimum yield point or yield strength, psi, of the type of steel being used
\( S \) = elastic section modulus

The section consequently must be proportioned so that

\[ F_y S \geq 1.30 \left[ D + \frac{5}{3} (L + I) \right] \]

Here, \( D \), \( L \), and \( I \) represent moments induced by the Service Loads.

For this relationship to be permitted, the following criteria must be satisfied:

1. Width-thickness ratio of the compression flange projections, when the compressive stress equals \( F_y \), should not exceed

\[ \frac{b'}{t} = \frac{2.200}{\sqrt{F_y}} \]

where \( b' \) = width of projecting flange element
\( t \) = flange thickness

When \( M < M_u \), \( b'/t \) may be increased in the ration \( \sqrt{M_u}/M \). Under superimposed dead and live loads, the \( b'/t \) requirement need not be satisfied for the top flange of composite girders.

2. Depth-thickness ratio of the web for an unsymmetrical section with one longitudinal stiffener should not exceed

\[ \frac{D}{t_w} = \frac{73,000}{\sqrt{F_y}} \]

where \( D \) = clear unsupported distance between flange components
\( t_w \) = web thickness

*Some of the criteria following do not apply to hybrid girders. For treatment of hybrid girders, refer to Chapter 7 and to AASHTO Specifications.
The longitudinal stiffener should be placed an average distance $2D_c / 5$ from the inner surface of the compression flange, where $D_c$ is the clear distance between the neutral axis and the compression flange. This distance should be adjusted to accommodate welding clearance from the flange.

When $D_c$ exceeds $D/2$, the ratio $D_c/t_w$ should not exceed

$$\frac{D_c}{t_w} = \frac{36,500}{\sqrt{F_y}}$$

It should be noted that in the inflection regions of continuous girders either flange can be a compression flange depending on the position of live load. To satisfy the requirements for longitudinally stiffened webs, it is necessary to overlap the top and bottom longitudinal stiffeners in these regions to provide stiffening adjacent to both flanges.

3. To insure that a section can reach $F_yS$, spacing of lateral bracing of the bottom compression flange should not exceed

$$L_b = \frac{20,000,000A_f}{F_y d}$$

where $A_f = \text{cross-sectional area of compression flange}$

$$d = \text{depth of girder}$$

At the top flange of composite girders, lateral buckling is prevented by rigid attachment of the roadway concrete slab to the flange with stud shear connectors.

4. Axial compression should not exceed

$$P = 0.15 F_y A$$

where $A = \text{cross-sectional area of girder}$

5. Shear should not exceed either of the following values:

$$V = \frac{3.5Et_w^3}{D}$$

$$V = 0.58 F_y D t_w$$

where $E = \text{steel modulus of elasticity}$

If a girder section acting together with the longitudinal slab reinforcing steel meets the preceding requirements it may be designed as a braced, noncompact section, though it is unsymmetrical about its horizontal centroidal axis. Its maximum strength is the moment inducing yielding at an extreme surface under Maximum Design Loads composed of the initial dead load, superimposed dead load, and live load plus impact, taking into account whether the construction is shored or unshored when the concrete slab is cast.

When a member does not meet Criterion 3 for spacing of lateral bracing of braced, noncompact sections, it is considered an unbraced section. For symmetrical sections, the calculated maximum strength is reduced to

$$M_u = F_y S \left[ 1 - \frac{3 F_y}{4 \pi^2 E} \left( \frac{L_b}{b'} \right)^2 \right]$$

When the ratio of the smaller moment to the larger moment at the ends of the
braced length $L_b$ is less than 0.7, this value of $M_u$ may be increased 20% but may not exceed $F_s S$.

For sections unsymmetrical about the horizontal axis but symmetrical about the vertical axis, along the web, maximum strength may be computed from the appropriate formula previously given, except that when the preceding formula for $M_u$ is used, $b'$ should be replaced by 0.9$b'$. Because the girder section with longitudinal slab steel is unsymmetrical about the horizontal axis, this modification applies to the calculation of maximum bending strength when the section does not qualify as braced, in accordance with Criterion 3. Details such as transverse or longitudinal web stiffeners are not considered, in this case, to be part of the geometric section nor to affect symmetry about the vertical axis.

The preceding AASHTO lateral buckling equation for maximum strength was developed for prismatic compression flanges. In the case where there is a transition in compression-flange width or thickness within an unbraced length, the compression-flange section throughout this length is no longer prismatic and the AASHTO lateral buckling requirements are not directly applicable. However, it can be shown that, by a modification of application, the AASHTO lateral buckling formula can be applied conservatively to girders with stepped flanges.* This can be done by rearranging the AASHTO formula and computing the critical buckling stress of the braced panel in which the transition occurs as that of the girder in the stepped-down region. This stress may then be increased by 20% providing the ratio of compression-flange axial forces at the ends of the braced panel are equal to or less than 0.7. Although concurrent axial forces are theoretically correct, maximum axial forces, as obtained from the moment envelopes, may conservatively be used.

The critical buckling stress is determined from the following rearrangement of the AASHTO buckling formula:

$$ F_{cr} = \frac{M_u}{S} = F_y \left[ 1 - \frac{3F_y}{4\pi^2E} \left( \frac{L_b}{b'} \right)^2 \right] $$

where $b'$ = projecting compression flange width of the girder in the stepped-down region

$S$ = section modulus of the steel section in the stepped-down region

The maximum strength at any point in the panel is expressed as:

$$ M_u = F_{cr} S_x $$

where $S_x$ = section modulus at the point considered.

**BEARING AND INTERMEDIATE STIFFENERS**

In the section of the AASHTO Specifications dealing with Load Factor Design, there are no provisions for bearing stiffeners, though intermediate transverse stiffeners and longitudinal stiffeners are covered. AASHTO Art. 1.7.73, however, requires stiffeners to be placed over bearings of welded plate girders. These stiffeners preferably should be made of plates and should satisfy the following requirements:

They should extend as nearly as practicable to the outer edges of the flange plates.

The plates should be placed on both sides of the web.

The stiffeners should be designed as columns. For stiffeners composed of a pair of

*United States Steel Research reviewed the basis for the AASHTO requirements and analyzed the buckling loads of stepped columns with various geometries. Based on these results, a design procedure was developed which relates the strength of a stepped flange to that of a prismatic flange. For additional information on this procedure contact a United States Steel Construction Representative through the nearest United States Steel Sales Office.
plates, the column section should be assumed to comprise those plates plus a centrally located strip of web with width not exceeding 18 times the web thickness.

The connection of the stiffeners to the web should be capable of transmitting the entire end reaction to the bearings.

The stiffeners should be ground to fit against the flange through which they receive their reaction or attached to the flange by full-penetration groove welds.

Only the portion of the stiffeners outside the flange-to-web plate welds should be considered effective in bearing.

Thickness of the stiffener plates should be at least

\[
t = b' \sqrt{\frac{F_y}{12 \times 33000}}
\]

AASHTO 1.7.134 contains Load Factor Design provisions for compression members. Presumably, these would apply to design of bearing stiffeners as columns, whereas the bearing pressure would be limited by the allowable stress in bearing. The total end reaction transmitted to the bearings and caused by the Maximum Design Loads, therefore, should not exceed the maximum strength of the bearing stiffeners as a column. By AASHTO 1.7.134, the maximum strength may be computed from

\[
P_u = 0.85 A_s F_{cr}
\]

where \( A_s \) = gross effective area of the column cross-section

\( F_{cr} \) = critical stress, determined from whichever of the following formulas is appropriate

\[
F_{cr} = F_u \left[ 1 - \frac{F_y}{4\pi^2 E} \left( \frac{KL_c}{r} \right)^2 \right]
\]

\[
F_{cr} = \frac{\pi^2 E}{(KL_c/r)^2}
\]

\[
\frac{KL_c}{r} \leq \sqrt{\frac{2\pi^2 E}{F_y}}
\]

\[
\frac{KL_c}{r} > \sqrt{\frac{2\pi^2 E}{F_y}}
\]

\( K \) = effective length factor, which may be taken as unity for bearing stiffeners

\( L_c \) = length of member between points of support = \( D \) for bearing stiffeners

\( r \) = radius of gyration of the column section in the plane of buckling

The shear capacity of girder webs with transverse stiffeners is given by

\[
V_u = V_p \left[ C + \frac{0.87(1-C)}{\sqrt{1+(d_o/D)^2}} \right]
\]

where \( V_p = 0.58F_yDt_w \)

\( d_o \) = distance between transverse stiffeners

\[
C = 18,000 \frac{t_w}{D} \sqrt{\frac{1+(D/d_o)^2}{F_y}} - 0.3 \leq 1.0
\]

The effect of shear on the bending strength of a stiffened girder can usually be ignored if the shear \( V \) within a transversely stiffened panel is less than \( 0.6V_u \). If \( V > 0.6V_u \), the moment-shear relationship at that section is defined as

\[
M = M_u (1.375 - 0.625 \frac{V}{V_u})
\]
where $M_u$ is the bending strength of the section unreduced for shear. Most designers prefer to use the full bending strength and limit the shear to $0.6V_u$.

Spacing of transverse stiffeners along a girder should not exceed the distance $d_o$ determined from the preceding formula for $V_u$ nor $1.5D$. At simply supported ends of girders, though, the first stiffener space may not be larger than $D$ nor

$$d_o = 14,500 \sqrt{\frac{D t_w^3}{V}}$$

Transverse stiffeners should be proportioned so that the width-thickness ratio does not exceed

$$\frac{b'}{t} = \frac{2,600}{\sqrt{F_u}}$$

Also, the gross cross-sectional area of each one-sided stiffener or pair of two-sided stiffeners should be at least

$$A = Y \left[ 0.15BDt_w (1-C) \frac{V}{V_u} - 18t_w^2 \right]$$

where $Y = \text{ratio of web yield strength to stiffener yield strength}$
- $B = 1.0$ for stiffener pairs
- $= 1.8$ for single angles
- $= 2.4$ for single plates

$C$ is the same as for the computation of $V_u$.

In addition, the required moment of inertia of each stiffener with respect to the mid-plane of the web is

$$I = d_o t_w^3 J$$

where $J = 2.5 \left( \frac{D}{d_o} \right)^2 - 2 \geq 0.5$

Transverse stiffeners need not bear on a tension flange, but the maximum distance between a face of that flange and the nearest web-to-stiffener weld should not exceed $4t_w$. When stiffeners are provided on only one side of the web, they should bear on the compression flange, but need not be attached to it.

The dimensions of the longitudinal stiffener should meet the following requirements:

$$\frac{b'}{t} \leq \frac{2,600}{\sqrt{F_y}}$$

The rigidity of the stiffener should not be less than

$$I = D t_w^3 \left[ 2.4 \left( \frac{d_o}{D} \right)^2 - 0.13 \right]$$

The radius of gyration of the stiffener should not be less than

$$r = \frac{d_o \sqrt{F_y}}{23,000}$$
In the computation of $I$ and $r$ for a stiffener, a centrally located web strip not more than $18t_w$ in width may be considered as part of the longitudinal stiffener.

**DESIGN FOR OVERLOAD**

To guard against objectionable deformation under occasional overloads, the following moment relationship must be observed for noncomposite sections and negative-bending of composite sections.

$$0.8F_y S \geq \left[ D + \frac{5}{3} (L+I) \right]$$

The negative-bending region is defined as that region in which the top flange of the girder is in tension. This region varies in extent in accordance with the position of the moving load.

For the same purpose, composite sections in positive bending must satisfy the relationship

$$0.95F_y S \geq \left[ D + \frac{5}{3} (L+I) \right]$$

The positive-bending region is defined as that region in which the top flange of the girder is in compression. This region also varies in extent with the position of the live load.

**DESIGN FOR SERVICE LOAD**

Fatigue is investigated in the same manner as in Working Stress Design, with Service Loads and the provisions of AASHTO Art. 1.7.3. The stress range in the longitudinal reinforcing steel is limited to 20 ksi.

**SHEAR CONNECTORS**

Provisions for shear connectors in Load Factor Design are identical with provisions for working stress design. These are illustrated in Chapter 3A.

**Design Example — Three Span Continuous Girder (273-350-273 Ft)**

**Composite Throughout**

To illustrate Load Factor Design, an interior girder of a three-span bridge will be designed. The section in positive-bending consists of the steel girder acting compositely with the concrete slab. In negative-bending, the section consists of the steel girder and the longitudinal reinforcing steel of the concrete slab. The following data apply to this design:

Roadway Section: See CROSS-SECTION OF MEDIUM-SPAN BRIDGE, p. II/5.1.

Loading: HS20-44 and Interstate (Military) Loading
Structural Steel: ASTM A588, Grade A, with $F_y = 50{,}000$ psi
Concrete: $f'_c = 4{,}000$ psi, modular ratio $n = 8$
Slab Reinforcing Steel: ASTM A615, Grade 40 with $F_y = 40{,}000$ psi
Loading Conditions:

Case 1—Weight of girder and slab \((DL_1)\) supported by the steel girder alone.
Case 2—Superimposed dead load \((DL_2)\) (curbs and railings) supported by the composite section with the modular ratio \(n=8\).
Case 3—Superimposed dead load \((DL_2)\) (curbs and railings) supported by the composite section with the increased modular ratio \(3n=3 \times 8=24\).
Case 4—Live load plus impact \((L + I)\) supported by the composite section with the modular ratio \(n=8\).

Loading Combinations:
Combination A = Case 1+3+4.
Combination B = Case 1+2+4.

Stress Cycles for Fatigue:
500,000 cycles of truck loading.
100,000 cycles of lane loading.

To obtain steel weights for girder design, the weights of stringers, cross frames and lateral bracing must be obtained.

**SUBSTRINGER DESIGN**

A sub-stringer is designed with Load Factor Design procedures similar to those used in Chapter 3A for composite wide-flange beams. The stringers, however, are non-composite and are designed as five-span, continuous members, all with 25-ft spans. Stringers are spliced at the quarter points of interior spans.

![SECTION AT SUBSTRINGER](image)

Dead, live and impact loads are obtained as follows:

**Dead Load**

\[
\begin{align*}
\text{Slab} & = (8/12) \times 0.15 \times 9.25 = 0.925 \\
\text{Haunch} & = 0.010 \\
\text{Wearing surface} & = 0.02 \times 9.25 = 0.185 \\
\text{Stringer} & = 0.062 \\
\text{Total} & = 1.182 \text{ kips per ft}
\end{align*}
\]
Live Load

The wheel distribution for a substringer for bending moment is

\[
\frac{S}{5.5} = \frac{9.25 - 5.5}{9.25} = 1.682 \text{ wheels}
\]

WHEEL LOADS PLACED FOR MAXIMUM END REACTION

The wheel distribution for maximum end reaction is

\[
1 + \frac{(9.25 - 4) + (9.25 - 6)}{9.25} = 1.919 \text{ wheels}
\]

Impact

For impact, the following fraction of the live load should be added to the live load:

\[
\frac{50}{L + 125} = \frac{50}{25 + 125} = 0.333
\]

Use 0.3.

Moments and shears are determined by elastic theory. Maximum moments for an end span and an interior span, maximum shears and maximum reactions are listed in a table. Because the spans are short, Interstate Loading governs at some sections. For fatigue, only HS20 loading is considered applied.

Maximum Bending Moments, Kip-Ft

<table>
<thead>
<tr>
<th></th>
<th>DL</th>
<th>HS20 $+ (L + I)$</th>
<th>Interstate $+ (L + I)$</th>
<th>HS20 $- (L + I)$</th>
<th>Interstate $- (L + I)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Positive</td>
<td>57.4</td>
<td>185.9</td>
<td>222.5</td>
<td>-28.0</td>
<td>-38.6</td>
</tr>
<tr>
<td>Moment in End Span</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Moment at First</td>
<td>-77.7</td>
<td>18.8</td>
<td>25.9</td>
<td>-161.2</td>
<td></td>
</tr>
<tr>
<td>Interior Support</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M$ at Center of</td>
<td>33.9</td>
<td>147.5</td>
<td>177.7</td>
<td>-28.1</td>
<td>-48.3</td>
</tr>
<tr>
<td>Typical Interior</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Span</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Maximum Shears, Kips

<table>
<thead>
<tr>
<th></th>
<th>DL</th>
<th>HS20 + (L + I)</th>
<th>Interstate + (L + I)</th>
<th>HS20 -(L + I)</th>
<th>Interstate -(L + I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear at End Support</td>
<td>13.0</td>
<td>51.7</td>
<td></td>
<td></td>
<td>-3.8</td>
</tr>
<tr>
<td>Shear at First Interior Support</td>
<td>-17.8</td>
<td>1.0</td>
<td>-54.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shear at Typical Interior Support</td>
<td>14.9</td>
<td>50.8</td>
<td></td>
<td></td>
<td>-5.3</td>
</tr>
</tbody>
</table>

Maximum Reaction, Kips

<table>
<thead>
<tr>
<th></th>
<th>DL</th>
<th>L + I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reaction at First Interior Support</td>
<td>33.4</td>
<td>61.1</td>
</tr>
<tr>
<td>Reaction at Typical Interior Support</td>
<td>28.6</td>
<td>58.0</td>
</tr>
</tbody>
</table>

Typical Interior Span — Positive Bending

The maximum positive moment due to Overload is

\[ M_o = D + \frac{5}{3} (L + I) = 33.9 + \frac{5}{3} \times 177.7 = 330 \text{ kip-ft} \]

The maximum positive moment due to Maximum Design Load is

\[ M = 1.30 \left[D + \frac{5}{3} (L + I)\right] = 1.30 \times 330 = 429 \text{ kip-ft} \]

Try a W24 x 55 of A588 steel, with plastic modulus \(Z = 134 \text{ in.}^3\) and section modulus \(S = 114 \text{ in.}^3\). It qualifies as a compact section:

The width-thickness ratio of the compression flange is

\[ \frac{b}{t} = \frac{3.5}{0.505} = 6.94 \]

The maximum permissible width-thickness ratio is

\[ \frac{b}{t} = \frac{1,600}{\sqrt{F_y}} = \frac{1,600}{\sqrt{50,000}} = 7.16 > 6.94 \]

The beam depth-thickness ratio is

\[ \frac{d}{t} = \frac{23.57}{0.395} = 59.7 \]

The maximum permissible depth-thickness ratio is

\[ \frac{d}{t} = \frac{13,300}{\sqrt{F_y}} = \frac{13,300}{\sqrt{50,000}} = 59.5 \approx 59.7 \]
The depth-thickness ratio of the beam is satisfactory. For positive-bending, the compression flange is fully braced along its length by the concrete slab. The section therefore satisfies all the requirements for compactness.

Because the section is compact, the moment capacity is given by

$$ M_u = F_y Z = \frac{50 \times 134}{12} = 558 > 429 \text{kip-ft} $$

For Overload,

$$ 0.80F_y S = 0.80 \times 50 \times 114/12 = 380 > 330 \text{kip-ft} $$

The allowable stress range for Case II loading (500,000 cycles of truck loading), AASHTO Category A for fatigue in base metal, is 36 ksi. The range of live-load stress in the interior span is

$$ f_b = \frac{(147.5 + 28.1)12}{114} = 18.5 < 36 \text{ksi} $$

The section is satisfactory in fatigue.

**Typical Interior Span — Negative-Bending**

In the negative-moment region, the compression flange is not braced. In accordance with AASHTO Table 1.7.1, Footnote (2), the unbraced length $L_e$ may be taken as the distance between a support and the dead-load contraflexure point.

$$ L_e = 0.3L = 0.3 \times 25 = 7.5 \text{ft} $$

The maximum negative-moment at the support due to Maximum Design Load is

$$ M = 1.30(-77.7 - \frac{5}{3} \times 161.2) = 450 \text{kip-ft} $$

At 7.5 ft from a support in the second span, the bending moment is

$$ M_2 = 5.6 - 80.1 = -74.5 \text{kip-ft} $$

The ratio of moments at the ends of the unbraced length of the subdivider then is

$$ \frac{M_2}{M_1} = \frac{-74.5}{-77.7 - 161.2} = 0.31 < 0.7 $$

The W24 x 55 is investigated as an unbraced, noncompact section. The maximum permissible width-thickness ratio of the compression flange of such a section, with $M_u/M$ taken as unity, is

$$ \frac{b'}{t} = \frac{2,200}{\sqrt{F_y}} \sqrt{\frac{M_u}{M}} = \frac{2,200}{\sqrt{50,000}} = 9.84 > 6.94 $$

Clear depth of web is $23.57 - 2 \times 0.505 = 22.56$ in. The web depth-thickness ratio is

$$ \frac{D}{t_w} = \frac{22.56}{0.395} = 57 < 150 $$

For the W24 x 55, $F_y S = 50 \times 114/12 = 475 \text{kip-ft}$. For an unbraced length of
7.5 ft, the moment capacity of the section is

\[ M_u = F_y S \left[ 1 - \frac{3F_y}{4\pi^2E} \left( \frac{L_b}{b^2} \right)^2 \right] = 475 \left[ 1 - \frac{3 \times 50}{4\pi^2 \times 29,000} \left( \frac{7.5 \times 12}{3.5} \right)^2 \right] = 434 \text{ kip-ft} \]

With \( M_2/M_1 < 0.7, M_u \) may be increased up to 20%, but not to more than \( F_y S = 475 \text{ kip-ft}. \) Inasmuch as \( 1.2 \times 434 = 520 \text{ kip-ft} \), use \( M_u = 475 > 450 \text{ kip-ft} \).

The stress range for fatigue is

\[ f_b = \frac{(18.8 + 161.2)12}{114} = 18.9 < 36 \text{ ksi} \]

Shear in the W24 x 55 under Maximum Design Load is

\[ V = 1.30 \left[ D + \frac{5}{3} (L + I) \right] = 1.30(-17.8 - \frac{5}{3} \times 54.2) = -141 \text{ kips} \]

The capacity of the section is the smaller of the following:

\[ V = \frac{3.5E t_w^3}{D} = \frac{3.5 \times 29,000(0.395)^3}{22.56} = 277 \text{ kips} \]

\[ V = 0.58F_y D t_w = 0.58 \times 50 \times 22.56 \times 0.395 = 258 > 141 \text{ kips} \]

The section is satisfactory in shear. Next, a check is made to see if a bearing stiffener is required. Under working-stress provisions, stiffeners are not needed if the actual shear is less than 75% of the 12-ksi allowable shear. The shear stress is

\[ f_v = \frac{17.8 + 54.2}{23.57 \times 0.395} = 7.73 < (0.75 \times 12 = 9 \text{ ksi}) \]

Bearing stiffeners are not required.

Use a W24 x 55 for interior spans.

**End Span**

For the end span, the maximum positive moment due to Overload is

\[ M_o = 57.5 + \frac{5}{3} \times 222.5 = 428 \text{ kip-ft} \]

The maximum positive-moment due to Maximum Design Load is

\[ M = 1.30 \times 428 = 557 \text{ kip-ft} \]

Try a W24 x 62 of A588 steel, with plastic modulus \( Z = 153 \text{ in.}^3 \) and section modulus \( S = 131 \text{ in.}^3 \). It qualifies as a compact section. Under Maximum Design Loads, its moment capacity is

\[ M_u = F_y Z = 50 \times 153/12 = 638 > 557 \text{ kip-ft} \]

For Overload,

\[ 0.80F_y S = 0.80 \times 50 \times 131.12 = 437 > 428 \text{ kip-ft} \]
The stress range for fatigue is

\[ f_b = \frac{(185.9 + 28.0)12}{131} = 19.6 < 36 \text{ ksi} \]

Use a W24 × 62 for the end span.

DESIGN OF AN INTERMEDIATE CROSS-FRAME

Cross-frames with diagonals forming an inverted V are spaced about 25 ft apart. They support the substringers and brace the flanges of the girders transversely. Assume that the cross-frames are about 10 ft deep.

Cross-Frame Diagonals

Length of a cross-frame diagonal is about

\[ L_c = \sqrt{10^2 + 9.25^2} = 13.62 \text{ ft} \]

The maximum reaction of a substringer under dead load is 33.4 kips and under live load plus impact, 61.1 kips. The reaction is provided by two compression diagonals of the cross-frame. Each diagonal may be assumed to carry half the load. For Maximum Design Load on a substringer, a diagonal must therefore support a vertical load of

\[ P_v = \frac{1}{2} \times 1.30(33.4 + \frac{5}{3} \times 61.1) = 88.0 \text{-kips} \]

The axial load on a diagonal then is

\[ P = 88.0 \times \frac{13.62}{10} = 120.0 \text{-kips} \]

The diagonal is connected in the cross-frame with 7/8-in.-dia, A325 bolts. For Overload, the bolts carry 120.0/1.30 = 92.1-kips. Shear capacity of a bolt is 12.63-kips. The number of bolts required is

\[ \frac{92.1}{12.63} = 7.3 \]

Use 8 bolts.

![Diagram](image)

**WT6 X 22.5**

Try a WT6 X 22.5 for the diagonal of the cross-frame. The effective depth of the
stem of the WT as a compression member is determined by

\[ \frac{d}{t_w} = \frac{1.625}{\sqrt{f_a}} \]

where \( d \) = effective depth
\( t_w \) = web thickness
\( f_a \) = compressive stress in the section

Because \( f_a \) is not known initially, \( d/t_w \) is assumed to be 12. The properties of the effective section and the actual stress are then computed. When the stress has been determined, the actual value of \( d/t_w \) is calculated and compared with the assumed value. Based on the assumed value of 12,

\[ d = 12 \times 0.335 = 4.02 \text{ in.} \]

### Effective WT Section

<table>
<thead>
<tr>
<th>Material</th>
<th>( A )</th>
<th>( d )</th>
<th>( Ad )</th>
<th>( Ad^2 )</th>
<th>( I_o )</th>
<th>( I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>WT6 × 22.5</td>
<td>6.61</td>
<td>-0.48</td>
<td>-4.18</td>
<td>2.01</td>
<td>16.6</td>
<td>16.6</td>
</tr>
<tr>
<td>Ineffective stem 1.43 × 0.335</td>
<td>-0.48</td>
<td>-0.48</td>
<td>-4.18</td>
<td>2.01</td>
<td>0.1</td>
<td>-8.3</td>
</tr>
<tr>
<td></td>
<td>6.13 in.³</td>
<td>2.01 in.³</td>
<td>8.3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \bar{y} = \frac{2.01}{6.13} = 0.328 \text{ in.} \]

\[ -0.328 \times 2.01 = -0.7 \]

\[ I_{NA} = 7.6 \text{ in.}^4 \]

The radius of gyration then is

\[ r = \sqrt{\frac{I}{A}} = \sqrt{\frac{7.6}{6.13}} = 1.11 \text{ in.} \]

For computation of the slenderness ratio \( KL_c/r \), assume \( K = 0.75 \).

\[ \frac{KL_c}{r} = \frac{0.75 \times 13.62 \times 12}{1.11} = 110 \]

\[ \sqrt{\frac{2\pi^2 E}{F_y}} = \sqrt{\frac{2\pi^2 \times 29,000}{50}} = 107 < 110 \]

Hence, the critical buckling stress is

\[ F_{cr} = \frac{\pi^2 E}{(KL_c/r)^2} = \frac{\pi^2 \times 29,000}{110^2} = 23.7 \text{ ksi} \]

The maximum strength of the WT therefore is

\[ P_u = 0.85A_xF_{cr} = 0.85 \times 6.13 \times 23.7 = 123.5 > 120 \text{ kips} \]

Finally, the effective depth assumed for the stem is checked with \( f_a = 120,000/6.13 = 19,600 \text{ psi} \).

\[ \frac{d}{t_w} = \frac{1.625}{\sqrt{19,600}} = 11.6 \]
This ratio is close enough to 12, initially assumed, that the WT section may be considered satisfactory.

**Bottom Strut of the Cross-Frame**

The bottom strut of the cross-frame is always in tension under the load transmitted by the cross-frame diagonals. The substringer dead load is the minimum load on the diagonal (See page 11).

\[
P = \frac{28.6}{2} \times \frac{9.25}{10} = 13.2 \text{ kips}
\]

Tension \(>10.73\)-kips

Compression

The maximum load on the strut is

\[
P = 120.0 \times \frac{9.25}{13.62} = 81.5 \text{ kips}
\]

Try a WT7 \(\times\) 19 for the bottom strut.

\[
\frac{L}{f} = \frac{18.0 \times 12}{1.55} = 139 < 200
\]

The horizontal component of the stress in the cross-frame diagonal is transferred from the diagonal through a gusset plate to the bottom strut. The gusset plate and strut are connected with a full-penetration butt weld, which must take the 81.5-kip horizontal component. Assume a 5/16-in.-thick gusset plate with a 21-in. weld along the strut. The horizontal shear stress on the weld is

\[
f_v = \frac{81.5}{0.313 \times 21} = 12.4 \text{ ksi}
\]

The vertical shear stress on the weld is

\[
f_b = \frac{M}{S} = \frac{81.5(7.06 - 1.55)}{0.313(21)^2 / 6} = 19.5 \text{ ksi}
\]

The resultant stress on the weld is

\[
f_r = \sqrt{12.4^2 + 19.5^2} = 23.1 < 50 \text{ ksi}
\]

The tensile strength of the strut, which has a cross-sectional area of 5.58 in.\(^2\), is

\[
P_u = 5.58 \times 50 = 279 > 81.5 \text{ kips}
\]

The WT7 \(\times\) 19 is satisfactory for the bottom strut.

**Cross-Frame Gusset Plates**

The diagonals of the cross-frame are connected to the top strut and to a stiffener on the girders through 5/16-in. gusset plates. These plates transmit the 88.0-kip vertical component of the diagonal load through 7/8-in.-dia bolts to the stiffener. For Overload, the bolts carry

\[
P_v = \frac{88.0}{1.3} = 67.7 \text{ kips}
\]

The number of bolts required in the gusset-plate connection therefore is

\[
\frac{67.7}{12.63} = 5.4
\]
Use 10 bolts to accommodate details.

**Top Strut of the Cross-Frame**

The top strut of the cross-frame is a secondary compression member. It should have a slenderness ratio not exceeding 140. The minimum radius of gyration for the unbraced length of 9 ft is

\[
r = \frac{9 \times 12}{140} = 0.771 \text{ in.}
\]

Use a 4 × 4 × 5/16-in. angle, with \( r_z = 0.791 > 0.771 \text{ in.} \).

**Cross-Frame at Interior Girder Support**

Because the cross-frame bottom strut must resist the entire wind reaction at each girder support, a heavier bottom strut is required at these locations than for the intermediate cross-frames. The bottom strut acts as a beam-column subject to eccentric loading and must satisfy the following interaction equations:

\[
\frac{P}{0.85A_y F_{cr}} + \frac{MC}{M_u (1 - P/A_y F_{cr})} \leq 1.0
\]

\[
\frac{P}{0.85A_y F_{cr}} + \frac{M}{M_p} \leq 1.0
\]

The average panel shear adjacent to the cross-frame is \( \frac{1}{2}(69.7 + 68.8) = 69.3 \) kips. (Loads are obtained from a wind analysis of the lateral bracing for the girders, presented later.) This shear produces tension in the bottom strut. The load due to wind causes 142.9 kips compression in the bottom strut. The total load on the strut therefore is 149.2 - 69.3 = 79.9 kips compression.

Try a WT9 × 38 of A588 steel, with area \( A_y = 11.2 \text{ in.}^2 \). The slenderness ratio for the Y-Y axis is

\[
\frac{KL_c}{r_y} = \frac{0.75 \times 18 \times 12}{2.61} = 62.1
\]

\[
\sqrt{\frac{2\pi^2 E}{F_{cr}}} = 107 > 79.4
\]

Hence, the critical buckling stress for the Y-Y axis is

\[
F_{cr} = F_y \left[ 1 - \frac{F_y}{4\pi^2 E} \left( \frac{KL_c}{r_y} \right)^2 \right] = 50 \left[ 1 - \frac{50}{4\pi^2 \times 29,000 \left( 62.1^2 \right)} \right] = 41.58 \text{ ksi}
\]

The capacity of the WT as an unbraced beam, with section modulus \( S = 13.8 \text{ in.}^3 \), is

\[
M_u = F_y S \left[ 1 - \frac{3F_y}{4\pi^2 E} \left( \frac{L_b}{0.9b'} \right)^2 \right] = 50 \times 10 \left[ 1 - \frac{3 \times 50}{4\pi^2 \times 29,000 \left( 0.9 \times 5.31 \right)^2} \right] = 388 \text{ kip-in.}
\]

The slenderness ratio for the X-X axis is

\[
\frac{KL_c}{r_x} = \frac{0.75 \times 18 \times 12}{2.54} = 63.8
\]
The Euler buckling stress for the X-X axis is

\[ F_e = \frac{\pi^2 E}{(KL_c/r_e)^2} = \frac{\pi^2 \times 29,000}{(63.8)^2} = 70.32 \text{ ksi} \]

\[ 0.680'' \quad 11.035'' \quad x = 0.502'' \]

\[ 9.105'' \quad 8.425'' \quad 0.425'' \]

WT9 X 38

The neutral axis of the WT9 X 38 for bending under Maximum Design Load is located at a distance \( y_p \) below the outer surface of the flange. The area of the section above the axis equals the area below.

\[ 11.035y_p = 8.425 \times 0.425 + 11.035(0.680 - y_p) \]

Solution of this equation yields \( y_p = 0.502 \text{ in.} \) and \( 0.680 - y_p = 0.178 \text{ in.} \)

The plastic section modulus then is

\[ Z = \frac{11.035(0.502)^2}{2} + \frac{11.035(0.178)^2}{2} + 8.425 \times 0.425 \times 4.391 = 17.29 \text{ in.}^3 \]

The capacity of the WT as a compact beam therefore is

\[ M_p = F_e Z = 50 \times 17.29 = 865 \text{ kip-in.} \]

The factored axial load is

\[ P = 1.30 \times 79.9 - 103.9 \text{ kips} \]

The maximum bending moment in the cross-frame bottom strut is

\[ M_{sec} = 103.9 \times 1.80 = 187.0 \]

\[ M_{DL} = (1.3 \times 0.038(18)^2/8)^{12} = -24.0 \]

\[ M = 163.0 \text{ kip-in.} \]

Substitution of the results of the preceding calculations in the interaction equations yields:

\[ \frac{103.9}{0.85 \times 11.2 \times 41.58} + \frac{163.0}{388 \left(1 - \frac{103.9}{11.2 \times 70.32}\right)} = 0.746 < 1.0 \]

\[ \frac{103.9}{0.85 \times 11.2 \times 50} + \frac{187.0}{865} = 0.434 < 1.0 \]

The WT9 X 38 is satisfactory.

The intermediate and support cross-frame details are shown in a drawing.
CROSS-FRAME

Transverse Beam at End Supports

Transverse beams are required above the cross-frames at the end supports of the girder to support the edge of the roadway slab. These beams span 8.5 ft between stiffeners on the main girders and stiffeners on the stringers. Also, a cantilever bracket is needed on the outer side of each exterior girder for the same purpose. The edge of the slab is haunched down to the transverse beams and the brackets.

TRANSVERSE BEAM AT EDGE SUPPORT
Dead Load on Transverse Beam

Slab: $8/12 \times 2 \times 0.150 = 0.20$
Haunch: $6/12 \times 1.75 \times 0.150 = 0.13$
Dam angle = 0.03
Wearing surface: $2 \times 0.020 = 0.04$
Beam = 0.02
Details = 0.01

$0.43 \text{ kips per ft}$

The live load consists of a pair of 16-kip wheels 6 ft apart. Impact is 30%. Hence, live load plus impact is $16 \times 1.30 = 20.8$ kips.

The maximum dead-load shear is

$$V_D = \frac{1}{2} \times 0.43 \times 8.5 = 1.8 \text{ kips}$$

The maximum shear due to live load plus impact is

$$V_L = 20.8 + 20.8 \times \frac{8.5 - 6}{8.5} = 26.9 \text{ kips}$$

The maximum dead-load bending moment occurs at midspan and is

$$M_D = \frac{0.43(8.5)^2}{8} = 3.9 \text{ kip-ft}$$

The maximum live-load bending moment with a 16-kip wheel at midspan is

$$M_L = \frac{20.8 \times 8.5}{4} = 44.2 \text{ kip-ft}$$

Under Maximum Design Load, the maximum shear is

$$V = 1.30(1.8 + \frac{5}{3} \times 26.9) = 60.7 \text{ kips}$$

and the maximum bending moment is

$$M = 1.30(3.9 + \frac{5}{3} \times 44.2) = 100.8 \text{ kip-ft}$$

The number of 7/8-in.-dia bolts required at the end connections of the beam is

$$\frac{60.7}{1.30 \times 12.63} = 3.7$$

Use 5 bolts.

Try a W12 x 22. It has a plastic section modulus $Z = 29.3 \text{ in.}^3$ and a section modulus $S = 25.4 \text{ in.}^3$ It qualifies as a compact section. The width-thickness ratio of the flanges is

$$\frac{b'}{t} = \frac{2.015}{0.425} = 4.7$$
Maximum permissible width-thickness ratio, as computed for the sub-stringers, is \( 7.16 > 4.7 \). The beam depth-thickness ratio is

\[
\frac{d}{t_w} = \frac{12.31}{0.26} = 47.3
\]

Maximum permissible depth-thickness ratio, as computed previously for the sub-stringers, is \( 59.5 > 47.3 \). The top flange is fully supported laterally by the concrete slab.

The shear capacity of the W12 × 22 is

\[ V_u = 0.55F_y d t_w = 0.55 \times 50 \times 12.31 \times 0.26 = 88.0 > 60.7 \text{ kips} \]

Bending strength of the beam is

\[ M_u = F_y Z = \frac{50 \times 29.3}{12} = 122.1 > 100.8 \text{ kip-ft} \]

The W12 × 22 is satisfactory.

Cross-Frame at End Support

The cross-frame below the transverse beam carries the end reaction of the W24 × 55 sub-stringer as well as the end reaction of the W12 × 22 transverse beam. The load, however, does not differ greatly from that computed for the intermediate cross-frames. Therefore, use the same sizes of diagonals and struts for the cross-frames at the end supports.

Assume that dam steel will support the dead loads along the edge of the slab and that only a wheel load will be supported by the bracket on the exterior girder. The maximum bending moment in the bracket then is

\[ M = 20.8 \times 1.5 = 31.2 \text{ kip-ft} \]
The bracket is connected to the girder with 7/8-in.-dia, A325 bolts. Allowable bolt load is 12.63 kips. For Overload, the design moment is $31.2 \times 5/3 = 52.0$ kip-ft.

**DETAIL OF BRACKET**

Assume that in the connection to the girder the web bolts will take the compressive force and the bolts in the flange connection will carry the tensile force of the moment, with a moment arm of 9 in. = 0.75 ft. The force in the flange bolts is

$$ F = \frac{52.0}{0.75} = 69.3 \text{ kips} $$

The number of bolts required for the flange connection therefore is

$$ \frac{69.3}{12.63} = 5.5 $$

Use 6 bolts.

The web bolts are subject to a horizontal force of 69.3 kips and a vertical force of $5/3 \times 20.8 = 34.7$ kips. The resultant force on the web bolts then is

$$ P = \sqrt{69.3^2 + 34.7^2} = 77.5 \text{ kips} $$

The number of bolts required for the web connection therefore is

$$ \frac{77.5}{12.63} = 6.1 $$

Use 8 bolts.

For the flange splice plate, try $1/2 \times 6$ in. With two rows of 7/8-in. bolts, the net area of the plate is

$$ A = \frac{1}{2} (6 - 2) = 2.0 \text{ in.}^2 $$

The tensile stress in the plate is

$$ f_t = \frac{69.3}{2.0} = 34.7 < 50 \text{ ksi} $$
For fatigue adjacent to a bolted connection, the allowable stress range is 27.5 ksi. The actual stress range with a Service Load of $69/(5/3) = 41.4$ kips is

$$f_s = \frac{41.4}{2.0} = 20.7 < 27.5 \text{ ksi}$$

The connection is satisfactory.

**LATERAL BRACING**

AASHTO Art. 1.7.21 requires lateral bracing of the bottom flange of girders when the span exceeds 125 ft, to assist in carrying wind loads. Because the girder spans in the design example are larger than 125 ft, lateral bracing is provided along the entire length of the girders.

A wind pressure of 50 psf is applied to the area of the bridge as seen in elevation and a wind pressure of 100 lb per lin ft is applied as wind on live load. The assumption is made that half the total wind load is taken by the lateral bracing and half is taken by the roadway slab.

<table>
<thead>
<tr>
<th>Depth of Structure, In.</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Parapet</td>
<td>34</td>
</tr>
<tr>
<td>Slab</td>
<td>8</td>
</tr>
<tr>
<td>Haunch</td>
<td>5</td>
</tr>
<tr>
<td>Web of girder</td>
<td>156</td>
</tr>
<tr>
<td>Bottom flange of girder</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>206 in.</td>
</tr>
</tbody>
</table>

For the 50-psf wind pressure, the wind load on the bridge profile is

$$w = 0.050 \times \frac{206}{12} = 0.858 \text{ kips per ft}$$

Group II loading includes 100% of this wind load. Group III loading combines 30% of this load with wind on live load, which is 0.1 kip per ft.

$$0.30 \times 0.858 + 0.1 = 0.357 < 0.858 \text{ kips per ft}$$

Hence, wind on live load may be ignored. Half the wind load, $0.858/2 = 0.429$ kips per ft, is assumed to be carried by the lateral bracing.

**WIND SHEARS ON LATERAL BRACING**

*A liberal interpretation of the provisions of AASHTO Art. 1.7.21 or a more rational analysis would permit designers to eliminate lateral bracing in the central portions (between dead-load points of contraflexure) of continuous spans for distances up to 125 ft. If the bracing is omitted, the remaining end portions of the girder, with bracings, must have sufficient stiffness to transmit the lateral forces to the bearings.
Maximum Shear on Lateral Bracing

The shear at the end of the girder is

\[ V_A = 0.367 \times 0.429 \times 273 = 43.0 \text{ kips} \]

The shear in the end span adjacent to the interior support is

\[ V_{BA} = -0.633 \times 0.429 \times 273 = -74.1 \text{ kips} \]

The shear in the center span adjacent to a support is

\[ V_{BA} = 0.641 \times 0.429 \times 273 = 75.1 \text{ kips} \]

The reaction \( R_b \) at the interior support equals \( 74.1 + 75.1 = 149.2 \text{ kips} \).

Warren Truss

A Warren truss of A588 steel is used for lateral bracing in each bay between the girders, just above the bottom flanges. (A K-bracing system was also studied and found to require about the same weight of structural steel as the Warren truss but was rejected because more connections would be required.) Two different sizes of lateral-bracing diagonals are employed in the Warren truss. A heavier section is required for the regions of high wind shear than for the section needed for the remainder of the structure. Panels in the 273-ft girder spans are 24.82 ft long and panels in the 350-ft girder span are 25 ft long.

LATERAL BRACING

Panel Wind Loads

\[ 0.429 \times 24.82 = 10.65 \text{ kips} \]
\[ 0.429 \times 25.0 = 10.73 \text{ kips} \]
Bracing for Region of High Shear

The diagonals of the lateral bracing system act as beam-columns and must satisfy the same interaction equations as the bottom strut of the cross-frames at the interior girder support.

The length of a lateral-bracing diagonal is

\[ L = \sqrt{18.5^2 + 25^2} = 31.1 \text{ ft} \]

The maximum slenderness ratio for a bracing member in compression is 140.

Hence, the radius of gyration of the diagonal should be at least

\[ r = \frac{31.1 \times 12}{140} = 2.67 \text{ in.} \]

Try a WT12 × 65.5 for the diagonal. It has a radius of gyration about the Y-Y axis \( r_y = 2.97 \), area \( A_s = 19.3 \text{ in.}^2 \) and section modulus \( S = 24.8 \text{ in.}^3 \) The slenderness ratio for the diagonal for the Y-Y axis is

\[ \frac{KL_c}{r_y} \cdot \frac{0.75 \times 31.1 \times 12}{2.97} = 94.2 \]

\[ \frac{2\pi^2 E}{F_y} = 107 > 94.2 \]

Hence, the critical buckling stress for the Y-Y axis is

\[ F_{cr} = F_y \left[ 1 - \frac{F_y}{4\pi^2 E} \left( \frac{KL_c}{r_y} \right)^2 \right] = 50 \left[ 1 - \frac{50}{4\pi^2 \times 29,000 \times (94.2)^2} \right] = 30.62 \text{ ksi} \]

The bending strength of the diagonal as an unbraced beam is

\[ M_u = F_y S \left[ 1 - \frac{3F_y}{4\pi^2 E} \left( \frac{L_b}{0.96} \right)^2 \right] = 50 \times 23.1 \]

\[ \left[ 1 - \frac{3 \times 50}{4\pi^2 \times 29,000} \left( \frac{31.1 \times 12}{0.9 \times 6.718} \right)^2 \right] = 461.4 \text{ kip-in.} \]

The slenderness ratio for the X-X axis is

\[ \frac{KL_c}{r_x} \cdot \frac{0.75 \times 31.1 \times 12}{3.52} = 79.5 \]
The Euler buckling stress for the X-X axis is

\[ F_e = \frac{\pi^2 E}{(KL_c/r_e)^2} = \frac{\pi^2 \times 29,000}{(79.5)^2} = 45.29 \text{ ksi} \]

The neutral axis of the WT for bending under Maximum Design Load is located at a distance \( y_p \) below the outer surface of the flange. The area of the section above the axis equals the area below.

\[ 12.855y_p = 11.28 \times 0.605 + 12.855(0.96 - y_p) \]

Solution of the equation yields \( y_p = 0.715 \)-in. and \( 0.90 - y_p = 0.185 \)-in.

The plastic section modulus then is

\[ Z = \frac{12.855(0.715)^2}{2} + \frac{12.855(0.185)^2}{2} + 11.28 \times 0.605 \times 5.825 = 43.3 \text{ in.}^3 \]

The capacity of the WT as a compact beam therefore is

\[ M_p = F_e Z = 50 \times 43.3 = 2,165 \text{ kip-in.} \]

The maximum shear on a panel of the Warren truss is 69.7 kips. Hence, the Maximum Design Load on the diagonal is

\[ P = 1.30 \times \frac{69.7}{2} \times \frac{31.1}{18.5} = 76.2 \text{ kips} \]

The maximum bending moment in the diagonal is

\[ M_{ecc} = 76.2 \times 2.65 = 201.9 \]

\[ M_{DL} = 1.30 \times 0.0655(31.1)^2 / 8 \times 12 = 123.5 \]

\[ M = 325.4 \text{ kip-in.} \]

Substitution of the preceding results in the interaction equation yields

\[ \frac{76.2}{0.85 \times 19.3 \times 30.62} + \frac{325.4}{461.4 \left(1 - \frac{76.2}{19.3 \times 45.29}\right)} = 0.924 < 1.0 \]

\[ \frac{76.2}{0.85 \times 19.3 \times 50} + \frac{201.9}{2,030} = 0.186 < 1.0 \]

The WT12 X 65.5 is satisfactory.
Bracing for Regions of Lower Shear

The lateral-bracing diagonals for regions of lower shear are designed in the same manner as for the high-shear region, but for a shear of 37.7 kips or less. A WT12 x 52 section is adequate. Additional reduction in member size could be made, if desired.

Bracing Connections

The lateral bracing of the girders is connected to the girder webs and the connections are designed to transmit the longitudinal components of the forces in the bracing members into the girders. The highest force occurs in the center span of the girder at the panel point adjacent to the interior support. The force is calculated for shears of 69.7 and 59.0 kips in the panels on each side of the panel point.

The maximum axial load in the diagonal of the first panel from the support is

\[ P_1 = \frac{69.7}{2} \times \frac{31.1}{18.5} = 58.6 \text{ kips} \]

The maximum axial load in the diagonal of the second panel from the support is

\[ P_2 = \frac{59.0}{2} \times \frac{31.1}{18.5} = 49.6 \text{ kips} \]

The maximum longitudinal load then this

\[ P_L = \left(58.6 + 49.6\right) \times \frac{25.0}{31.1} = 87.0 \text{ kips} \]

A design is made for the lateral bracing on the assumption that a gusset plate is welded to the girder web for connection of the bracing. Fatigue for the connection should be checked for AASHTO Category E in the design of the girder. The welded connection is low cost and could be used appropriately at locations along the girder where the permissible stress range of Category E would not be exceeded.
Welded Gusset Plate Connections

The connections of bracing near the interior supports are investigated first. The connection to a girder web is made with two fillet welds along the edge of the gusset plate next to the web. The capacity of a fillet weld is \(0.45F_u = 0.45 \times 70 = 31.5 \text{ ksi}\).

Try a 1/4-in. fillet weld. The connected edge of the gusset plate is about 38 in. long. For two 1/4-in. fillet welds carrying \(F_L = 87.0 \text{ kips}\), the stress on a weld is

\[
f_w = \frac{1.3 \times 87.0}{38 \times 2 \times 0.707 \times 1/4} = 8.42 < 31.5 \text{ ksi}
\]

The weld should be checked for fatigue under 100,000 cycles of loading for AASHTO Category F, for which the allowable stress range is 15 ksi. The actual stress range for reversal of wind direction is

\[
f_{sr} = \frac{2 \times 87.0}{38 \times 2 \times 0.707 \times 1/4} = 12.95 < 15 \text{ ksi}
\]

The diagonals are connected to the gusset plate with 7/8-in.-dia, A325 bolts. Each bolt has a capacity of 12.63 kips. Hence, the number of bolts required for a diagonal is

\[
\frac{1.3 \times 58.6}{12.63} = 6.0
\]

Use 6 bolts.

A similar design is made for the lateral bracing connection at the ends of the girders, where the longitudinal shear is 25.3 kips. The gusset plate is about 18 in. long along the connection to the girder web. For two 1/4-in. fillet welds, the stress in a weld is

\[
f_w = \frac{1.3 \times 25.3}{18 \times 2 \times 0.707 \times 1/4} = 5.17 < 31.5 \text{ ksi}
\]

![BRACING CONNECTION WITH WELDED GUSSET PLATE AT END OF GIRDER](image)

For fatigue, the stress range is

\[
f_{sr} = \frac{2 \times 25.3}{18 \times 2 \times 0.707 \times 1/4} = 7.95 < 15 \text{ ksi}
\]
The maximum load on the bolts connecting the bottom strut of a cross-frame to a gusset plate is 18.9 kips. The number of bolts required in the strut is

\[
\frac{1.3 \times 18.9}{12.63} = 1.9
\]

Use 3 bolts.

**Bolted Gusset Plate Connection**

As an alternative, the connections of the lateral bracing to the girder webs can be made with bolts. The connections can be designed with the same general principles as those used for the welded connections. The girder at the bolted connections should be checked for fatigue for AASHTO Category B. (They have substantially better fatigue characteristics than the welded connections.) The allowable girder stress ranges using bolted connections are 45.0 ksi (lane loading) and 27.5 ksi (truck loading) compared with corresponding ranges of 21.0 and 12.5 ksi, respectively, using welded connections. Bolted connections could be used appropriately at locations along the girder where the permissible girder stress range of fatigue Category E (welded connections) is exceeded.

---

**TYPICAL BOLTED CONNECTION FOR LATERAL BRACING**

The bolted connections of the lateral bracing at a girder are made to the stem of a WT18 X 67.5, which is connected with flange bolts to the girder web. Bolts are 7/8-in. in diameter and have a capacity of 12.63 kips each. Maximum permissible length of the unsupported edge of the gusset is

\[
L = \frac{11,000t}{\sqrt{F_y}} = \frac{11,000 \times 0.600}{\sqrt{50,000}} = 29.4 \text{ in.}
\]

The unsupported edge of the gusset is about 12 in. < 29.5 in.
For the maximum load $P_t = 87.0$ kips on the gusset, the maximum number of bolts required for the connection to the girder web is

$$\frac{87.0}{12.63} = 6.9$$

Use 12 bolts.

The bolts in the connection to the girder web are subject to combined tension and shear. The tensile forces are caused by a direct pull and by prying action. The maximum direct tensile force is

$$P_t = 58.6 \times \frac{18.5}{31.1} = 34.9 \text{ kips}$$

Divided among six bolts, the tension is $T - 34.9/6 = 5.8 \text{ kips per bolt}$. Praying action results from both the tension on the bolts and distortion of the connected parts. The lever arms involved are the distance $a = 3$ in. from the center of the bolts to the edge of the flange and the distance $b = 1.7/8$ in. from the toe of the fillet between flange and web of the WT and the center of the bolts. Thickness of the WT flange is 0.794 in. and the thickness of the girder web may be assumed to be 0.5 in. $< 0.794$ in. The prying force on the bolts then is

$$Q = \left( 3b - \frac{r^3}{8a} \right) T = \left( \frac{3 \times 1.875}{8 \times 3} - \frac{(0.794)^3}{20} \right) 5.8 = 1.2 \text{ kips per bolt}.$$ 

The total tension on the bolts $= 5.8 + 1.2 = 7.0 \text{ kips per bolt}$. The tensile stress in each bolt therefore is

$$f_t = \frac{7.0}{0.601} = 11.6 \text{ ksi}$$

The shear stress in each bolt is

$$f_v = \frac{58.6 \times 25/31.1}{6 \times 0.601} = 13.1 \text{ ksi}$$

The allowable shear stress is

$$f_v = 21 - 0.35f_t = 21 - (0.35)(11.8) = 16.9 > 13.1 \text{ ksi}$$

The connection is satisfactory.

![Diagram of a bolted gusset connection](image)
For a bolted gusset connection for the lateral bracing at the end of the girder, a WT15 x 49.5 is selected. The maximum longitudinal shear is

\[ P_L = 31.7 \times \frac{25.0}{31.1} = 25.5 \text{ kips} \]

The maximum transverse shear is \( 37.7/2 = 18.9 \) kips. The number of bolts required in the connection of the WT to the girder web is

\[ \frac{25.5}{12.63} = 2.0 \]

Use 6 bolts. The number of bolts required in the connection of the bottom strut of the cross-frame to the gusset is

\[ \frac{18.9}{12.63} = 1.5 \]

Use three bolts.

**DESIGN OF INTERIOR GIRDER**

The initial dead load \( DL_1 \) on the interior girder consists of an estimated weight for the girder and framing details plus the weight of the concrete roadway slab. The dead load \( DL_2 \) carried by the composite section is made up of the weight of the parapet and a 20-psf future wearing surface. The live load is AASHTO HS20-44 truck and lane loading with impact.

**Dead Load Carried by Steel Section**

- Slab \( \frac{8}{12} \times 18.5 \times 0.150 = 1.850 \)
- Concrete haunches on girder and stringer = 0.106
- Estimated girder weight = 0.650
- Stringer, cross-frames and lateral bracing = 0.187

\[ 2.793 \text{ k/ft} \]

**Dead Load Carried by Composite Section**

- Parapets = \( 0.435 \times 2/3 = 0.290 \)
- Future wearing surface = \( 0.020 \times 44/3 = 0.293 \)

\[ 0.583 \text{ k/ft} \]

**LIVE LOAD PLACED FOR MAXIMUM LOAD ON INTERIOR GIRDER**
Live Load

When the interior girder receives load from three lanes of traffic, the total load may be reduced 10%. In this case, the maximum load on the girder is

\[ R_2 = 0.90 \times \frac{P}{18.5} (18.5 + 12.5 + 6.5 + 0.5 + 14.5 + 8.5) = 2.968P \]

where \( P \) = wheel load.

When the interior girder receives load from two lanes of traffic, the maximum load is

\[ R_1 = \frac{P}{18.5} (18.5 + 12.5 + 14.5 + 8.5) = 2.919P < 2.968P \]

Therefore, the maximum load on the girder is 2.968\( P \) or 1.484 axles.

Impact

Positive-moment in 273-ft span: \[ I = \frac{50}{273 + 125} = 0.126 \]

Positive-moment in 350-ft span: \[ I = \frac{50}{350 + 125} = 0.105 \]

Negative-moments: \[ I = \frac{50}{0.5(273 + 350) + 125} = 0.115 \]

Curves for maximum moments and shears may be determined by any convenient method. The curves presented in this chapter were obtained from the final design cycle with variable moments of inertia in each span.
STIFFENED WEB

Studies of variation of girder weight with changes in girder depth indicate that, for the spans of this example, a clear girder depth \( D \) of 156 in. is the most efficient depth. Transverse stiffeners and one longitudinal stiffener are used over the full length of the girder. With these stiffeners, the maximum permissible web depth-thickness ratio is

\[
\frac{D}{t_w} = \frac{73,000}{\sqrt{F_y}}
\]

In the inflection regions the longitudinal stiffeners are carried beyond the dead load inflection points to locations at which the moments resulting from

\[
1.30 \left[ D + \frac{5}{3} \left\{ + (L + I) \right\} \right] \quad \text{and} \quad 1.30 \left[ D + \frac{5}{3} \left\{ -(L + I) \right\} \right]
\]

are zero.

Hence, the web thickness for \( D = 156 \) in. must be at least

\[
t_w = \frac{156 \sqrt{50,000}}{73,000} = 0.478 \text{ in.}
\]

The shear capacity of a transversely stiffened web depends on the web thickness and the stiffener spacing. The moment capacity of the section, however, must be reduced if the shear exceeds 60% of the shear capacity of the web. A combination of web thickness and stiffener spacing should therefore be selected to furnish both the necessary shear and moment capacities. It is necessary, though, to place a stiffener at the location of each cross-frame for connection purposes.

A 9/16-in. web is selected for use in the portion of the girder near the interior support.
Stiffener Spacing Near Interior Support

A trial stiffener spacing of \( d_o = 74.5 \) in., equal to one-fourth the cross-frame spacing in the end span, is investigated for the end span near the interior support.

### Maximum Shear at Interior Support

<table>
<thead>
<tr>
<th>Shear, kips</th>
<th>( DL_1 )</th>
<th>( DL_2 )</th>
<th>( L + I )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>479</td>
<td>102</td>
<td>247</td>
</tr>
</tbody>
</table>

For determination of the shear capacity of the web near the interior support,

\[
C = 18,000 \cdot \frac{t_w}{D} \cdot \sqrt{\frac{1 + (D/d_o)^2}{F_y}} - 0.3 = 18,000 \cdot \frac{9/16}{156} \cdot \sqrt{\frac{1 + (156/74.5)^2}{50,000}} - 0.3 = 0.374
\]

\[
V_p = 0.58 \times 50 \times 156 \times 9/16 = 2,545 \text{ kips}
\]

The shear capacity of the web near the interior support then is

\[
V_u = V_p \cdot C + \sqrt{\frac{0.87(1 - C)}{1 + (d_o/D)^2}} = 2,545 \cdot 0.374 + \sqrt{\frac{0.87(1 - 0.374)}{1 + (74.5/156)^2}} = 2,203 \text{ kips}
\]

The maximum shear at the interior support due to Maximum Design Load is

\[
V = 1.30 \cdot 479 + 102 + \frac{5}{3} \times 247 = 1.290 < (0.6 \times 2,203 = 1,322 \text{ kips})
\]

Consequently, a reduction in moment capacity is not required with a stiffener spacing of 74.5 in. near the interior support.

### Web Thickness Transition

The 9/16-in. web could be extended to the field splices 81 and 87 ft on either side of the interior support. But a 9/16-in. plate in the length required could not be furnished and would have to be fabricated by splicing together two or three plates with available lengths. Instead, a transition in web thickness to a 1/2-in. plate is made at a point about where a butt-welded splice would normally be required for the 9/16-in. plate. In the end spans, the transition in web thickness is located 46 ft from the interior support. In the interior span, the transition is located 52 ft from the interior support.

### Stiffener Spacing Near Web Thickness Transition

The transition to a 1/2-in. web does not require a reduction in moment capacity if at least two stiffeners are placed between the cross-frame connections on either side of the transition. (Cross-frames in the end span are 24.82 ft apart.) For example, the diagram of maximum design shears shows that the maximum design shear in the end span about 50 ft from the interior support is 937 kips. For a transverse
MAXIMUM DESIGN SHEARS

stiffener spacing \( d_o = 24.82 \times 12/3 = 99.3 \) in. and a 1/2-in. web,

\[
C = 18,000 \times \frac{0.5}{156} \sqrt{\frac{1 + (156/99.3)^2}{50,000}} = 0.3 = 0.180
\]

\( V_p = 0.58 \times 50 \times 156 \times 0.5 = 2,262 \) kips

The shear capacity of the 1/2-in. web near the transition then is

\[
V_u = 2,262 \left[ 0.180 + \sqrt{\frac{0.87(1 - 0.180)}{1 + (99.3/156)^2}} \right] = 1,768 \) kips

\( 0.6V_u = 0.6 \times 1,768 = 1,061 > 937 \) kips

Hence, the shear does not cause a reduction in moment capacity with a stiffener spacing of 99.3 in.

A similar calculation (not shown here) indicates that the web transition in the interior span does not require any reduction in moment capacity if two stiffeners are placed between the cross-frame connections on either side of the transition.

Stiffener Spacing at End Bearing

At the end bearing, the web is braced transversely by a transverse beam and a cross-frame. Clear unsupported depth \( D \) of the web may be taken as 123 in. Shear at the end support due to maximum design load is \( V = 775 \) kips.

The end shear capacity is dependent on the first stiffener space in accordance with the formula

\[
V = 1.2 \times 10^5 \left[ 1 + \left( \frac{D}{d_o} \right)^2 \right] \frac{t_w^3}{D} \leq V_p
\]

With a 1/2-in. web and an assumed first stiffener space of 53 in.,

\( V_p = 0.58 \times 50 \times 123 \times 0.5 = 1,784 \) kips
\[ V = 1.2 \times 10^4 \left[ 1 + \left( \frac{123}{53} \right)^2 \right] \left( \frac{0.5}{123} \right)^3 = 779 \text{ kips} < 1,784 \]

The 53 in. stiffener spacing is satisfactory with the 1/2-in. web.

**General Stiffener Spacing**

Transverse-stiffener spacing in the rest of the girder is established in a manner similar to the preceding. To avoid repetitious calculations, additional stiffener calculations are not presented. Overall stiffener spacing, however, is shown in a drawing at the end of the chapter. Design of stiffeners is discussed later in the chapter.

**FLANGE-THICKNESS REQUIREMENTS**

For \( F_y = 50 \text{ ksi} \), the criterion for width-thickness ratio of projecting compression flanges becomes \( b/t \leq 9.8 \).

<table>
<thead>
<tr>
<th>Flange Width, In.</th>
<th>Projection ( b ), In.</th>
<th>Minimum ( t ), In.</th>
<th>Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>11</td>
<td>11/9.8 = 1.12</td>
<td>1-1/8</td>
</tr>
<tr>
<td>30</td>
<td>15</td>
<td>15/9.8 = 1.53</td>
<td>1-9/16</td>
</tr>
<tr>
<td>36</td>
<td>18</td>
<td>18/9.8 = 1.84</td>
<td>1-7/8</td>
</tr>
</tbody>
</table>

**FATIGUE REQUIREMENTS**

Before the start of the design of positive-moment and negative-moment sections, a determination should be made of what fatigue checks are required. For welded plate girders of the type in the design example, fatigue due to tension or stress reversal should be checked at the following locations:

1. Base metal in the girder web adjacent to fillet welds of lateral-bracing connection plates. (Category E)
2. Base metal in the girder web adjacent to the end of a longitudinal stiffener. (Category E)
3. Base metal in the girder web at the toe of fillet welds of transverse stiffeners or cross-frame connection plates. (Category C)
4. Base metal adjacent to stud-type shear connectors. (Category C)
5. Base metal adjacent to flange transitions with full-penetration groove welds.
6. Stress range in slab reinforcement limited to 20.0 ksi in negative bending region. (Category B)

AASHTO Specifications assign the following allowable ranges of stress:

<table>
<thead>
<tr>
<th></th>
<th>500,000 Cycles (Truck Loading)</th>
<th>100,000 Cycles (Lane Loading)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Category B</td>
<td>27.5 ksi</td>
<td>45.0 ksi</td>
</tr>
<tr>
<td>Category C</td>
<td>19.0 ksi</td>
<td>32.0 ksi</td>
</tr>
<tr>
<td>Category E</td>
<td>12.5 ksi</td>
<td>21.0 ksi</td>
</tr>
</tbody>
</table>

**CRITICAL NEGATIVE MOMENT AT INTERIOR SUPPORT**

The girder section at the interior support is noncompact. For Maximum Design
Load, the section should satisfy

\[ F_y S \geq 1.30 \left[ D + \frac{5}{3}(L + I) \right] \]

and for Overload,

\[ 0.80F_y S \geq \left[ D + \frac{5}{3}(L + I) \right] \text{ or } F_y S \geq 1.25 \left[ D + \frac{5}{3}(L + I) \right] \]

By inspection, the relationship for Maximum Design Load governs.

A section made up of a 3 X 36-in. top flange, a 3-1/4 X 36-in. bottom flange and a 9/16 X 156-in. web plate is tried. The slab contains 22 No. 5 longitudinal reinforcing bars within the effective slab width.

**CRITICAL-NEGATIVE-MOMENT SECTION**

With cross-frames spaced at 25 ft to brace the girder, the unbraced length of compression flange exceeds the maximum permissible unbraced length for a braced section.

\[
L_b = \frac{20,000,000A_f}{F_y d} = \frac{20,000,000 \times 3.25 \times 36}{50,000 \times 162.25} = 288 \text{ in.} = 24.0 < 25 \text{ ft}
\]

The negative-moment section, therefore, is an unbraced, noncompact section. As a result, the maximum bending strength is defined by

\[
M_u = F_y S \left[ 1 - \frac{3F_y}{4\pi^2E} \left( \frac{L_b}{0.9b_1} \right)^2 \right]
\]

or, dividing through by \( S \) gives the critical lateral buckling stress for the compression flange as

\[
F_{cr} = \frac{M_u}{S} = F_y \left[ 1 - \frac{3F_y}{4\pi^2E} \left( \frac{L_b}{0.9b_1} \right)^2 \right]
\]

Because the compression flange does not change in width within the braced length, the critical allowable compression stress becomes

\[
F_{cr} = F_y \left[ 1 - \frac{3 \times 50}{4\pi^2 \times 29,000} \left( \frac{25 \times 12}{0.9 \times 18} \right)^2 \right] = 0.955F_y
\]
If the ratio of moments at the two ends of the unbraced length is less than 0.7, this stress may be increased by 20% but not to more than $F_y$.

**Maximum Negative Moments, Kip-Ft**

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$-(L + I)$</th>
<th>Total</th>
<th>$+(L + I)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>At Interior Support</td>
<td>-30,543</td>
<td>-6,410</td>
<td>-15,351</td>
<td>-52,304</td>
<td>2,090</td>
</tr>
<tr>
<td>At Cross-Frame</td>
<td>-20,640</td>
<td>-4,220</td>
<td>-10,440</td>
<td>-35,300</td>
<td></td>
</tr>
</tbody>
</table>

The ratio of the total moments is

$$ R = \frac{35,300}{52,304} = 0.67 < 0.7 $$

Therefore, the allowable stress may be increased up to 20%.

$$ 1.20 \times 0.955F_y = 1.15F_y > F_y $$

Use $F_{cr} = F_y = 50$ ksi for the allowable stress in the compression flange.

Section properties are calculated for the girder section alone and for the girder section plus the longitudinal reinforcing bars in the concrete slab.

**Steel Section at Interior Support**

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_o$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Flg. 3 x 36</td>
<td>108.00</td>
<td>79.5</td>
<td>8,586</td>
<td>682,600</td>
<td>682,600</td>
<td></td>
</tr>
<tr>
<td>Web 9/16 x 156</td>
<td>87.75</td>
<td>79.5</td>
<td>8,586</td>
<td>682,600</td>
<td>682,600</td>
<td></td>
</tr>
<tr>
<td>Bot. Flg. 3-1/4 x 36</td>
<td>117.00</td>
<td>-79.63</td>
<td>-9,317</td>
<td>741,900</td>
<td>741,900</td>
<td></td>
</tr>
</tbody>
</table>

312.75 in.$^2$         -731 in.$^3$ 1,602,500

$$ d_s = \frac{-731}{312.75} = -2.34 \text{ in.} $$

$$ -2.34 \times 731 = -1,700 $$

$$ I_{NA} = 1,600,800 \text{ in.}^4 $$

$$ d_{Top \text{ of steel}} = 81.0 + 2.34 = 83.34 \text{ in.} $$

$$ d_{Bot. \text{ of steel}} = 81.25 - 2.34 = 78.91 \text{ in.} $$

$$ S_{Top \text{ of steel}} = \frac{1,600,800}{83.34} = 19,208 \text{ in.}^3 $$

$$ S_{Bot. \text{ of steel}} = \frac{1,600,800}{78.91} = 20,286 \text{ in.}^3 $$

**Steel Section with Reinforcing Steel at Interior Support**

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_o$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>312.75</td>
<td></td>
<td>-731</td>
<td></td>
<td>1,602,500</td>
<td></td>
</tr>
<tr>
<td>Reinf. 22 No. 5</td>
<td>6.82</td>
<td>86.02</td>
<td>587</td>
<td>50,500</td>
<td>50,500</td>
<td></td>
</tr>
</tbody>
</table>

319.57 in.$^2$         -144 in.$^3$ 1,653,000

$$ d_s = \frac{-144}{319.57} = -0.45 \text{ in.} $$

$$ -0.45 \times 144 \approx -100 $$

$$ I_{NA} = 1,652,900 \text{ in.}^4 $$
\[ d_{\text{Top of steel}} = 81.0 + 0.45 = 81.45 \quad d_{\text{Bot. of steel}} = 81.25 - 0.45 = 80.80 \text{ in.} \]

\[ S_{\text{Top of steel}} = \frac{1,652,900}{81.45} = 20,293 \text{ in}^3 \quad S_{\text{Bot. of steel}} = \frac{1,652,900}{80.80} = 20,457 \text{ in}^3 \]

\[ d_{\text{Reinf.}} = 86.02 + 0.45 = 86.47 \text{ in.} \]

\[ S_{\text{Reinf.}} = \frac{1,652,900}{86.47} = 19,115 \text{ in}^3 \]

Steel Stresses for Critical Negative Moment Due to Maximum Design Loads

<table>
<thead>
<tr>
<th>Top of Steel (Tension)</th>
<th>Bottom of Steel (Compression)</th>
</tr>
</thead>
<tbody>
<tr>
<td>For ( DL_1 ): ( F_b = \frac{30,543 \times 12}{19,208} \times 1.30 = 24.8 )</td>
<td>( F_b = \frac{30,543 \times 12}{20,286} \times 1.30 = 23.5 )</td>
</tr>
<tr>
<td>For ( DL_2 ): ( F_b = \frac{6,410 \times 12}{20,293} \times 1.30 = 4.9 )</td>
<td>( F_b = \frac{6,410 \times 12}{20,457} \times 1.30 = 4.9 )</td>
</tr>
<tr>
<td>For ( L + I ): ( F_b = \frac{15,351 \times 12}{20,293} \times 1.30 \times \frac{5}{3} = 19.7 )</td>
<td>( F_b = \frac{15,351 \times 12}{20,457} \times 1.30 \times \frac{5}{3} = 19.5 )</td>
</tr>
</tbody>
</table>

49.4 < 50 ksi

50 ksi > 47.9

Reinforcing Steel Stress (Tension)

\[ F_b = \frac{6,410 + (5/3)15,351}{19,115} \times 12 \times 1.30 = 26.1 < 40 \text{ ksi} \]

The fatigue stress range in the reinforcing steel is limited to 20 ksi. The actual live-load stress range is

\[ f_{sr} = \frac{(15,351 + 2,090)12}{19,115} = 11.0 < 20 \text{ ksi} \]

In addition to the check for Maximum Design Load, the critical negative-moment section should be checked for fatigue at the weld of the stud shear connector. On the assumption that a row of connectors will be placed on the top flange near the interior support, the live-load stress range at that section is determined to be

\[ f_{sr} = \frac{(15,351 + 2,090)12}{20,293} = 10.3 < 32 \text{ ksi (Lane load controls)} \]

The section is satisfactory in fatigue near the interior support.

MAXIMUM-POSITIVE-MOMENT — END SPAN

The maximum-positive-moment section in the end span qualifies as a braced, noncompact section, because the compression flange is braced throughout by the concrete slab. A trial section comprising a 1-1/16 x 22-in. top flange, 1/2 x 156-in. web and 2-1/4 x 22-in. bottom flange is selected. Properties are calculated for the steel section alone, the composite section with modular ratio \( 3\pi = 24 \), and the composite section with modular ratio \( \pi = 8 \).
**COMPOSITE SECTION**

**Effective Flange Width**

\[
\text{1/4 span} = \frac{1}{4} \times 273 \times 12 = 819 \text{ in.}
\]

Girder spacing, \(c\) to \(c\) = 111 in.

\(12 \times \text{slab thickness} = 12 \times 7.5 = 90 \text{ in. (governs)}\)

### Steel Section

<table>
<thead>
<tr>
<th>Material</th>
<th>(A)</th>
<th>(d)</th>
<th>(Ad)</th>
<th>(Ad^2)</th>
<th>(I_o)</th>
<th>(I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Flg. 1-1/16 X 22</td>
<td>23.38</td>
<td>78.53</td>
<td>1,836</td>
<td>144,200</td>
<td>144,200</td>
<td></td>
</tr>
<tr>
<td>Web 1/2 X 156</td>
<td>78.00</td>
<td>-79.13</td>
<td>-3,917</td>
<td>309,900</td>
<td>309,900</td>
<td></td>
</tr>
<tr>
<td>Bot. Flg. 2-1/4 X 22</td>
<td>49.50</td>
<td>150.88</td>
<td>-2,081</td>
<td>612,300</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
d_s = \frac{-2,081}{150.88} = -13.79 \text{ in.}
\]

\[
-13.79 \times 2,081 = \frac{-28,700}{I_{NA}} = 583,600 \text{ in.}^4
\]

\[
d_{Top \ of \ steel} = 79.06 + 13.79 = 92.85 \text{ in.} \quad d_{Bot. \ of \ steel} = 80.25 - 13.79 = 66.46 \text{ in.}
\]

\[
S_{Top \ of \ steel} = \frac{583,600}{92.75} = 6,285 \text{ in.}^3 \quad S_{Bot. \ of \ steel} = \frac{583,600}{66.46} = 8,781 \text{ in.}^3
\]

### Composite Section, \(n = 8\)

<table>
<thead>
<tr>
<th>Material</th>
<th>(A)</th>
<th>(d)</th>
<th>(Ad)</th>
<th>(Ad^2)</th>
<th>(I_o)</th>
<th>(I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>150.88</td>
<td>2,081</td>
<td>331 in.²</td>
<td>612,300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conc. 90 X 7.5/24</td>
<td>28.13</td>
<td>85.75</td>
<td>2,412</td>
<td>206,800</td>
<td>206,900</td>
<td></td>
</tr>
</tbody>
</table>

\[
d_{24} = \frac{331}{179.01} = 1.85 \text{ in.}
\]

\[
-1.85 \times 331 = \frac{-600}{I_{NA}} = 818,600 \text{ in.}^4
\]

\[
d_{Top \ of \ steel} = 79.06 - 1.85 = 77.21 \text{ in.} \quad d_{Bot. \ of \ steel} = 80.25 + 1.85 = 82.10 \text{ in.}
\]

\[
S_{Bot. \ of \ steel} = \frac{818,600}{77.21} = 10,602 \text{ in.}^3 \quad S_{Bot. \ of \ steel} = \frac{818,600}{82.10} = 9,971 \text{ in.}^3
\]

### Composite Section, \(n = 8\)

<table>
<thead>
<tr>
<th>Material</th>
<th>(A)</th>
<th>(d)</th>
<th>(Ad)</th>
<th>(Ad^2)</th>
<th>(I_o)</th>
<th>(I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>150.88</td>
<td>2,081</td>
<td>235.26 in.²</td>
<td>612,300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conc. 90 X 7.5/8</td>
<td>84.38</td>
<td>85.75</td>
<td>7,236</td>
<td>620,500</td>
<td>620,900</td>
<td></td>
</tr>
</tbody>
</table>

\[
d_s = \frac{5,155}{235.26} = 21.91 \text{ in.}
\]

\[
-21.91 \times 5,155 = \frac{-113,000}{I_{NA}} = 1,120,200
\]

II/5.40 12/73
\[ d_{\text{Top of steel}} = 79.06 - 21.91 = 57.15 \text{ in.} \quad d_{\text{Bot. of steel}} = 80.25 + 21.91 = 102.16 \text{ in.} \]
\[ S_{\text{Top of steel}} = \frac{1,120,200}{57.15} = 19,601 \text{ in.}^3 \quad S_{\text{Bot. of steel}} = \frac{1,120,200}{102.16} = 10,965 \text{ in.}^3 \]

For Maximum Design Load, the section should satisfy
\[ F_y S \geq 1.30 \left[ D + \frac{5}{3}(L + I) \right] \]
and, for Overload,
\[ 0.95F_y S \geq \left[ D + \frac{5}{3}(L + I) \right] \text{ or } F_y S \geq 1.053 \left[ D + \frac{5}{3}(L + I) \right] \]

By inspection, Maximum Design Load governs, and the allowable stress is \( F_y = 50 \text{ ksi.} \) Maximum positive-moment occurs at about \( 0.4L = 0.4 \times 273 = 109.2 \text{ ft} \) from the end support.

**Bending Moments 109.2 Ft from End Support**

<table>
<thead>
<tr>
<th></th>
<th>DL(_1)</th>
<th>DL(_2)</th>
<th>( + (L + I) )</th>
<th>(- (L + I) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M, \text{ kip-ft} )</td>
<td>11,510</td>
<td>2,650</td>
<td>9,748</td>
<td>-3,583</td>
</tr>
</tbody>
</table>

Steel Stresses Due to Maximum Design Loads

For \( DL_1 \):
\[ F_b = \frac{11,510 \times 12}{6,285} \times 1.30 = 28.6 \quad F_b = \frac{11,510 \times 12}{8,781} \times 1.30 = 20.4 \]

For \( DL_2 \):
\[ F_b = \frac{2,650 \times 12}{10,602} \times 1.30 = 3.9 \quad F_b = \frac{2,650 \times 12}{9,971} \times 1.30 = 4.1 \]

For \( L + I \):
\[ F_b = \frac{9,748 \times 12}{19,601} \times 1.30 \times \frac{5}{3} = 12.9 \quad F_b = \frac{9,748 \times 12}{10,965} \times 1.30 \times \frac{5}{3} = 23.1 \]

In addition to the check for Maximum Design Load, the section used for maximum positive-moment should also be investigated for fatigue at the toes of the fillet welds of the transverse stiffeners. On the assumption that these fillet welds are terminated at a distance of \( 4t_w \) from the tension flange, the maximum bending stress at the toe of a weld is given by

\[ f_b = \frac{M \left[ y_L - (4t_w + t_f) \right]}{I} \]

where \( y_L \) = distance from neutral axis to outer surface of tension flange
\[ t_w \] = web thickness
\[ t_f \] = flange thickness

A transverse stiffener is welded to the girder web in the positive-moment region of the end span at 99.3 ft from the end bearing. Also, a lateral-bracing connection plate is welded to the girder web at this section. If the bottom of the connection plate is located 15 in. above the top of the bottom flange, the bending stress in the web at the connection plate is given by

\[ f_b = \frac{M \left[ y_L - (15 + t_f) \right]}{I} \]
SECTION 99.3 FT FROM END SUPPORT

The ranges of Service Load moments at 99.3 ft from the end bearing are

Truck LL range = 6,000 + 1,600 = 7,600 kip-ft
Lane LL range = 9,350 + 3,200 = 12,550 kip-ft

The range of tensile stress at the fillet weld of the transverse stiffener is then determined to be:

For truck loading,

$$f_{sr} = \frac{7,600 \times 12(102.16 - 4 \times 0.5 - 2.25)}{1,120,200} = 8.0 < 19.0 \text{ ksi}$$

For lane loading,

$$f_{sr} = \frac{12,550 \times 12(102.16 - 4 \times 0.5 - 2.25)}{1,120,200} = 13.2 < 32.0 \text{ ksi}$$

The range of tensile stress at the lateral-bracing connection plate is:

For truck loading:

$$f_{sr} = \frac{7,600 \times 12(102.16 - 15 - 2.25)}{1,120,200} = 6.9 < 12.5 \text{ ksi}$$

For lane loading,

$$f_{sr} = \frac{12,550 \times 12(102.16 - 15 - 2.25)}{1,120,200} = 11.4 < 21.0 \text{ ksi}$$

The trial section therefore is satisfactory for maximum positive-moment.

MAXIMUM-POSITIVE-MOMENT — CENTER SPAN

The maximum-positive-moment section in the center span is designed in the same way as the end span. The trial section is composed of a 1 x 22-in. top flange, a 1/2 x 156-in. web and a 2-1/8 x 22-in. bottom flange.

Immediately after placement of the concrete deck, the girder top flange in the positive-moment region is not supported against local buckling until the concrete hardens. For the maximum-positive-moment section, with a 1-in.-thick top flange, b/t = 11/1 = 11, exceeding the maximum permissible of 9.8 for $F_v = 50$ ksi. Because the moment capacity of the section exceeds the applied moment under $DL_1$
loading, however, the allowable $b'/t$ ratio can be increased by the factor $\sqrt{\frac{M}{M}} = \sqrt{\frac{F_g}{F_b}}$.

### Steel Section

<table>
<thead>
<tr>
<th></th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_o$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Flg. 1 x 22</td>
<td>22.00</td>
<td>78.50</td>
<td>1,727</td>
<td>135,600</td>
<td>135,600</td>
<td></td>
</tr>
<tr>
<td>Web 1/2 x 156</td>
<td>78.00</td>
<td>-79.06</td>
<td>-3,696</td>
<td>292,200</td>
<td>292,200</td>
<td></td>
</tr>
<tr>
<td>Bot. Flg. 2-1/8 x 22</td>
<td>46.75</td>
<td>92.42</td>
<td>6,066</td>
<td>560,600</td>
<td>560,600</td>
<td></td>
</tr>
</tbody>
</table>

\[
d_s = \frac{-1.969}{146.75} = -13.42 \text{ in.}
\]

\[
d_{Top \ of \ steel} = 79.00 + 13.42 = 92.42 \text{ in.}
\]

\[
s_{Top \ of \ steel} = \frac{560,600}{92.42} = 6,066 \text{ in.}^3
\]

\[
d_{Bot \ of \ steel} = 80.13 - 13.42 = 66.71 \text{ in.}
\]

\[
s_{Bot \ of \ steel} = \frac{560,600}{66.71} = 8,404 \text{ in.}^3
\]

### Composite Section, $n = 24$

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_o$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>146.75</td>
<td>-1,969</td>
<td>2,412</td>
<td>206,800</td>
<td>100</td>
<td>206,900</td>
</tr>
<tr>
<td>Conc. 90 x 7.5/24</td>
<td>28.13</td>
<td>85.75</td>
<td>443</td>
<td>793,900</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
d_{24} = \frac{443}{174.88} = 2.53 \text{ in.}
\]

\[
d_{Top \ of \ steel} = 79.00 - 2.53 = 76.47 \text{ in.}
\]

\[
s_{Top \ of \ steel} = \frac{792,800}{76.47} = 10,370 \text{ in.}^3
\]

\[
d_{Bot \ of \ steel} = 80.13 + 2.53 = 82.66 \text{ in.}
\]

\[
s_{Bot \ of \ steel} = \frac{792,800}{82.66} = 9,591 \text{ in.}^3
\]

### Composite Section, $n = 8$

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_o$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>146.75</td>
<td>-1,969</td>
<td>7,236</td>
<td>620,500</td>
<td>400</td>
<td>620,900</td>
</tr>
<tr>
<td>Conc. 90 x 7.5/8</td>
<td>84.38</td>
<td>85.75</td>
<td>5,267</td>
<td>1,207,900</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
d_s = \frac{5,267}{231.13} = 22.79 \text{ in.}
\]

\[
d_{Top \ of \ steel} = 79.00 - 22.79 = 56.21 \text{ in.}
\]

\[
s_{Top \ of \ steel} = \frac{1,087,900}{56.21} = 19,354 \text{ in.}^3
\]

\[
d_{Bot \ of \ steel} = 80.13 + 22.79 = 102.92 \text{ in.}
\]

\[
s_{Bot \ of \ steel} = \frac{1,087,900}{102.92} = 10,570 \text{ in.}^3
\]
Bending Moments at Midspan

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$L + I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M, \text{ kip-ft}$</td>
<td>9,968</td>
<td>2,517</td>
<td>9,799</td>
</tr>
</tbody>
</table>

Steel Stresses Due to Maximum Design Loads

Top of Steel (Compression)  
Bottom of Steel (Tension)

For $DL_1$:  
$F_b = \frac{9,968 \times 12}{6,066} \times 1.30 = 25.6$  
$F_b = \frac{9,968 \times 12}{8,404} \times 1.30 = 18.5$

For $DL_2$:  
$F_b = \frac{2,517 \times 12}{10,370} \times 1.30 = 3.8$  
$F_b = \frac{2,517 \times 12}{9,591} \times 1.30 = 4.1$

For $L + I$:  
$F_b = \frac{9,799 \times 12}{19,354} \times 1.30 \times \frac{5}{3} = 13.2$  
$F_b = \frac{9,799 \times 12}{10,570} \times 1.30 \times \frac{5}{3} = 24.1$

42.6 $< 50 \text{ ksi}$  
50 ksi $> 46.7$

A check is made of $b'/t$ for the top flange and the 1-in. thickness is found to be satisfactory.

$$\frac{b'}{t} = \frac{2,200}{\sqrt{50,000}} \sqrt{\frac{50,000}{25,600}} = 13.8 > 11$$

Next, fatigue is investigated at the fillet weld between a transverse stiffener and the web at midspan. A lateral-bracing connection plate is also located at midspan, and fatigue is checked at the plate-to-web fillet weld. In both cases, the computed stress range is less than the allowable.

The ranges of Service Load moment at midspan are:

- Truck LL range $= 6,057 + 1,189 = 7,246$ kip-ft
- Lane LL range $= 9,799 + 4,303 = 14,102$ kip-ft

The range of tensile stress at the transverse-stiffener fillet weld is then determined to be:

For truck loading,

$$f_{sr} = \frac{7,246 \times 12(102.92 - 4 \times 0.5 - 2.12)}{1,087,900} = 7.9 < 19.0 \text{ ksi}$$

For lane loading,

$$f_{sr} = \frac{14,102 \times 12(102.92 - 4 \times 0.5 - 2.12)}{1,087,900} = 15.4 < 32.0 \text{ ksi}$$

The range of tensile stress at the lateral-bracing connection plate is:

For truck loading,

$$f_{sr} = \frac{7,246 \times 12(102.92 - 15 - 2.12)}{1,087,900} = 6.9 < 12.5 \text{ ksi}$$

For lane loading,

$$f_{sr} = \frac{14,102 \times 12(102.92 - 15 - 2.12)}{1,087,900} = 13.4 < 21.0 \text{ ksi}$$
FLANGE-PLATE TRANSITION 63 FT FROM END SUPPORT

At 63 ft from the end bearing, the flange thickness used for the maximum-positive-moment section of the end span are reduced. The top flange changes from 1.1/16 to 1 in., and the bottom flange decreases from 2-1/4 to 1-11/16 in. Properties are calculated for the steel and composite sections.

For the 1-in. top flange, $b'/t = 11/1 = 11$, exceeding the maximum permissible of 9.8 for $F_y^* = 50$ ksi. But the allowable $b'/t$ may be increased by the factor $\sqrt{M_u/M} = \sqrt{F_y/F_y^*}$, where $F_y$ = top flange stress, psi, caused by DL, loading.

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_o$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Flg. 1 X 22</td>
<td>22.00</td>
<td>78.5</td>
<td>1,727</td>
<td>135,600</td>
<td>135,600</td>
<td></td>
</tr>
<tr>
<td>Web 1/2 X 156</td>
<td>78.00</td>
<td>-78.84</td>
<td>-2,927</td>
<td>230,800</td>
<td>230,800</td>
<td></td>
</tr>
<tr>
<td>Bot. Flg. 1-11/16 X 22</td>
<td>37.13</td>
<td>78.84</td>
<td>-2,927</td>
<td>230,800</td>
<td>230,800</td>
<td></td>
</tr>
</tbody>
</table>

$$d_s = \frac{-1.200}{137.13} = -8.75 \text{ in.}$$

$$d_{Top \text{ of steel}} = 79.0 + 8.75 = 87.75 \text{ in.}$$

$$d_{Bot. \text{ of steel}} = 79.69 - 8.75 = 70.94 \text{ in.}$$

$$S_{Top \text{ of steel}} = \frac{514,100}{87.75} = 5,859 \text{ in.}^2$$

$$S_{Bot. \text{ of steel}} = \frac{514,100}{70.94} = 7,247 \text{ in.}^2$$

Composite Section, $3n = 24$

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_o$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>137.13</td>
<td>85.75</td>
<td>2,412</td>
<td>206,800</td>
<td>100</td>
<td>206,900</td>
</tr>
</tbody>
</table>

$$d_{24} = \frac{1,212}{165.26} = 7.33 \text{ in.}$$

$$d_{Top \text{ of steel}} = 79.0 - 7.33 = 71.67 \text{ in.}$$

$$d_{Bot. \text{ of steel}} = 79.69 + 7.33 = 87.02$$

$$S_{Top \text{ of steel}} = \frac{722,600}{71.67} = 10,082 \text{ in.}^2$$

$$S_{Bot. \text{ of steel}} = \frac{722,600}{87.02} = 8,304 \text{ in.}^2$$

Composite Section, $n = 8$

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_o$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>137.13</td>
<td>85.75</td>
<td>7,236</td>
<td>620,500</td>
<td>400</td>
<td>620,900</td>
</tr>
</tbody>
</table>

$$d_8 = \frac{6,036}{221.51} = 27.25$$

$$d_{Top \text{ of steel}} = 79.0 - 27.25 = 51.75 \text{ in.}$$

$$d_{Bot. \text{ of steel}} = 79.69 + 27.25 = 106.94$$

$$S_{Top \text{ of steel}} = \frac{722,600}{51.75} = 13,808 \text{ in.}^2$$

$$S_{Bot. \text{ of steel}} = \frac{722,600}{106.94} = 6,746 \text{ in.}^2$$

$$d_{Top \text{ of steel}} = 84.38 - 13.75 = 70.63 \text{ in.}$$

$$d_{Bot. \text{ of steel}} = 84.38 + 13.75 = 98.13$$

$$S_{Top \text{ of steel}} = \frac{620,500}{70.63} = 8,765 \text{ in.}^2$$

$$S_{Bot. \text{ of steel}} = \frac{620,500}{98.13} = 6,294 \text{ in.}^2$$

$$I_{NA} = \frac{1,145,500}{981,400} \text{ in.}^4$$
$d_{\text{Top of steel}} = 79.0 - 27.25 = 51.75 \text{ in.}$  
$d_{\text{Bot. of steel}} = 79.69 + 27.25 = 106.94 \text{ in.}$

$S_{\text{Top of steel}} = \frac{981,000}{51.75} = 18,956 \text{ in.}^3$  
$S_{\text{Bot. of steel}} = \frac{981,000}{106.94} = 9,173 \text{ in.}^3$

As with previous sections investigated, the strength of this section is checked for Maximum Design Load, which is more critical than Overload. Also, fatigue in the web at the toe of the transverse-stiffener welds and fatigue in base metal adjacent to the butt-welded flange transition are checked. In addition, fatigue at lateral-bracing connection plates is checked at the cross-frame 49.6 ft from the end support.

### Bending Moments 63 Ft from End Support

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$+(L + I)$</th>
<th>$-(L + I)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M, \text{kip-ft}$</td>
<td>10,400</td>
<td>2,350</td>
<td>7,600</td>
<td>-2,070</td>
</tr>
</tbody>
</table>

### Steel Stresses Due to Maximum Design Loads

Top of Steel (Compression)  
Bottom of Steel (Tension)

For $DL_1$:  
$F_b = \frac{10,400 \times 12}{5,859} \times 1.30 = 27.7$  
$F_b = \frac{10,400 \times 12}{7,247} \times 1.30 = 22.4$

For $DL_2$:  
$F_b = \frac{2,350 \times 12}{10,082} \times 1.30 = 3.6$  
$F_b = \frac{2,350 \times 12}{8,304} \times 1.30 = 4.4$

For $L + I$:  
$F_b = \frac{7,600 \times 12}{18,956} \times 1.30 \times \frac{5}{3} = 10.4$  
$F_b = \frac{7,600 \times 12}{9,173} \times 1.30 \times \frac{5}{3} = 21.5$

$41.7 < 50 \text{ ksi}$  
$50 \text{ ksi} > 48.3$

A check is made of $b'/t$ for the top flange and the 1-in. thickness is found to be satisfactory.

$$\frac{b'}{t} = \frac{2,200}{\sqrt{50,000}} \sqrt{\frac{50,000}{27,700}} = 13.2 > 11$$

### Check of Fatigue at 63 Ft from End Support

The ranges of live-load moments at the flange transition are:

Truck LL range = 5,000 + 1,000 = 6,000 kip-ft
Lane LL range = 7,600 + 2,070 = 9,670 kip-ft

The range of tensile stress in the bottom flange at the transition is:

For truck loading,

$$f_{sr} = \frac{6,000 \times 12}{9,173} = 7.8 < 27.5 \text{ ksi}$$

For lane loading,

$$f_{sr} = \frac{9,670 \times 12}{9,173} = 12.7 < 45.0 \text{ ksi}$$

At the transverse-stiffener fillet weld adjacent to the flange transition, the range of tensile stress is found to be:
For truck loading,
\[
f_{sr} = \frac{6,000 \times 12(106.94 - 4 \times 0.5 - 1.69)}{981,000} = 7.6 < 19.0 \text{ ksi}
\]
For lane loading,
\[
f_{sr} = \frac{9,670 \times 12(106.94 - 4 \times 0.5 - 1.69)}{981,000} = 12.2 < 32.0 \text{ ksi}
\]

Check for Fatigue at 49.6 Ft from End Support

The ranges of live-load moments at the lateral-bracing connection 49.6 ft from the end support are:

- Truck LL range = 4,300 + 800 = 5,100 kip-ft
- Lane LL range = 6,450 + 1,650 = 8,100 kip-ft

At the lateral-bracing connection plate, the range of tensile stress is:

For truck loading,
\[
f_{sr} = \frac{5,100 \times 12(106.94 - 15 - 1.69)}{981,000} = 5.6 < 12.5 \text{ ksi}
\]
For lane loading,
\[
f_{sr} = \frac{8,100 \times 12(106.94 - 15 - 1.69)}{981,000} = 8.9 < 21.0 \text{ ksi}
\]

FLANGE-PLATE TRANSITION 35 FT FROM END SUPPORT

At 35 ft from the end bearing, the bottom-flange thickness is reduced from 1.11/16 to 7/8-in. Steel and composite section properties are calculated. It is not necessary to check b/ft for the top flange at the transition, because the 1-in. flange thickness was found to be satisfactory 63 ft from the end support, where DL stresses are larger.

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad^2</th>
<th>I_o</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Flg. 1 x 22</td>
<td>22.00</td>
<td>78.50</td>
<td>1,727</td>
<td>135,600</td>
<td>135,600</td>
<td></td>
</tr>
<tr>
<td>Web 1/2 x 156</td>
<td>78.00</td>
<td></td>
<td></td>
<td></td>
<td>158,200</td>
<td>158,200</td>
</tr>
<tr>
<td>Bot. Flg. 7/8 x 22</td>
<td>19.25</td>
<td>-78.44</td>
<td>-1,510</td>
<td>118,400</td>
<td>118,400</td>
<td></td>
</tr>
</tbody>
</table>

\[ds = \frac{217}{119.25} = 1.82 \text{ in.}\]

\[d_{Top \ of \ steel} = 79.0 - 1.82 = 77.18 \text{ in.}\]

\[d_{Bot. \ of \ steel} = 78.88 - 1.82 = 80.70 \text{ in.}\]

\[S_{Top \ of \ steel} = \frac{411,800}{77.18} = 5,336 \text{ in.}^3\]

\[S_{Bot. \ of \ steel} = \frac{411,800}{80.70} = 5,103 \text{ in.}^3\]
Composite Section, $3n = 24$

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_o$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>119.25</td>
<td>28.13</td>
<td>217</td>
<td>2,412</td>
<td>206,800</td>
<td>100</td>
</tr>
<tr>
<td>Conc. 90 x 7.5/24</td>
<td>85.75</td>
<td>2,629</td>
<td>147.38</td>
<td>2,629 in.$^3$</td>
<td>619,100</td>
<td></td>
</tr>
</tbody>
</table>

\[ d_{24} = \frac{2,629}{147.38} = 17.84 \text{ in.} \quad -17.84 \times 2,629 = -46,900 \]

\[ I_{NA} = 572,200 \text{ in.}^4 \]

\[ d_{\text{Top of steel}} = 79.0 - 17.84 = 61.16 \text{ in.} \quad d_{\text{Bot. of steel}} = 78.88 + 17.84 = 96.72 \text{ in.} \]

\[ S_{\text{Top of steel}} = \frac{572,200}{61.16} = 9,356 \text{ in.}^3 \quad S_{\text{Bot. of steel}} = \frac{572,200}{96.72} = 5,916 \text{ in.}^3 \]

Composite Section, $n = 8$

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_o$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>119.25</td>
<td>84.38</td>
<td>217</td>
<td>7,236</td>
<td>620,500</td>
<td>400</td>
</tr>
<tr>
<td>Conc. 90 x 7.5/8</td>
<td>85.75</td>
<td>7,453</td>
<td>203.63</td>
<td>7,453 in.$^3$</td>
<td>1,033,100</td>
<td></td>
</tr>
</tbody>
</table>

\[ d_{8} = \frac{7,453}{203.63} = 36.60 \text{ in.} \quad -36.60 \times 7,453 = -272,800 \]

\[ I_{NA} = 760,300 \text{ in.}^4 \]

\[ d_{\text{Top of steel}} = 79.0 - 36.60 = 42.40 \text{ in.} \quad d_{\text{Bot. of steel}} = 78.88 + 36.60 = 115.48 \text{ in.} \]

\[ S_{\text{Top of steel}} = \frac{760,300}{42.40} = 17,932 \text{ in.}^3 \quad S_{\text{Bot. of steel}} = \frac{760,300}{115.48} = 6,584 \text{ in.}^3 \]

<table>
<thead>
<tr>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>+(L + I)</th>
<th>-(L + I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$, kip-ft</td>
<td>6,950</td>
<td>1,550</td>
<td>4,850</td>
</tr>
</tbody>
</table>

Bending Moments 35 Ft from End Support

Steel Stresses Due to Maximum Design Loads

Top of Steel (Compression) \quad \text{Bottom of Steel (Tension)}

For $DL_1$: \[ F_b = \frac{6,950 \times 12}{5,336} \times 1.30 = 20.3 \quad F_b = \frac{6,950 \times 12}{5,103} \times 1.30 = 21.2 \]

For $DL_2$: \[ F_b = \frac{1,550 \times 12}{9,356} \times 1.30 = 2.6 \quad F_b = \frac{1,550 \times 12}{5,916} \times 1.30 = 4.1 \]

For $L + I$: \[ F_b = \frac{4,850 \times 12}{17,932} \times 1.30 \times \frac{5}{3} = 7.0 \quad F_b = \frac{4,850 \times 12}{6,584} \times 1.30 \times \frac{5}{3} = 19.2 \]

29.9 < 50 ksi \quad 50 ksi > 44.5

II/5.48 \quad 12/78
Fatigue is investigated at the toe of the fillet weld between a transverse stiffener and the web, near the flange transition. Fatigue is also checked in base metal adjacent to the butt-welded flange transition. Lateral-bracing connections to the portions of the girder with the 7/8-in. bottom flange occur only at the end bearings, where fatigue is not critical.

The ranges of live-load moment at the transition are:

- Truck LL range = 3,300 + 600 = 3,900 kip-ft
- Lane LL range = 4,850 + 1,150 = 6,000 kip-ft

At the toe of the transverse-stiffener fillet weld, the range of tensile stress is:

For truck loading,

\[ f_{sr} = \frac{3,900 \times 12(115.48 - 4 \times 0.5 - 0.88)}{760,300} = 6.9 < 19.0 \text{ ksi} \]

For lane loading,

\[ f_{sr} = \frac{6,000 \times 12(115.48 - 4 \times 0.5 - 0.88)}{760,300} = 10.7 < 32.0 \text{ ksi} \]

At the bottom-flange transition, the range of tensile stress is:

For truck loading,

\[ f_{sr} = \frac{3,900 \times 12}{6,584} = 7.1 < 27.5 \text{ ksi} \]

For lane loading,

\[ f_{sr} = \frac{6,000 \times 12}{6,584} = 10.9 < 45.0 \text{ ksi} \]

**FLANGE TRANSITION 133 FT FROM END SUPPORT**

At 133 ft from the end bearing, the flange thicknesses used for the maximum-positive-moment section of the end span are reduced. The top flange changes from 1-1/16 to 1 in., and the bottom flange decreases from 2-1/4 to 2-1/8 in. Fatigue under Service Load is investigated at the bottom-flange butt-welded transition, at the toe of a transverse-stiffener fillet weld and at the lateral-bracing connection plate at the cross-frame 148.9 ft from the end support. It is not necessary to check \( b/t \) for the top flange at the transition, because the 1-in. flange thickness was found to be satisfactory 63 ft. from the end support, where \( DL_1 \) stresses are larger.

<table>
<thead>
<tr>
<th>Material</th>
<th>( A )</th>
<th>( d )</th>
<th>( Ad )</th>
<th>( Ad^2 )</th>
<th>( I_o )</th>
<th>( I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Flg. 1 X 22</td>
<td>22.00</td>
<td>78.50</td>
<td>1,727</td>
<td>135,600</td>
<td>135,600</td>
<td></td>
</tr>
<tr>
<td>Web 1/2 X 156</td>
<td>78.00</td>
<td></td>
<td></td>
<td></td>
<td>158,200</td>
<td>158,200</td>
</tr>
<tr>
<td>Bot. Flg. 2-1/8 X 22</td>
<td>46.75</td>
<td>-79.06</td>
<td>-3,696</td>
<td>292,200</td>
<td>292,200</td>
<td></td>
</tr>
</tbody>
</table>

\[ d_s = \frac{-1.969}{146.75} = 13.42 \text{ in.} \]

\[ -13.42 \times 1.969 = -26.400 \]

\[ I_{NA} = 559,600 \text{ in.}^4 \]
\[ d_{Top\ of\ steel} = 79.0 + 13.42 = 92.42 \text{ in.} \quad d_{Bot.\ of\ steel} = 80.13 - 13.42 = 66.71 \text{ in.} \]

\[ S_{Top\ of\ steel} = \frac{559,600}{92.42} = 6,055 \text{ in.}^3 \quad S_{Bot.\ of\ steel} = \frac{559,600}{66.71} = 8,389 \text{ in.}^3 \]

### Composite Section, 3n = 24

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>146.75</td>
<td>28.13</td>
<td>1,969</td>
<td>2,412</td>
<td>586,000</td>
<td>792,900</td>
</tr>
<tr>
<td>Conc. 90 x 7.5/24</td>
<td>85.75</td>
<td>443</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ d_{24} = \frac{443}{174.88} = 2.53 \text{ in.} \quad I_{NA} = \frac{-2.53 \times 443}{-1,100} = 791,800 \text{ in.}^4 \]

\[ d_{Top\ of\ steel} = 79.0 - 2.53 = 76.47 \text{ in.} \quad d_{Bot.\ of\ steel} = 80.13 + 2.53 = 82.66 \text{ in.} \]

\[ S_{Top\ of\ steel} = \frac{791,800}{76.47} = 10,354 \text{ in.}^3 \quad S_{Bot.\ of\ steel} = \frac{791,800}{82.66} = 9,579 \text{ in.}^3 \]

### Composite Section, n = 8

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
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<th>Ad²</th>
<th>I₀</th>
<th>I</th>
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</thead>
<tbody>
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<td>Steel Section</td>
<td>146.75</td>
<td>84.38</td>
<td>-1,969</td>
<td>7,236</td>
<td>586,000</td>
<td>620,900</td>
</tr>
<tr>
<td>Conc. 90 x 7.5/8</td>
<td>85.75</td>
<td>5,267</td>
<td></td>
<td>620,500</td>
<td>400</td>
<td>1,206,900</td>
</tr>
</tbody>
</table>

\[ d_{8} = \frac{5,267}{231.13} = 22.79 \text{ in.} \quad I_{NA} = \frac{-22.79 \times 5,267}{-120,000} = 1,086,900 \text{ in.}^4 \]

\[ d_{Top\ of\ steel} = 79.0 - 22.79 = 56.21 \text{ in.} \quad d_{Bot.\ of\ steel} = 80.13 + 22.79 = 102.92 \text{ in.} \]

\[ S_{Top\ of\ steel} = \frac{1,086,900}{56.21} = 19,336 \text{ in.}^3 \quad S_{Bot.\ of\ steel} = \frac{1,086,900}{102.92} = 10,560 \text{ in.}^3 \]

### Bending Moments 133 Ft from End Support

<table>
<thead>
<tr>
<th></th>
<th>(DL_1)</th>
<th>(DL_2)</th>
<th>(+ (L + I))</th>
<th>(- (L + I))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M, \text{ kip-ft})</td>
<td>9,700</td>
<td>2,300</td>
<td>9,850</td>
<td>-4,360</td>
</tr>
</tbody>
</table>

### Steel Stresses Due to Maximum Design Loads

**Top of Steel (Compression)**

\[ F_b = \frac{9,700 \times 12}{6,055} \times 1.30 = 25.0 \]

**Bottom of Steel (Tension)**

\[ F_b = \frac{9,700 \times 12}{8,389} \times 1.30 = 18.0 \]

\[ F_b = \frac{2,300 \times 12}{10,354} \times 1.30 = 3.5 \]

\[ F_b = \frac{2,300 \times 12}{9,579} \times 1.30 = 3.7 \]

\[ F_b = \frac{9,850 \times 12}{19,336} \times 1.30 = 13.2 \]

\[ F_b = \frac{9,850 \times 12}{10,560} \times 1.30 = 24.3 \]

41.7 < 50 ksi

50 ksi > 46.0
The range of Service Load moments at the transition are:

Truck LL range = 6,150 + 2,150 = 8,300 kip-ft
Lane LL range = 9,850 + 4,360 = 14,210 kip-ft

At the bottom-flange transition butt-weld, the range of tensile stress is:

For truck loading,

\[ f_{sr} = \frac{8,300 \times 12}{10,560} = 9.4 < 27.5 \text{ ksi} \]

For lane loading,

\[ f_{sr} = \frac{14,210 \times 12}{10,560} = 16.1 < 45.0 \text{ ksi} \]

By observation, it can be determined that the stress range at the toe of the nearby transverse-stiffener fillet weld are less than the allowable ranges for both truck and lane loadings.

The ranges of Service Load moments at the lateral-bracing connection 148.9 ft from the end support are:

Truck LL range = 5,700 + 2,250 = 7,950 kip-ft
Lane LL range = 9,600 + 4,900 = 14,500 kip-ft

At the lateral-bracing connection plate, the range of tensile stress is:

For truck loading,

\[ f_{sr} = \frac{7,950 \times 12(102.92 - 15 - 2.12)}{1,086,900} = 7.5 < 12.5 \text{ ksi} \]

For lane loading,

\[ f_{sr} = \frac{14,500 \times 12(102.92 - 15 - 2.12)}{1,086,900} = 13.7 < 21.0 \text{ ksi} \]

FLANGE TRANSITION 158 FT FROM END SUPPORT

At 158 ft from the end bearing, the bottom flange is reduced in thickness from 2-1/8 to 1-3/8 in. The section is found to be satisfactory for bending under Maximum Design Load and for resistance to fatigue under Service Load.

It is not necessary to check \( b'/t \) for the top flange, because the 1-in. thickness was found to be satisfactory at sections subjected to larger stresses.

A lateral-bracing connection plate is not located near this flange transition. Therefore, a fatigue check at a connection weld need not be made.

### Steel Section

<table>
<thead>
<tr>
<th>Material</th>
<th>( A )</th>
<th>( d )</th>
<th>( Ad )</th>
<th>( Ad^2 )</th>
<th>( I_o )</th>
<th>( I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Flg. 1 x 22</td>
<td>22.00</td>
<td>78.50</td>
<td>1,727</td>
<td>135,600</td>
<td>135,600</td>
<td>135,600</td>
</tr>
<tr>
<td>Web 1/2 x 156</td>
<td>78.00</td>
<td>30.25</td>
<td>-78.69</td>
<td>-2,380</td>
<td>187,300</td>
<td>187,300</td>
</tr>
</tbody>
</table>

\[ d_s = \frac{-653}{130.25} = -5.01 \text{ in.} \]

\[ -5.01 \times 653 = \frac{-3,300}{477,800 \text{ in.}^4} \]

\[ I_{NA} \]
\[ d_{\text{Top of steel}} = 79.0 + 5.01 = 84.01 \text{ in.} \]
\[ d_{\text{Bot of steel}} = 79.38 - 5.01 = 74.37 \text{ in.} \]

\[ S_{\text{Top of steel}} = \frac{477,800}{84.01} = 5,687 \text{ in}^3 \]
\[ S_{\text{Bot of steel}} = \frac{477,800}{74.37} = 6,425 \text{ in}^3 \]

**Composite Section, 3n = 24**

<table>
<thead>
<tr>
<th>Material</th>
<th>( A )</th>
<th>( d )</th>
<th>( Ad )</th>
<th>( Ad^2 )</th>
<th>( I_o )</th>
<th>( I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>130.25</td>
<td>85.75</td>
<td>2,412</td>
<td>206,800</td>
<td>100</td>
<td>206,900</td>
</tr>
<tr>
<td>Conc. 90 x 7.5/24</td>
<td>28.13</td>
<td>85.75</td>
<td>-653</td>
<td>1,759</td>
<td>158.38</td>
<td>688,000</td>
</tr>
</tbody>
</table>

\[ d_{24} = \frac{1,759}{158.38} = 11.11 \text{ in.} \]
\[ I_{NA} = 668,500 \text{ in}^4 \]

**Composite Section, n = 8**

<table>
<thead>
<tr>
<th>Material</th>
<th>( A )</th>
<th>( d )</th>
<th>( Ad )</th>
<th>( Ad^2 )</th>
<th>( I_o )</th>
<th>( I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>130.25</td>
<td>85.75</td>
<td>2,412</td>
<td>206,800</td>
<td>100</td>
<td>206,900</td>
</tr>
<tr>
<td>Conc. 90 x 7.5/8</td>
<td>84.38</td>
<td>85.75</td>
<td>7,236</td>
<td>620,500</td>
<td>400</td>
<td>620,900</td>
</tr>
</tbody>
</table>

\[ d_{8} = \frac{6,583}{214.63} = 30.67 \text{ in.} \]
\[ I_{NA} = 900,100 \text{ in}^4 \]

**Bending Moments 158 Ft from End Support**

<table>
<thead>
<tr>
<th></th>
<th>( DL_1 )</th>
<th>( DL_2 )</th>
<th>( + (L + I) )</th>
<th>( -(L + I) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M, \text{kip-ft} )</td>
<td>6,250</td>
<td>1,550</td>
<td>9,050</td>
<td>-5,150</td>
</tr>
</tbody>
</table>

**Steel Stresses Due to Maximum Design Loads**

For \( DL_1 \):
\[ F_b = \frac{6,250 \times 12}{5,687} \times 1.30 = 17.1 \]
\[ F_b = \frac{6,250 \times 12}{6,425} \times 1.30 = 15.2 \]

For \( DL_2 \):
\[ F_b = \frac{1,550 \times 12}{9,487} \times 1.30 = 2.5 \]
\[ F_b = \frac{1,550 \times 12}{7,388} \times 1.30 = 3.3 \]

For \( L + I \):
\[ F_b = \frac{9,050 \times 12}{18,624} \times 1.30 \times \frac{5}{3} = 12.6 \]
\[ F_b = \frac{9,050 \times 12}{8,179} \times 1.30 \times \frac{5}{3} = 28.8 \]

\[ 32.2 < 50 \text{ ksi} \quad 50 \text{ ksi} > 47.3 \]
Live-load moment ranges at the transition 158 ft from the end support are:

Truck LL range = 5,500 + 2,500 = 8,000 kip-ft
Lane LL range = 9,050 + 5,150 = 14,200 kip-ft

At the bottom-flange transition butt-weld, the range of tensile stress is:

For truck loading,
\[ f_{sr} = \frac{8,000 \times 12}{8,179} = 11.7 < 27.5 \text{ ksi} \]

For lane loading,
\[ f_{sr} = \frac{14,200 \times 12}{8,179} = 20.8 < 45.0 \text{ ksi} \]

At the toe of a nearby transverse-stiffener fillet weld, the range of tensile stress is:

For truck loading,
\[ f_{sr} = \frac{8,000 \times 12(110.05 - 4 \times 0.5 - 1.38)}{900,100} = 11.4 < 19.0 \text{ ksi} \]

For lane loading,
\[ f_{sr} = \frac{14,200 \times 12(110.05 - 4 \times 0.5 - 1.38)}{900,100} = 20.2 < 32.0 \text{ ksi} \]

**FLANGE TRANSITION 244 FT FROM END SUPPORT**

At 244 ft from the end bearing, the flange thicknesses and widths of the maximum-negative-moment section are reduced. The top flange changes from 3 × 36 to 1-15/16 × 30 in., and the bottom flange decreases from 3-1/4 × 36 to 2-1/4 × 36 in. Properties are calculated for the steel section alone and for the steel section plus the longitudinal reinforcing in the slab.

### Steel Section

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>Io</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Flg. 1-15/16 × 30</td>
<td>58.13</td>
<td>78.97</td>
<td>4,591</td>
<td>362,500</td>
<td>362,500</td>
<td></td>
</tr>
<tr>
<td>Web 9/16 × 156</td>
<td>87.75</td>
<td>-79.13</td>
<td>-5,341</td>
<td>178,000</td>
<td>178,000</td>
<td></td>
</tr>
<tr>
<td>Bot. Flg. 2-1/4 × 30</td>
<td>67.50</td>
<td>79.13</td>
<td>422,700</td>
<td>422,700</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ d_s = \frac{-750}{213.38} = -3.51 \quad -3.51 \times 750 = -2,600 \quad I_{NA} = 960,600 \text{ in.}^4 \]

\[ d_{Top \ of \ steel} = 79.94 + 3.51 = 83.45 \text{ in.} \quad d_{Bot. \ of \ steel} = 80.25 - 3.51 = 76.74 \text{ in.} \]

\[ S_{Top \ of \ steel} = \frac{960,600}{83.45} = 11,511 \text{ in.}^3 \quad S_{Bot. \ of \ steel} = \frac{960,600}{76.74} = 12,518 \text{ in.}^3 \]
Steel Section with Reinforcing Steel

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_o$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>213.38</td>
<td>86.02</td>
<td>-750</td>
<td>50,500</td>
<td>963,200</td>
<td></td>
</tr>
<tr>
<td>Reinf. 22 No. 5</td>
<td>6.82</td>
<td>587</td>
<td>50,500</td>
<td>50,500</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$d_s = \frac{-163}{220.20} = -0.74 \text{ in.}$

$I_{NA} = 1,013,600 \text{ in.}^4$

$d_{Top \ of \ steel} = 79.94 + 0.74 = 80.68 \text{ in.}$  
$d_{Bot. \ of \ steel} = 80.25 - 0.74 = 79.51 \text{ in.}$

$S_{Top \ of \ steel} = \frac{1,013,600}{80.68} = 12,563 \text{ in.}^3$

$S_{Bot. \ of \ steel} = \frac{1,013,600}{79.51} = 12,748 \text{ in.}^3$

$d_{Reinf.} = 80.68 - 1.94 + 8.02 = 86.76 \text{ in.}$

$S_{Reinf.} = \frac{1,013,600}{86.76} = 11,683 \text{ in.}^3$

For determination of the critical allowable compression stress, the assumption is made that the compression flange has a constant width of 30 in. between the cross-frames on opposite sides of the flange transition. The allowable stress then becomes

$$F_{cr} = F_y \left[ 1 - \frac{3F_y}{4\pi^2 E} \left( \frac{L_b}{0.9b} \right)^2 \right] = 50 \left[ 1 - \frac{3 \times 50}{4\pi^2 \times 29,000} \left( \frac{24.82}{0.9 \times 15} \right)^2 \right] = 46.8 \text{ ksi}$$

If the ratio of the moments at the ends of the unbraced flange length is less than 0.7, however, $F_{cr}$ may be increased 20%, but not to more than $F_y$. The bending moments at the cross-frames are given in a table.

### Bending Moments, Kip-Ft, at Cross-Frames

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$L + I$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>At 223.36 Ft from End Support</td>
<td>-10,000</td>
<td>-2000</td>
<td>-7,600</td>
<td>-19,600</td>
</tr>
<tr>
<td>At 248.18 Ft from End Support</td>
<td>-20,640</td>
<td>-4,220</td>
<td>-10,440</td>
<td>-35,300</td>
</tr>
</tbody>
</table>

The ratio of the total moments then is:

$$R = \frac{19,600}{35,300} = 0.56 < 0.7$$

Therefore, $F_{cr}$ may be increased. But because $1.2 \times 46.8 = 56.2 > 50 \text{ ksi}$, use $F_{cr} = 50 \text{ ksi}$.

### Bending Moments 244 Ft from End Support

<table>
<thead>
<tr>
<th>$M$, kip-ft</th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$+(L + I)$</th>
<th>$-(L + I)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>-17,800</td>
<td>-3,700</td>
<td>2,600</td>
<td>-9,900</td>
</tr>
</tbody>
</table>

II/5.54 12/78
Steel Stresses Due to Maximum Design Loads

Top of Steel (Tension) 
For \( DL_1: F_b = \frac{17,800 \times 12}{11,511} \times 1.30 = 24.1 \)
\[
F_b = \frac{17,800 \times 12}{12,518} \times 1.30 = 22.2
\]

For \( DL_2: F_b = \frac{3,700 \times 12}{12,563} \times 1.30 = 4.6 \)
\[
F_b = \frac{3,700 \times 12}{12,748} \times 1.30 = 4.5
\]

For \( L + I: F_b = \frac{9,900 \times 12}{12,563} \times 1.30 \times \frac{5}{3} = 20.5 \)
\[
F_b = \frac{9,900 \times 12}{12,748} \times 1.30 \times \frac{5}{3} = 20.2
\]

\[
49.2 < 50 \text{ ksi} \quad 50 \text{ ksi} > 46.9
\]

Bottom of Steel (Compression)

Reinforcing Steel Stress Tension
\[
F_b = \frac{1.30 \times 12 \left[3,700 + (5/3) \times 9,900\right]}{11,683} = 27.0 < 40 \text{ ksi}
\]

Fatigue stress is then checked in the top flange, where shear connectors are likely to be attached.

Truck LL range = 1,400 + 3,700 = 5,100 kip-ft
Lane LL range = 2,600 + 9,900 = 12,500 kip-ft

The range of live-load tensile stress in the top flange is:

For truck loading,
\[
f_{sr} = \frac{5,100 \times 12}{12,563} = 4.9 < 19.0 \text{ ksi}
\]

For lane loading,
\[
f_{sr} = \frac{12,500 \times 12}{12,563} = 11.9 < 32.0 \text{ ksi}
\]

The fatigue stress range in the reinforcing steel is:
\[
f_{sr} = \frac{12,500 \times 12}{11,683} = 12.8 < 20 \text{ ksi}
\]

FLANGE TRANSITION 219 FT FROM END SUPPORT

At 219 ft from the end bearing, the thickness of the top (tension) flange decreases from 1-15/16 to 1 in., and the bottom (compression) flange reduces from 2-1/4 to 1-3/8 in. The 9/16-in. thickness used for the web throughout most of the negative-moment region is reduced to 1/2-in. at a distance of 227 ft from the end support. By inspection of the shear and moment diagrams, it is determined that the transition in web thickness at this location is satisfactory.

Properties of the reduced section are calculated for the steel section alone and for the steel section plus the longitudinal reinforcing in the slab.
Steel Section

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Flg. 1 x 30</td>
<td>30.00</td>
<td>78.50</td>
<td>2,355</td>
<td>184,900</td>
<td>184,900</td>
<td></td>
</tr>
<tr>
<td>Web 1/2 x 156</td>
<td>78.00</td>
<td>-78.69</td>
<td>-3,246</td>
<td>255,400</td>
<td>255,400</td>
<td></td>
</tr>
<tr>
<td>Bot. Flg. 1-3/8 x 30</td>
<td>41.25</td>
<td>-78.69</td>
<td>-3,246</td>
<td>255,400</td>
<td>255,400</td>
<td></td>
</tr>
</tbody>
</table>

\[
d_s = \frac{-891}{149.25} = -5.97 \text{ in.} \quad \text{and} \quad -5.97 \times 891 = -5,300 \quad I_{NA} = 593,200 \text{ in.}^4
\]

\[
d_{Top \ of \ steel} = 79.0 + 5.97 = 84.97 \text{ in.} \quad d_{Bot. \ of \ steel} = 79.38 - 5.97 = 73.41 \text{ in.}
\]

\[
S_{Top \ of \ steel} = \frac{593,200}{84.97} = 6,981 \text{ in.}^3 \quad S_{Bot. \ of \ steel} = \frac{593,200}{73.41} = 8,081 \text{ in.}^3
\]

Steel Section with Reinforcing Steel

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>149.25</td>
<td>-891</td>
<td>-891</td>
<td>50,500</td>
<td>50,500</td>
<td></td>
</tr>
<tr>
<td>Reinf. 22 No. 5</td>
<td>6.82</td>
<td>86.02</td>
<td>587</td>
<td>50,500</td>
<td>50,500</td>
<td></td>
</tr>
</tbody>
</table>

\[
d_s = \frac{-304}{156.07} = -1.95 \quad -1.95 \times 304 = -600 \quad I_{NA} = 648,400 \text{ in.}^4
\]

\[
d_{Top \ of \ steel} = 79.0 + 1.95 = 80.95 \text{ in.} \quad d_{Bot. \ of \ steel} = 79.38 - 1.95 = 77.43 \text{ in.}
\]

\[
S_{Top \ of \ steel} = \frac{648,400}{80.95} = 8,010 \text{ in.}^3 \quad S_{Bot. \ of \ steel} = \frac{648,400}{77.43} = 8,374 \text{ in.}^3
\]

\[
d_{Reinf.} = 80.95 - 1.0 + 8.02 = 87.97 \text{ in.}
\]

\[
S_{Reinf.} = \frac{648,400}{87.97} = 7,371 \text{ in.}^3
\]

The critical allowable compression stress for the 30-in. wide compression flange was previously found to be \(F_{cr} = 46.8 \text{ ksi}\). The ratio of moments at the ends of the unbraced length is computed and found to be less than 0.7.

Bending Moments, Kip-Ft, at Cross-Frames

<table>
<thead>
<tr>
<th></th>
<th>(DL_1)</th>
<th>(DL_2)</th>
<th>(L + I)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>At 198.54 Ft from End Support</td>
<td>-2,200</td>
<td>-400</td>
<td>-6,500</td>
<td>-9,100</td>
</tr>
<tr>
<td>At 223.36 Ft from End Support</td>
<td>-10,000</td>
<td>-2,000</td>
<td>-7,600</td>
<td>-19,600</td>
</tr>
</tbody>
</table>

\[
R = \frac{9,100}{19,600} = 0.464 < 0.7
\]

Consequently, as for the transition 244 ft from the end support, \(F_{cr}\) can be increased to 50 ksi.
Bending Moments 219 Ft from End Support

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$+(L + I)$</th>
<th>$-(L + I)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$, kip-ft</td>
<td>-8,400</td>
<td>-1,650</td>
<td>4,270</td>
<td>-7,250</td>
</tr>
</tbody>
</table>

Steel Stresses Due to Maximum Design Loads

<table>
<thead>
<tr>
<th>For</th>
<th>$DL_1$: $F_b$ =</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$DL_1$:</td>
<td>$F_b$ = ( \frac{8,400 \times 12}{6,981} \times 1.30 = 18.8 )</td>
<td>$F_b$ = ( \frac{8,400 \times 12}{8,081} \times 1.30 = 18.2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$DL_2$:</td>
<td>$F_b$ = ( \frac{1,650 \times 12}{8,010} \times 1.30 = 3.2 )</td>
<td>$F_b$ = ( \frac{1,650 \times 12}{8,374} \times 1.30 = 3.1 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L + I$:</td>
<td>$F_b$ = ( \frac{7,250 \times 12}{8,010} \times 1.30 \times \frac{5}{3} = 23.5 )</td>
<td>$F_b$ = ( \frac{7,250 \times 12}{8,374} \times 1.30 \times \frac{5}{3} = 22.5 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

45.5 < 50 ksi
50 ksi > 41.8

Reinforcing Steel Stress Tension

$$F_b = \frac{1.30 \times 12[1,650 + (5/3)7,250]}{7,371} = 29.1 < 40 \text{ ksi}$$

The 1.3/8 x 30-in. bottom flange has adequate resistance to lateral buckling. But the 1.3/8 in. thickness should be checked for local buckling. The permissible flange width-thickness ratio $b'/t$ for $F_y = 50$ ksi is 9.8, whereas, for this flange, $b'/t = 15/1.38 = 10.9 < 9.8$. But the allowable ratio may be increased by the factor $\sqrt{M_u/M} = \sqrt{F_y/F_b}$.

$$\frac{b'}{t} = \frac{2,200}{\sqrt{\frac{50,000}{41,800}}} = 10.8 \approx 10.9$$

The top flange is then checked for fatigue at the shear connectors.

Truck LL range = 2,800 + 3,350 = 6,150 kip-ft
Lane LL range = 4,270 + 7,250 = 11,520 kip-ft

The range of live-load tensile stress in the top flange is:

For truck loading,

$$f_{sr} = \frac{6,150 \times 12}{8,010} = 9.2 < 19.0 \text{ ksi}$$

For lane loading,

$$f_{sr} = \frac{11,520 \times 12}{8,010} = 17.3 < 32.0 \text{ ksi}$$

The fatigue stress range in the reinforcing steel is:

$$f_{sr} = \frac{11,520 \times 12}{7,371} = 18.8 < 20.0 \text{ ksi}$$
BOLTED FIELD SPICE — END SPAN

A bolted field splice is located at 192 ft from the end support. At this point, the girder top flange consists of a 1 x 22-in. plate on the positive-moment side of the splice and a 1 x 30-in. plate on the negative-moment side of the splice. The girder bottom flange changes from a 1-3/8 x 22-in. plate on the positive-moment side of the splice to a 1-3/8 x 30-in. plate on the negative-moment side of the splice.

Design calculations for splice plates and an analysis of the net section are not presented here. Calculations for a similar splice are given in Chapters 4 and 4A.

At a section near the dead-load point of contraflexure, flange bending stresses are small. Fatigue stresses, however, may be significant. For the gross section on the positive-moment side of the field splice (net section is considered in the splice-plate design), fatigue should be investigated at the following locations:

1. In the top flange at stud shear connectors. (Category C)
2. In the web at interruptions or terminations of longitudinal web stiffeners. (Category E)
3. In the web at the top and bottom ends of the transverse-stiffener fillet weld to the web. (Category C)
4. In the slab reinforcing steel. (20 ksi allowable)

Gross section properties for the positive-moment side of the field splice were previously computed in the investigation of the section at the flange transition 158 ft from the end support. Properties for the steel section plus slab reinforcing steel are calculated in the following:

### Steel Section with Reinforcing Steel

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I_o</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>130.25</td>
<td>-653</td>
<td>587</td>
<td>50,500</td>
<td>481,100</td>
<td></td>
</tr>
<tr>
<td>Reinf. 22 No. 5</td>
<td>6.82</td>
<td>86.02</td>
<td>50,500</td>
<td>50,500</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
d_s = \frac{-66}{137.07} = 0.48 \text{ in.}
\]

\[
-0.48 \times 66 = 0
\]

\[
I_{NA} = 531,600 \text{ in.}^4
\]

\[
d_{Top \ of \ steel} = 79.0 + 0.48 = 79.48 \text{ in.}
\]

\[
d_{Bot. \ of \ steel} = 79.38 - 0.48 = 78.90 \text{ in.}
\]

\[
S_{Top \ of \ steel} = \frac{531,600}{79.48} = 6,688 \text{ in.}^3
\]

\[
S_{Bot. \ of \ steel} = \frac{531,600}{78.90} = 6,738 \text{ in.}^3
\]

\[
d_{Reinf.} = 79.48 - 1.0 + 8.02 = 86.50 \text{ in.}
\]

\[
d_{Reinf.} = \frac{531,600}{86.50} = 6,146 \text{ in.}^3
\]

As a simplification, the bending moments at the centerline of the field splice are used for fatigue checks.

### Bending Moments 192 Ft from End Support

<table>
<thead>
<tr>
<th>DL₁</th>
<th>DL₂</th>
<th>+(L+I) Truck</th>
<th>-(L+I) Truck</th>
<th>+(L+I) Lane</th>
<th>-(L+I) Lane</th>
</tr>
</thead>
<tbody>
<tr>
<td>M, kip-ft</td>
<td>-650</td>
<td>20</td>
<td>4,200</td>
<td>-3,000</td>
<td>6,800</td>
</tr>
</tbody>
</table>

II/5.58 12/78
At the top-flange shear connectors, the live-load stress ranges are:

For truck loading,

\[ f_{sr} = \frac{4,200 \times 12}{18,624} + \frac{3,000 \times 12}{6,688} = 8.1 < 19 \text{ ksi} \]

For lane loading,

\[ f_{sr} = \frac{6,800 \times 12}{18,624} + \frac{6,300 \times 12}{6,688} = 15.7 < 32 \text{ ksi} \]

At the interruption of the bottom longitudinal stiffener at the web splice, the live-load stress ranges are:

For truck loading,

\[ f_{sr} = \frac{4,200 \times 12 (110.05 - 1.38 - 156/5)}{900,100} + \frac{3,000 \times 12 (78.90 - 1.38 - 156/5)}{531.70} = 7.4 < 12.5 \text{ ksi} \]

For lane loading,

\[ f_{sr} = \frac{6,800 \times 12 (110.05 - 1.38 - 156/5)}{900,100} + \frac{6,300 \times 12 (78.90 - 1.38 - 156/5)}{531,600} = 13.6 < 13.6 \text{ ksi} \]

At the bottom end of the fillet weld of a transverse stiffener to the web, the live-load stress ranges are:

For truck loading

\[ f_{sr} = \frac{4,200 \times 12 (110.05 - 4 \times 0.5 - 1.38)}{900,100} + \frac{3,000 \times 12 (78.90 - 4 \times 0.5 - 1.38)}{531,600} = 11.1 < 19 \text{ ksi} \]

For lane loading,

\[ f_{sr} = \frac{6,800 \times 12 (110.05 - 4 \times 0.5 - 1.38)}{900,100} + \frac{6,300 \times 12 (78.90 - 4 \times 0.5 - 1.38)}{531,600} = 20.4 < 32 \text{ ksi} \]

Fatigue stress range in the reinforcing steel is:

\[ f_{sr} = \frac{6,300 \times 12}{6,146} = 12.3 < 20 \text{ ksi} \]

(Neglect the compression in positive bending.)

The longitudinal stiffeners are actually carried well beyond the field splices to points at which compressive stresses on the "tension" side of the web are near zero. In the positive bending region the bottom longitudinal stiffener is terminated 33 ft from the splice. Bending moments are tabulated and live load stress ranges computed.

<table>
<thead>
<tr>
<th></th>
<th>DL1</th>
<th>DL2</th>
<th>+{(L+I)\text{Truck}}</th>
<th>-(L+I)\text{Truck}</th>
<th>+(L+I)\text{Lane}</th>
<th>-(L+I)\text{Lane}</th>
</tr>
</thead>
<tbody>
<tr>
<td>M, kip-ft</td>
<td>6,300</td>
<td>1,600</td>
<td>5,500</td>
<td>-2,500</td>
<td>9,000</td>
<td>-5,300</td>
</tr>
</tbody>
</table>
For truck loading,
\[
f_{tr} = \frac{5,500 \times 12}{900,100} \left( 110.05 - 1.38 - 156/5 \right) + \frac{2,500 \times 12}{531,600} \left( 78.90 - 1.38 - 156/5 \right) - 8.3 < 12.5 \text{ ksi}
\]

For lane loading,
\[
f_{lr} = \frac{9,000 \times 12}{900,100} \left( 110.05 - 1.38 - 156/5 \right) + \frac{5,300 \times 12}{531,600} \left( 78.90 - 1.38 - 156/5 \right) = 14.8 < 21.0 \text{ ksi}
\]

Other longitudinal stiffener terminations are checked for fatigue in the same manner.

On the interior girder longitudinal stiffeners are interrupted at each cross-frame connection plate, and must accordingly be checked for fatigue at the interruptions. Transverse web stiffeners are placed on the opposite web face. At exterior girders the longitudinal stiffeners are placed on the outside web face, with transverse stiffeners and cross-frame connection plates on the inside face. Thus no interruptions of longitudinal stiffeners occur at cross-frames on the exterior girders.

Longitudinal stiffeners with radiused terminations, as shown on the plans at the end of the example, constitute a Category C detail and may be used if the square-terminated detail does not satisfy fatigue requirements. The square-cut detail is more economical where it can be used.

FLANGE TRANSITION 136 FT FROM SUPPORT — CENTER SPAN

In the center span, at 136 ft from a support, thicknesses of flanges of the maximum-positive-moment section are changed. The top flange decreases from 1 to 15/16 in., and the bottom flange reduces from 2-1/8 to 1-1/2 in. Properties are calculated for the steel and composite sections.

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I_o</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Flg. 15/16 x 22</td>
<td>20.63</td>
<td>78.47</td>
<td>1,619</td>
<td>127,000</td>
<td>127,000</td>
<td></td>
</tr>
<tr>
<td>Web 1/2 x 156</td>
<td>78.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bot. Flg. 1-1/2 x 22</td>
<td>33.00</td>
<td>-78.75</td>
<td>-2,599</td>
<td>204,700</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

131.63 in.², -980 in.³

\[ d_s = \frac{-980}{131.63} = -7.45 \text{ in.} \]

\[ -7.45 \times 980 = -7,300 \]

\[ I_{NA} = 482,600 \text{ in.}^4 \]

\[ d_{Top \ of \ steel} = 78.94 + 7.45 = 86.39 \text{ in.} \]

\[ d_{Bot. \ of \ steel} = 79.50 - 7.45 = 72.05 \text{ in.} \]

\[ S_{Top \ of \ steel} = \frac{482,600}{86.39} = 5,586 \text{ in.}^3 \]

\[ S_{Bot. \ of \ steel} = \frac{482,600}{72.05} = 6,698 \text{ in.}^3 \]

Composite Section, 3n = 24

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I_o</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>131.63</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conc. 90 x 7.5/24</td>
<td>28.13</td>
<td>85.75</td>
<td>2,412</td>
<td>206,800</td>
<td>100</td>
<td>206,900</td>
</tr>
</tbody>
</table>

159.76 in.², 1,432 in.³

\[ d_{24} = \frac{1,432}{159.76} = 8.96 \text{ in.} \]

\[ -8.96 \times 1,432 = -12,800 \]

\[ I_{NA} = 684,000 \text{ in.}^4 \]

II/5.60
\[ d_{\text{Top of steel}} = 79.94 - 8.96 = 69.98 \text{ in.} \quad d_{\text{Bot of steel}} = 79.50 + 8.96 = 88.46 \text{ in.} \]

\[ S_{\text{Top of steel}} = \frac{684,000}{69.98} = 9,774 \text{ in.}^3 \quad S_{\text{Bot of steel}} = \frac{684,000}{88.46} = 7.732 \text{ in.}^3 \]

**Composite Section, \( n = 8 \)**

<table>
<thead>
<tr>
<th>Material</th>
<th>( A )</th>
<th>( d )</th>
<th>( Ad )</th>
<th>( Ad^2 )</th>
<th>( I_o )</th>
<th>( I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>131.63</td>
<td>84.38</td>
<td>85.75</td>
<td>-980</td>
<td>620,500</td>
<td>400</td>
</tr>
<tr>
<td>Conc. 90 x 7.5/8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ d_s = \frac{6.256}{216.01} = 28.96 \text{ in.} \]

\[ -28.96 \times 6.256 = -181,200 \]

\[ I_{NA} = 929,600 \text{ in.}^4 \]

\[ d_{\text{Top of steel}} = 78.94 - 28.96 = 49.98 \text{ in.} \quad d_{\text{Bot of steel}} = 79.50 + 28.96 = 108.46 \text{ in.} \]

\[ S_{\text{Top of steel}} = \frac{929,600}{49.98} = 18,600 \text{ in.}^3 \quad S_{\text{Bot of steel}} = \frac{929,600}{108.46} = 8,571 \text{ in.}^3 \]

**Bending Moments in Center Span at 136 Ft from Support**

<table>
<thead>
<tr>
<th></th>
<th>( DL_1 )</th>
<th>( DL_2 )</th>
<th>( + (L + I) )</th>
<th>( -(L + I) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M, \text{ kip-ft} )</td>
<td>7,920</td>
<td>1,990</td>
<td>8,780</td>
<td>-4,300</td>
</tr>
</tbody>
</table>

**Steel Stresses Due to Maximum Design Loads**

\[ F_b = \frac{7,920 \times 12}{5,586} \times 1.30 = 22.1 \quad F_b = \frac{7,920 \times 12}{6,698} \times 1.30 = 18.4 \]

\[ F_b = \frac{1,990 \times 12}{9,774} \times 1.30 = 3.2 \quad F_b = \frac{1,990 \times 12}{7,732} \times 1.30 = 4.0 \]

\[ F_b = \frac{8,780 \times 12}{18,500} \times 1.30 \times \frac{5}{3} = 12.3 \quad F_b = \frac{8,780 \times 12}{8,571} \times 1.30 \times \frac{5}{3} = 26.6 \]

\[ 37.6 < 50 \text{ ksi} \quad 50 \text{ ksi} > 49.0 \]

At this transition, fatigue is investigated in the web at the toe of the transverse-stiffener fillet welds and in base metal adjacent to the butt-welded flange transition. Fatigue also is checked at the lateral-bracing connection plates 125 ft from the interior support.

The ranges of live-load moment at the transition are:

- Truck LL range = 5,500 + 1,750 = 7,250 kip-ft
- Lane LL range = 8,780 + 4,300 = 13,080 kip-ft

The range in tensile stress in the web at the bottom of the transverse-stiffener fillet welds is:

For truck loading,

\[ f_{sr} = \frac{7,250 \times 12(108.46 - 4 \times 0.5 - 1.50)}{929,600} = 9.8 < 19 \text{ ksi} \]
For lane loading,

\[ f_{sr} = \frac{13,080 \times 12(108.46 - 4 \times 0.5 - 1.50)}{929,600} = 17.7 < 32 \text{ ksi} \]

At the butt-weld of the bottom-flange transition, the range of tensile stress in the base metal is:

For truck loading,

\[ f_{sr} = \frac{7,250 \times 12}{8,571} = 10.2 < 27.5 \text{ ksi} \]

For lane loading,

\[ f_{sr} = \frac{13,080 \times 12}{8,571} = 18.3 < 45 \text{ ksi} \]

The ranges of live-load moment at the lateral-bracing connection 125 ft from the interior support are:

\[
\begin{align*}
\text{Truck LL range} & = 4,200 + 1,900 = 6,100 \text{ kip-ft} \\
\text{Lane LL range} & = 8,100 + 4,300 = 12,400 \text{ kip-ft}
\end{align*}
\]

The range of tensile stress at the lateral-bracing connection plate is:

For truck loading,

\[ f_{sr} = \frac{6,100 \times 12(108.46 - 15 - 1.50)}{929,600} = 7.2 < 12.5 \text{ ksi} \]

For lane loading,

\[ f_{sr} = \frac{12,400 \times 12(108.46 - 15 - 1.50)}{929,600} = 14.7 < 21 \text{ ksi} \]

The 15/16-in. top flange is then checked for local buckling under DL₁ loading. The flange width-thickness ratio \( b'/t = 11/0.94 = 11.7 \).

Maximum allowable \( b'/t \) is found to be:

\[
\frac{b'}{t} = \frac{2,200}{\sqrt{50,000 \times 22,100}} = 14.8 > 11.7
\]

The section is satisfactory

**FLANGE TRANSITION 30 FT FROM SUPPORT — CENTER SPAN**

In the center span, at 30 ft from a support, the thicknesses and widths of the flanges used for the maximum-negative-moment section are reduced. The top flange changes from 3 × 36 to 1-7/8 × 30 in., and the bottom flange decreases from 3-1/4 × 36 to 2 × 30 in. Properties are calculated for the steel section alone and for the steel section plus the longitudinal reinforcing in the slab.
Steel Section

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad^2</th>
<th>I_o</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Flg. 1-7/8 x 30</td>
<td>56.25</td>
<td>78.94</td>
<td>4,440</td>
<td>350,500</td>
<td>350,500</td>
<td></td>
</tr>
<tr>
<td>Web 9/16 x 156</td>
<td>87.75</td>
<td>-79.00</td>
<td>-4,740</td>
<td>374,500</td>
<td>178,000</td>
<td></td>
</tr>
<tr>
<td>Bot. Flg. 2 x 30</td>
<td>60.00</td>
<td>-79.00</td>
<td>-4,740</td>
<td>374,500</td>
<td>374,500</td>
<td></td>
</tr>
</tbody>
</table>

\[ d_s = \frac{-300}{204} = -1.47 \text{ in.} \]
\[ I_{NA} = 902,600 \text{ in.}^4 \]
\[ d_{Top \ of \ steel} = 79.88 + 1.47 = 81.35 \text{ in.} \]
\[ d_{Bot. \ of \ steel} = 80.0 - 1.47 = 78.53 \text{ in.} \]
\[ S_{Top \ of \ steel} = \frac{902,600}{81.35} = 11,095 \text{ in.}^3 \]
\[ S_{Bot. \ of \ steel} = \frac{902,600}{78.53} = 11,474 \text{ in.}^3 \]

Steel Section with Reinforcing Steel

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad^2</th>
<th>I_o</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>204.00</td>
<td>86.02</td>
<td>587</td>
<td>50,500</td>
<td>903,000</td>
<td></td>
</tr>
<tr>
<td>Reinf. 22 No. 5</td>
<td>6.82</td>
<td>-300</td>
<td>-1.36</td>
<td>-400</td>
<td>50,500</td>
<td></td>
</tr>
</tbody>
</table>

\[ d_s = \frac{287}{210.82} = 1.36 \text{ in.} \]
\[ I_{NA} = 953,100 \text{ in.}^4 \]
\[ d_{Top \ of \ steel} = 79.88 - 1.36 = 78.52 \text{ in.} \]
\[ d_{Bot. \ of \ steel} = 80.00 + 1.36 = 81.36 \text{ in.} \]
\[ S_{Top \ of \ steel} = \frac{953,100}{78.52} = 12,138 \text{ in.}^3 \]
\[ S_{Bot. \ of \ steel} = \frac{953,100}{81.36} = 11,715 \text{ in.}^3 \]
\[ d_{Reinf.} = 86.02 - 1.36 = 84.66 \text{ in.} \]
\[ S_{Reinf.} = \frac{953,100}{84.66} = 11,258 \text{ in.}^3 \]

On the assumption that the compression flange maintains a 30-in. width between cross-frames on either side of the flange transition, the critical allowable compression stress is \( F_{cr} = 46.8 \text{ ksi} \), as determined previously at a corresponding flange transition in the end span. If the ratio of the moments at the end of the unbraced length is less than 0.7, however, \( F_{cr} \) may be increased 20%, but not to more than \( F_y = 50 \text{ ksi} \).

Bending Moments, Kip-Ft, at Cross-Frames

<table>
<thead>
<tr>
<th></th>
<th>( DL_1 )</th>
<th>( DL_2 )</th>
<th>( L + I )</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>At 25 Ft from Interior Support</td>
<td>-20,640</td>
<td>-4,220</td>
<td>-10,440</td>
<td>-35,300</td>
</tr>
<tr>
<td>At 50 Ft from Interior Support</td>
<td>-10,900</td>
<td>-2,200</td>
<td>-6,660</td>
<td>-19,760</td>
</tr>
</tbody>
</table>

\[ R = \frac{19.760}{35,300} = 0.56 < 0.7 \]

Therefore, \( F_{cr} \) may be increased. But because \( 1.2 \times 46.8 = 56.2 > 50 \text{ ksi} \), use \( F_{cr} = 50 \text{ ksi} \).
Bending Moments 30 Ft from Support — Center Span

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$-(L + l)$</th>
<th>$(L + l)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$, kip-ft</td>
<td>-18,400</td>
<td>-3,660</td>
<td>-9,600</td>
<td>2,000</td>
</tr>
</tbody>
</table>

Steel Stresses Due to Maximum Design Loads

Top of Steel (Tension) | Bottom of Steel (Compression)

For $DL_1$: $F_b = \frac{18,400 \times 12}{11,095} \times 1.30 = 25.9 \quad F_b = \frac{18,400 \times 12}{11,494} \times 1.30 = 25.0$

For $DL_2$: $F_b = \frac{3,660 \times 12}{12,138} \times 1.30 = 4.7 \quad F_b = \frac{3,660 \times 12}{11,715} \times 1.30 = 4.9$

For $L + l$: $F_b = \frac{9,660 \times 12}{12,138} \times 1.30 \times \frac{5}{3} = 20.7 \quad F_b = \frac{9,660 \times 12}{11,715} \times 1.30 \times \frac{5}{3} = 21.4$

51.3 \approx 50 \text{ ksi} \quad 50 \text{ ksi} \approx 51.3

Both flanges are judged to be satisfactory with a 2.6% over-stress.

Reinforcing Steel Stress (Tension)

$$f_b = \frac{1.30 \times 12 \times [3,660 + (5/3)9,660]}{11,258} = 27.4 < 40 \text{ ksi}$$

Fatigue is then checked in the top flange where shear connectors are likely to be attached.

Truck LL range = 1,200 + 3,200 = 4,400 kip-ft
Lane LL range = 2,000 + 9,600 = 11,660 kip-ft

The range of live-load tensile stress in the top flange is:

For truck loading,

$$f_{sr} = \frac{4,400 \times 12}{12,138} = 4.3 < 19 \text{ ksi}$$

For lane loading,

$$f_{sr} = \frac{11,660 \times 12}{12,138} = 11.5 < 32 \text{ ksi}$$

The fatigue stress range in the reinforcing steel is:

$$f_{sr} = \frac{11,660 \times 12}{11,258} = 12.4 < 20 \text{ ksi}$$

FLANGE TRANSITION 51 FT FROM SUPPORT — CENTER SPAN

In the center span, at 51 ft from a support, the thickness of the top flange decreases from 1-7/8 to 15/16 in., and the bottom flange changes from 2 to 1-1/2 in. The 9/16-in. thickness used for the web throughout most of the negative-moment region is reduced to 1/2 in. at a distance of 52 ft from the support.

Properties of the reduced section at 52 ft from the interior support are calculated
for the steel section alone and for the steel section plus the longitudinal reinforcing in the slab. These properties are then conservatively used to determine stresses at 51 ft from the interior support, where the web actually is thicker.

### Steel Section

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_o$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Flg. 15/16 X 30</td>
<td>28.13</td>
<td>78.47</td>
<td>2,207</td>
<td>173,200</td>
<td>173,200</td>
<td></td>
</tr>
<tr>
<td>Web 1/2 X 156</td>
<td>78.00</td>
<td>78.00</td>
<td></td>
<td></td>
<td>158,200</td>
<td>158,200</td>
</tr>
<tr>
<td>Bot. Flg. 1-1/2 X 30</td>
<td>45.00</td>
<td>-78.75</td>
<td>-3,544</td>
<td>279,100</td>
<td>279,100</td>
<td></td>
</tr>
</tbody>
</table>

$151.13$ in.$^2$ $-1,337$ in.$^3$ $610,500$

$d_s = \frac{-1,337}{151.13} = -8.85$ in.  

$-8.85 \times 1,337 = -11,800$

$I_{NA} = 598,700$ in.$^4$

$d_{Top\ of\ steel} = 78.94 + 8.85 = 87.79$ in.  

$d_{Bot\ of\ steel} = 79.50 - 8.85 = 70.65$ in.

$S_{Top\ of\ steel} = \frac{598,700}{87.79} = 6,820$ in.$^3$  

$S_{Bot\ of\ steel} = \frac{598,700}{70.65} = 8,474$ in.$^3$

### Steel Section with Reinforcing Steel

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_o$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>151.13</td>
<td>6.82</td>
<td>86.02</td>
<td>-1,337</td>
<td>610,500</td>
<td></td>
</tr>
<tr>
<td>Reinf. 22 No. 5</td>
<td>587</td>
<td>587</td>
<td>50,500</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$157.95$ in.$^2$ $-750$ in.$^3$ $661,000$

$d_s = \frac{-750}{157.95} = -4.75$ in.  

$-4.75 \times 750 = -3,600$

$I_{NA} = 657,400$ in.$^4$

$d_{Top\ of\ steel} = 78.94 + 4.75 = 83.69$ in.  

$d_{Bot\ of\ steel} = 79.50 - 4.75 = 74.75$ in.

$S_{Top\ of\ steel} = \frac{657,400}{83.69} = 7,855$ in.$^3$  

$S_{Bot\ of\ steel} = \frac{657,400}{90.77} = 7,242$ in.$^3$

The critical allowable compression stress for the 30-in.-wide compression flange was previously found to be $F_{cr} = 46.8$ ksi, but an increase is permissible if the ratio of the moments at the ends of the unbraced length is less than 0.7.

### Bending Moments, Kip-Ft, at Cross-Frames

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$L + I$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>At 50 Ft from Interior Support</td>
<td>-10,900</td>
<td>-2,200</td>
<td>-6,600</td>
<td>-19,760</td>
</tr>
<tr>
<td>At 75 Ft from Interior Support</td>
<td>-3,440</td>
<td>640</td>
<td>5,020</td>
<td>9,100</td>
</tr>
</tbody>
</table>

$R = \frac{9,100}{19,760} = 0.46 < 0.7$

The allowable compression stress therefore can be increased to 50 ksi.
Bending Moments 51 Ft from Support — Center Span

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$-(L + I)$</th>
<th>$(L + I)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$, kip-ft</td>
<td>-10,560</td>
<td>-2,160</td>
<td>-6,700</td>
<td>2,550</td>
</tr>
</tbody>
</table>

Steel Stresses Due to Maximum Design Loads

**Top of Steel (Tension)**

For $DL_1$: $F_b = \frac{10,560 \times 12}{6,820} \times 1.30 = 24.2$

For $DL_2$: $F_b = \frac{2,160 \times 12}{7,855} \times 1.30 = 4.3$

For $L + I$: $F_b = \frac{6,700 \times 12}{7,855} \times 1.30 \times \frac{5}{3} = 22.2$

**Bottom of Steel (Compression)**

For $DL_1$: $F_b = \frac{10,560 \times 12}{8,474} \times 1.30 = 19.4$

For $DL_2$: $F_b = \frac{2,160 \times 12}{8,795} \times 1.30 = 3.8$

For $L + I$: $F_b = \frac{6,700 \times 12}{8,795} \times 1.30 \times \frac{5}{3} = 19.8$


50.7 ≈ 50 ksi

50 ksi > 43.0

The top flange is considered satisfactory with 1.4% overstress. A check of the 1 1/2-in.-thick bottom flange for local buckling indicates that the flange is satisfactory. Its width-thickness ratio $b'/t$ is 15/1.5 = 10. The maximum permissible ratio is:

$$\frac{b'}{t} = \frac{2,200}{\sqrt{50,000}} \sqrt{\frac{50,000}{43,000}} = 10.6 > 10$$

The top flange is then checked for fatigue at the stud shear connectors.

Truck LL range = 2,150 + 2,800 = 4,950 kip-ft

Lane LL range = 2,550 + 6,700 = 9,250 kip-ft

The range of live-load tensile stress in the top-flange becomes:

For truck loading,

$$f_{sr} = \frac{4,950 \times 12}{7,855} = 7.6 < 19 \text{ ksi}$$

For lane loading,

$$f_{sr} = \frac{9,250 \times 12}{7,855} = 14.1 < 32 \text{ ksi}$$

The fatigue stress range in the reinforcing steel is:

$$f_{sr} = \frac{9,250 \times 12}{7,242} = 15.3 < 20 \text{ ksi}$$

**BOLTED FIELD SPLICE — CENTER SPAN**

A bolted field splice is located in the center span near the dead-load point of contraflexure, 87 ft from a support. At this point, the girder top flange consists of a 15/16 × 30-in. plate on the negative-moment side of the splice and a 15/16 × 22-in. plate on the positive-moment side of the splice. The girder bottom flange consists of a 1-1/2 × 30-in. plate on the negative-moment side of the splice and a 1-1/2 × 22-in.
plate on the positive-moment side of the splice. As for the field splices in the end spans, fatigue is investigated near the positive-moment side of the splice. Splice plate design calculations, however, are not presented in this chapter, because they are similar to those in Chapters 4 and 4A.

Gross section properties for the steel and composite sections on the positive-moment side of the splice were previously computed for the section at the flange transition 136 ft from the support. Section properties for the steel section plus slab reinforcing steel are calculated in the following.

### Steel Section with Reinforcing Steel

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_o$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>131.63</td>
<td>86.02</td>
<td>-980</td>
<td>50,500</td>
<td>489,900</td>
<td></td>
</tr>
<tr>
<td>Reinf. No. 5</td>
<td>6.82</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>50,500</td>
</tr>
</tbody>
</table>

\[ d_s = \frac{-393}{138.45} = -2.84 \text{ in.} \]
\[ d_{Top \ of \ steel} = 78.94 + 2.84 = 81.78 \text{ in.} \]
\[ d_{Bot \ of \ steel} = 79.5 - 2.84 = 76.66 \text{ in.} \]
\[ S_{Top \ of \ steel} = \frac{539,300}{81.78} = 6,595 \text{ in.}^3 \]
\[ S_{Bot \ of \ steel} = \frac{539,300}{76.66} = 7,035 \text{ in.}^3 \]

\[ d_{Reinf.} = 86.02 + 2.84 = 88.86 \text{ in.} \]
\[ S_{Reinf.} = \frac{539,300}{88.86} = 6,069 \text{ in.}^3 \]

Bending moments at the centerline of splice are used for fatigue checks in the splice region.

### Bending Moments 87 Ft from Support — Center Span

<table>
<thead>
<tr>
<th>$M$, kip-ft</th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$(L + I)\text{Truck}$</th>
<th>$-(L + I)\text{Truck}$</th>
<th>$(L + I)\text{Lane}$</th>
<th>$-(L + I)\text{Lane}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-600</td>
<td>200</td>
<td>3,800</td>
<td>-2,300</td>
<td>5,300</td>
<td>-4,500</td>
<td></td>
</tr>
</tbody>
</table>

At the top-flange shear connectors, the live-load stress ranges are:

For truck loading,

\[ f_{sr} = \frac{3,800 \times 12}{18,600} + \frac{2,300 \times 12}{6,595} = 6.7 < 19 \text{ ksi} \]

For lane loading,

\[ f_{sr} = \frac{5,300 \times 12}{18,600} + \frac{4,500 \times 12}{6,595} = 11.6 < 32 \text{ ksi} \]

At the interruption of the bottom longitudinal stiffener at the web splice, the live-load stress ranges are:

For truck loading,

\[ f_{sr} = \frac{3,800 \times 12 (108.46 - 1.50 - 156/5)}{929,600} + \frac{2,300 \times 12 (76.66 - 1.50 - 156/5)}{539,300} = 5.9 < 12.5 \]
For lane loading,
\[ f_{sr} = \frac{5,300 \times 12 (108.46 - 1.50 - 156/5)}{929,600} + \frac{4,500 (76.66 - 1.50 - 156/5)}{539,300} = 9.6 < 21 \text{ ksi} \]

At the bottom end of the fillet weld of the transverse stiffener to the web, the live-load stress ranges are:

For truck loading,
\[ f_{sr} = \frac{3,800 \times 12 (108.46 - 4 \times 0.5 - 1.50)}{929,600} + \frac{2,300 \times 12 (76.66 - 4 \times 0.5 - 1.50)}{539,300} = 8.8 < 19 \text{ ksi} \]

For lane loading,
\[ f_{sr} = \frac{5,300 \times 12 (108.46 - 4 \times 0.5 - 1.50)}{929,600} + \frac{4,500 \times 12 (76.66 - 4 \times 0.5 - 1.50)}{539,300} = 14.5 < 32 \text{ ksi} \]

Fatigue stress in the reinforcing steel is:
\[ f_{sr} = \frac{4,500 \times 12}{6,069} = 8.9 < 20 \text{ ksi} \]

Resistance to fatigue at the field splice in the center span at 87 ft from the interior support is satisfactory.

**SHEAR CONNECTORS**

Welded studs, 7/8-in. in diameter by 6 in. high, are used as shear connectors. They are placed three to a row along the top flanges of the girders. Calculations for connector spacing are similar to those shown in previous chapters and are not presented here.

![Diagram of shear connector spacing](image)
SHEAR-CONNECTOR SPACING

As can be seen from the shear-connector spacing diagram, fatigue and maximum permissible spacing (24 in.) govern the connector spacing.

TRANSVERSE AND LONGITUDINAL WEB STIFFENERS

Each transverse stiffener consists of an A588 steel plate welded to one side of the girder web. Longitudinal stiffeners also are made of A588 steel and are welded to the opposite side of the web.

A 9-1/2-in.-wide plate is assumed initially for the transverse stiffeners. The minimum required thickness of a stiffener is:

\[ t = \frac{b' \sqrt{F_y}}{2,600} = \frac{9.5 \sqrt{50,000}}{2,600} = 0.817 \]

Try a 7/8 x 9-1/2-in. plate.

The cross-sectional area of the plate is:

\[ A = \frac{7}{8} \times 9.5 = 8.31 \text{ in}^2 \]

The moment of inertia of the plate is:

\[ I = \frac{1}{3} \times \frac{7}{8} (9.5)^3 = 250 \text{ in}^4 \]

Area and moment-of-inertia requirements are checked for the stiffener near the interior support where shear is largest. At that section, the shear \( V \) is 1,290 kips under Maximum Design Load. The distance of the stiffener from the support was previously set at \( d_o = 74.5 \text{ in} \). The minimum stiffener area required by AASHTO Specifications is computed from

\[ A = Y \left[ 0.15BDt_w (1 - C) \frac{V}{V_w} - 18t_w^2 \right] \]
where \( Y = 1 \)
\( B = 2.4 \)
\( D = \) depth of subpanel = 123 in.
\( t_w = \) web thickness = 9/16 in.
\( V_u = \) shear capacity of web = 2,203 kips (previously computed)
\( C = 0.374 \) (previously computed)

Substitution of the preceding values in the area formula gives:
\[
A = 1.0 \left[ 0.15 \times 2.4 \times 123 \times \frac{9}{16} \left( 1 - 0.374 \right) \frac{1.290}{2,203} - 18 \left( \frac{9}{16} \right)^2 \right] = 3.43 < 8.31
\]

For computation of the required moment of inertia of the stiffener,
\[
J = 2.5 \left( \frac{D}{d_o} \right)^2 - 2 = 2.5 \left( \frac{123}{74.5} \right)^2 - 2 = 4.8
\]

The required moment of inertia then is:
\[
I = d_o t_w^3 J = 74.5 \left( \frac{9}{16} \right)^3 4.8 = 63.6 < 250 \text{ in.}^4
\]

Because a longitudinal stiffener is used in addition to the transverse stiffeners, the transverse stiffeners must also satisfy a requirement for section modulus. A 3/4 \( \times \) 8-in. plate is selected for use as a longitudinal stiffener in the negative-moment regions. AASHTO Specifications require that the section modulus of the transverse stiffener be at least
\[
S_t = \frac{1}{3} \left( \frac{D}{d_o} \right) S_i
\]

where \( D = \) total web depth
\( S_i = \) section modulus of longitudinal stiffener = 0.75(8)^2 / 3 = 16 in.\(^3\)

Therefore,
\[
S_t = \frac{1}{3} \times \frac{156}{74.5} \times 16 = 11.2 \text{ in.}^3
\]

The actual section modulus of the transverse stiffener is:
\[
S_t = \frac{1}{3} \times \frac{7}{8} (9.5)^2 = 26.3 > 11.2 \text{ in.}^3
\]

Use 7/8 \( \times \) 9-1/2-in. transverse stiffeners.

Longitudinal Stiffener for Negative Bending

The 3/4 \( \times \) 8-in. longitudinal stiffener in the negative-moment region has a width-thickness ratio \( b/t = 8/0.75 = 10.7 \). The maximum permissible ratio is:
\[
\frac{b}{t} = \frac{2,600}{\sqrt{50,000}} = 11.6 > 10.7
\]
For use in computing section properties, a centrally located web strip with width equal to $18t_w$ is considered to be part of the longitudinal stiffener. Properties are then determined to be:

$$A = 0.75 \times 8 + 18 \left( \frac{9}{16} \right)^2 = 11.7 \text{ in.}^2$$

$$I = \frac{1}{6} \times 0.75(8)^3 = 128.0 \text{ in.}^4$$

$$r = \frac{\sqrt{I}}{A} = \sqrt{\frac{128}{11.7}} = 3.31 \text{ in.}$$

The minimum moment of inertia required for a stiffener is:

$$I = Dt_w^3 \left[ 2.4 \left( \frac{d_o}{D} \right)^2 - 0.13 \right] = 156 \left( \frac{9}{16} \right)^3 \left[ 2.4 \left( \frac{74.5}{156} \right)^2 - 0.13 \right] = 11.6 < 128.0 \text{ in.}^4$$

And the minimum required radius of gyration is:

$$r = \frac{d_o \sqrt{F_y}}{23,000} = \frac{74.5 \sqrt{50,000}}{23,000} = 0.724 < 3.31 \text{ in.}$$

Use a $3/4 \times 8$-in. longitudinal stiffener in the negative-moment regions.

**Longitudinal Stiffener for Positive Bending**

In the positive-moment regions, a $7/8 \times 10$-in. longitudinal stiffener is employed.

$$\frac{b}{t} = \frac{10}{0.875} = 11.4 < 11.6$$

Other stiffener properties are as follows:

$$A = 0.875 \times 10 + 18(0.5)^2 = 13.25 \text{ in.}^2$$

$$I = \frac{1}{3} \times 0.875(10)^3 = 292 \text{ in.}^4$$

$$r = \frac{\sqrt{I}}{A} = \sqrt{\frac{292}{13.25}} = 4.69 \text{ in.}$$

In the positive-moment regions, the web is $1/2$-in. thick, and the maximum stiffener spacing is $150$ in. The minimum moment of inertia required for a longitudinal stiffener then is:

$$I = 156 \left( \frac{1}{2} \right)^3 \left[ 2.4 \left( \frac{150}{156} \right)^2 - 0.13 \right] = 40.7 < 292 \text{ in.}^4$$

And the minimum required radius of gyration is:

$$r = \frac{150 \sqrt{50,000}}{23,000} = 1.46 < 4.69 \text{ in.}$$

Use a $7/8 \times 10$-in. longitudinal stiffener in positive-moment regions.
BEARING STIFFENER AT END SUPPORT

Bearing stiffeners are designed as columns to carry the reactions at girder supports. A stiffener consisting of two 10-3/8-in. wide plates welded to opposite sides of the girder web is investigated at the end supports. Minimum required stiffener thickness is:

\[ t = b' \sqrt{\frac{F_y}{33,000}} = \frac{10.38}{12} \sqrt{\frac{50,000}{33,000}} = 1.06 \text{ in.} \]

Use a 1-1/8 X 10-3/8-in. stiffener.

**Reactions at End Support**

<table>
<thead>
<tr>
<th></th>
<th>DL₁</th>
<th>DL₂</th>
<th>L + I</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>R, kips</td>
<td>247</td>
<td>56</td>
<td>176</td>
<td>479</td>
</tr>
</tbody>
</table>

Bearing stress = \( \frac{479}{10 \times 1.125 \times 2} = 21.3 < 40 \text{ ksi} \)

As a column, the stiffener consists of the two 1-1/8 X 10-3/8 in. plates and a centrally located length of web equal to \( 18t_w = 18 \times 1/2 = 9 \) in. The area of the equivalent column is:

\[ A = 2 \times 1.125 \times 10.375 + 0.5 \times 9 = 27.8 \text{ in.}^2 \]

The moment of inertia of the equivalent column is:

\[ I = \frac{1.12 (10.38 + 0.35 + 10.38)^3}{12} = 897 \text{ in.}^4 \]

and the radius of gyration is:

\[ r = \sqrt{\frac{897}{27.8}} = 5.68 \text{ in.} \]

Consequently, the slenderness ratio of the stiffener equals:

\[ \frac{D}{r} = \frac{156}{5.68} = 27.5 \]

The allowable stress then is:

\[ F_{cr} = F_y \left[ 1 - \frac{F_y}{4\pi^2E} \left( \frac{D}{r} \right)^2 \right] = 50 \left[ 1 - \frac{50}{4\pi^2 \times 29,000} (27.5)^2 \right] = 48.3 \text{ ksi} \]

The Maximum-Design Load on the bearing stiffener is 775 kips. The capacity of the equivalent column is:

\[ P_u = 0.85AF_{cr} = 0.85 \times 27.8 \times 48.3 = 1,141 > 775 \text{ kips} \]

Therefore, the two 1-1/8 X 10-3/8-in. plates are satisfactory as bearing stiffeners at the end supports.
BEARING STIFFENERS AT INTERIOR SUPPORT

The bearing stiffener at the interior supports is designed in the same way as those at the end supports. A stiffener consisting of two 17-1/4-in. wide plates welded to opposite sides of the girder web is investigated at the interior supports. Minimum required thickness is:

\[ t = \frac{17.25}{12} \sqrt[3]{\frac{50,000}{33,000}} = 1.77 \text{ in.} \]

Use 1-15/16 × 17-1/4-in. stiffeners.

<table>
<thead>
<tr>
<th>Reactions at Interior Support</th>
</tr>
</thead>
<tbody>
<tr>
<td>DL₁</td>
</tr>
<tr>
<td>R, kips</td>
</tr>
</tbody>
</table>

The reaction due to Maximum Design Load is:

\[ R = 1.30(971 + 205 + \frac{5}{3} \times 486) = 2,582 \text{ kips} \]

The stiffener column consists of the two 1-15/16 × 17-1/4-in. plates and a centrally located portion of the web with length equal to \( 18t_w = 18 \times 9/16 = 10.12 \) in. The area of the equivalent column is:

\[ A = 3 \times 1.94 \times 17.25 + 0.56 \times 10.12 = 72.6 \text{ in.} \]

Moment of inertia of the equivalent column is:

\[ I = \frac{1.94(17.25 + 0.56 + 17.25)^3}{12} = 6,967 \text{ in.}^3 \]

and the radius of gyration is:

\[ r = \sqrt[3]{\frac{6,967}{72.6}} = 9.80 \text{ in.} \]

Therefore, the slenderness ratio is:

\[ \frac{D}{r} = \frac{156}{9.80} = 15.9 \]

The allowable compression stress then is:

\[ F_{cr} = 50 \left[ 1 - \frac{50}{4\pi^2 \times 29,000} (15.9)^2 \right] = 49.4 \text{ ksi} \]

The capacity of the equivalent column is:

\[ P_u = 0.85 \times 72.6 \times 49.4 = 3,048 > 2,582 \text{ kips} \]

The two 1-15/16 × 17-1/4-in. plates are satisfactory as bearing stiffeners at the interior supports.

DETAILS OF BOLTED FIELD SPLICE

The bolted field splice shown in the following drawing was designed in a manner
similar to that for previously illustrated splice designs. The splice is satisfactory for use as both end-span and interior-span splices. All splice material is A588 steel. All connectors are 7/8-in.-dia, A325 bolts.

BOLTED FIELD SPLICE

FINAL DESIGN

An elevation view of the interior girder designed in this example is shown on a following drawing.

The exterior girder, with a dead load of 2.838 kips per ft and a live-load distribution factor of 1.19 lanes, as compared with 3.371 kips per ft and 1.485 lanes, respectively, for the interior girder, will be lighter than the interior girder and warrants separate design. A design was made for the exterior girder with the same procedures illustrated in the example, but the detailed calculations are not given here. An elevation view of the exterior girder is shown with the elevation view of the interior girder.

An alternative design drawing also is presented (without detailed calculations) for interior and exterior girders with webs 144 in. deep in the spans and parabolic haunches 204 in. deep over the interior supports. Preliminary studies indicated that the haunched design would require somewhat less structural steel than the constant-depth design. It was assumed, however, that this savings would be offset by higher fabrication and handling costs. Actually, final design weights indicate 0.4% less weight for the constant-depth design than for the haunched design. Despite such apparent savings, however, the small weight difference indicates that, from a practical standpoint, the designs are equivalent.
In general, a haunched girder is shallower in the positive-moment regions than a constant-depth girder. Clearance requirements, therefore, might indicate which scheme would provide the better solution in this span range.

The drawings on following pages show details for the haunched and constant-depth designs, including bridge cross-section, lateral bracing and cross-frames, shear-connector layout and bolted field splices. Welded field splices are assumed impractical for girders of this size and are therefore not shown.
DETERMINING THE NEED FOR LATERAL WIND BRACING IN PLATE GIRDER BRIDGES

A Supplement to Chapter 5, Volume II,
USS Highway Structures Design Handbook
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COMPOSITE MEDIUM SPAN—
WELDED PLATE GIRDER—
LOAD FACTOR DESIGN

Until 1979, all plate-girder bridges, that have spans of 125 ft or longer, were required (per Article 1.7.17 of the AASHTO Standard Specifications for Highway Bridges) to place lateral wind bracing at the bottom flange level. This was followed by a less arbitrary stipulation in the 1979 AASHTO Interim Specification, that called for a more "rational" approach in determining the need for bottom lateral bracing. At present, the 1982 Interim Specification offers a sophisticated method for making this determination; the method is based upon rigorous computer solutions for a large number of hypothetical bridges, performed at the University of Maryland.1

In each case, the hypothetical bridges were examined both with and without bottom lateral wind bracing. The resulting maximum bottom flange stresses were determined, and these stresses were then compared with stresses developed in a continuous isolated bottom flange on fixed supports that was subjected to similar wind loading. As a result of these tests, the 1982 AASHTO Interim Specification now states:

"Bottom lateral wind bracing may or may not be provided at the discretion of the Engineer providing the stresses in the bottom flange due to wind loading are accounted for as specified in Article 1.7.17(B)."

Article 1.7.17(B) states:

"(B) Stresses Due to Wind Loading When Top Flanges are Continuously Supported.

(1) Flanges. The maximum induced stresses (F) in the bottom flange of each girder in the system can be computed from the following:

\[ F = R F_{cb} \]

where:

\[ R = \begin{cases} 0.2272L-11 & S_d^{-0.5} \\ 0.059L-0.64 & S_d^{-1} \end{cases} \]  
when no bottom lateral bracing is provided

\[ F_{cb} = \frac{72M_{cb}}{t_rb^2} \text{ (psi)} \]

\[ M_{cb} = 0.08WS_d^2 \text{ (ft-lbs)} \]

\[ W = \text{Wind loading along the exterior flange (lbs/ft)} \]

\[ S_d = \text{Diaphragm Spacing (ft)} \]

\[ L = \text{Span Length (ft)} \]

\[ t_r = \text{Thickness of Flange (inches)} \]

\[ b_r = \text{Width of Flange (inches)} \]

The latest (1982) provisions furnish a means of eliminating bottom lateral bracing in most plate-girder bridges; a factor that should result in substantial savings. Aside from its added weight, lateral bracing is costly because of two other factors: fabrication and erection. There is yet another positive aspect to this procedure: the girders' fatigue strength will be improved since the elimination of lateral bracing obviates the need for connection attachments.

Illustrative application of the provisions of Article 1.7.17 of the 1982 Interim Specifications is given in the calculations of the example that follows. In this case,
the structure investigated is the same as the constant
depth design given in Chapter 5, Vol. II of the USS
Highway Structures Design Handbook. As originally
presented, this bridge design met the requirements of
AASHTO Specifications prior to 1979, hence lateral
bracing was provided. Recalculation in accordance
with the 1982 provisions of Article 1.7.17 will show that
the bracing is unnecessary and may be eliminated.

The wind load is defined in AASHTO Article 1.2.14 as
acting on the lower half of the exposed profile of the
bridge. Stresses caused by such wind loading are con-
sidered only in exterior girders. The sum of flange
stresses caused by Group II Loading (wind and longi-
tudinal girder dead load bending) and Group III Load-
ing (wind plus dead and live load) are checked against
tension and compression limit stresses. The Load Fac-
tor Design method is used. For tension, the limit stress
is FY. For compression, the limit stress is the lower of
the stresses for yielding or buckling. Buckling limit
stress for a particular flange geometry is easily ob-
tained by rewriting the existing AASHTO equations
for flange local buckling—using the same concept of an
effective stress that is used in the development of the
effective plastic moment. For Load Factor Design, the
equation in Article 1.7.59B(1)(a) can be rewritten as
FYE = (2,200 \frac{t_r}{b_f})^2 ≤ FYE

The limit stress for tension flanges is FY. The limit
stress for compression flanges is the smallest of:

1) FY = yielding limit stress, ksi

2) FYE = \left( \frac{2200 t_r^2}{b_f^2} \right) = local buckling limit
stress, psi

3) \[ FYL = FY \left[ 1 - \frac{3FY}{4\pi^2E} \left( \frac{12S_{fl}}{b_f^2} \right)^2 \right], ksi \]

   = lateral torsional
   buckling limit stress

Terminology for these equations and the relevant
dimensions for the aforementioned structure are given
below:

E = modulus of elasticity of steel, ksi
L = span length, ft
S_{fl} = crossframe spacing, ft
W = wind load on bottom flange, kips/ft
b_f = projecting width of bottom flange, by \frac{2}{\pi} inches

F_{DL} = stress produced in bottom flange by vertical
dead load, ksi
F_{L+1} = live plus impact load stress in bottom flange,
ksi
FY = yield stress of bottom flange, ksi

---

CALCULATIONS

The design from Chapter 5 (Fig. 1) is developed by
using the Load Factor method. The lateral wind stress
is combined with the vertical dead and live load stresses
and factored as specified for Group II and Group III
loading, Table 1.2.22:

Group II: 1.3 (F_{DL} + F_{W})
Group III: 1.3 (F_{DL} + F_{L+1} + 0.3F_{W})^*

The limit stress for tension flanges is FY. The limit
stress for compression flanges is the smallest of:

1) FY = yielding limit stress, ksi

2) FYE = \left( \frac{2200 t_r^2}{b_f^2} \right) = local buckling limit
stress, psi

3) \[ FYL = FY \left[ 1 - \frac{3FY}{4\pi^2E} \left( \frac{12S_{fl}}{b_f^2} \right)^2 \right], ksi \]

   = lateral torsional
   buckling limit stress

Terminology for these equations and the relevant
dimensions for the aforementioned structure are given
below:

E = modulus of elasticity of steel, ksi
L = span length, ft
S_{fl} = crossframe spacing, ft
W = wind load on bottom flange, kips/ft
b_f = projecting width of bottom flange, by \frac{2}{\pi} inches

F_{DL} = stress produced in bottom flange by vertical
dead load, ksi
F_{L+1} = live plus impact load stress in bottom flange,
ksi
FY = yield stress of bottom flange, ksi

---

3"Autostress Design of Steel Bridges," G. Haaijer, P.S. Carskadon,
and M.A. Grubb, ASCE Reprint 80-519, October 1980.

*WL (wind on the live load) in Group III loading is assumed to be zero
for the bottom flange because wind on the live load is transferred to
and carried directly by the deck slab.
Lateral torsional buckling limit stress, FYL, is applicable only if \( S_d > \frac{20,000A_t}{FY_d} \), where \( A_t \) is the compression flange area, and \( d \) is the overall depth of the girder. For the structure in the given example, this limit can be expressed as:

\[
S_d > \frac{20,000 \text{ br} t_r}{12FY (156 + t_r + t_f)}.
\]

Also, if the ratio of moments at the end of the unbraced length \( (S_d) \) is less than 0.7, FYL may be increased up to 20 percent, but not to exceed FY.

Analysis is now made on the assumption that the bridge has no lateral bracing. A moving 50 lb per sq ft wind load is uniformly distributed along the fascia.

Referring to the elevation view of the exterior girder (Fig. 2), and the cross-section of the exterior girder (Fig. 3), the first section to investigate is the lower flange transition 34 ft from Support 1, the end bearing.

Section 34 ft Rt. of Support 1
Limit stress for tension: \( FY = 50 \) ksi
\( L = 273 \) ft
\( S_d = 24.82 \) ft
\( t_r = 0.75 \) inches
\( b_r = 18 \) inches
\( t_f = 0.6875 \) inches
\( F_{DL} = 20.00 \) ksi (from Chapter 5 design)
\( F_{L+1} = 7.94 \) ksi (from Chapter 5 design)

Half of the wind load is applied in the plane of the bottom flange.

\[
W = \frac{\left( 16.67 + \frac{0.75}{12} \right) \left( \frac{50}{1000} \right)}{2} = 0.418 \text{ kips/ft}
\]

\[
R = \frac{(0.2272L-11)S_d^{3/4}}{(0.2272)(273)-11} \times 24.82 = 5.997
\]

\[
M_{cb} = 0.08WS_d^2 = (0.08)(0.418)(24.82)^2 = 20.60 \text{ ft-kips}
\]

\[
F_{cb} = \frac{72M_{cb}}{t_r b_r^2} = \frac{(72)(20.6)}{(0.75)(18)^2} = 6.10 \text{ ksi}
\]

\[
F_W = RF_{cb}(5.997)(6.10) = 36.58 \text{ ksi}
\]

Total factored stress =

Group II:
\[
1.3(F_{DL} + F_W) = 1.3(20.00 + 36.58) = 73.56 \text{ ksi} > 50 \text{ ksi}
\]

Group III:
\[
1.3(F_{DL} + F_{L+1} + 0.3F_W) = 1.3(20.00 + 7.94 + (0.3)(36.58)) = 50.59 \text{ ksi} > 50 \text{ ksi}
\]

Since, at this location, the stress in the bottom flange is tension, the limit stress is FY. The flange stress exceeds the limit stress for both Group II and Group III loadings. Corrective options outlined above will not be considered until other critical sections along the girder have been investigated.

(Investigation of all sections will not be illustrated because the calculations would be repetitive.)

The next illustration is that of the flange transition section, 129 ft from the end bearing. At this location the bottom flange is again in tension and, as shown below, the Group II and Group III stresses are well below the limit stress.

Section 129 ft Rt. of Support 1

Limit stress for tension: \( FY = 50 \) ksi
\( L = 273 \) ft
\( S_d = 24.82 \) ft
\( t_r = 1.875 \) inches
\( b_r = 18 \) inches
\( t_f = 0.8125 \) inches
\( F_{DL} = 18.60 \) ksi (from Chapter 5 design)
\( F_{L+1} = 10.85 \) ksi (from Chapter 5 design)

\[
W = \frac{\left( 16.67 + \frac{1.875}{12} \right)((0.050)}{2} = 0.421 \text{ kips/ft}
\]

\[
R = 5.997 \text{ (Same as for Section 34' Rt. of Support 1)}
\]

\[
M_{cb} = 0.08WS_d^2 = (0.08)(0.421)(24.82)^2 = 20.75 \text{ ft-kips}
\]

\[
F_{cb} = \frac{72M_{cb}}{t_r b_r^2} = \frac{(72)(20.75)}{(1.875)(18)^2} = 2.46 \text{ ksi}
\]

\[
F_W = RF_{cb}(5.997)(2.46) = 14.75 \text{ ksi}
\]

Total factored stress =

Group II:
\[
1.3(F_{DL} + F_W) = 1.3(18.60 + 14.75) = 43.35 \text{ ksi} < 50 \text{ ksi}
\]

Group III:
\[
1.3(F_{DL} + F_{L+1} + 0.3F_W) = 1.3(18.60 + 10.85 + (0.3)(14.75)) = 44.04 \text{ ksi} < 50 \text{ ksi}
\]

Since \( L \) and \( S_d \) are the same as in the previous calculation, \( R \) is the same as for the previous section and therefore is not recalculated. Wind load, \( W \), varies from that of the previous section only because of the slightly different wind area resulting from a different flange thickness. (For all practical purposes, \( W \) could be treated as constant for the entire girder.)

The bottom flange transition section 219 ft from the end bearing, is now illustrated. At this location the bottom flange is a compression flange. Local buckling is determined to be the one that governs the limit stress.

Section 219 ft Rt. of Support 1

Limit Stress: \( FY = 50 \) ksi
\( L = 273 \) ft
\( S_d = 24.82 \) ft
\( t_r = 1.25 \) inches
\( b_r = 26 \) inches
\( t_f = 0.8125 \) inches
\( F_{DL} = -15.90 \) ksi (from Chapter 5 design)
\( F_{L+1} = -10.00 \) ksi (from Chapter 5 design)

\[
W = \frac{\left( 16.67 + \frac{1.25}{12} \right)(0.050)}{2} = 0.419 \text{ kips/ft}
\]
ELEVATION OF EXTERIOR GIRDER
ORIGINAL BRIDGE DESIGN
Since FYL is not the governing limit stress, the possibility of a 20 percent increase is not checked here. But, this provision should be investigated when FYL would otherwise yield the lowest limit stress, and when this limit stress is lower than the actual stress.

The bottom flange at Support 2 is now illustrated:

**Figure 3.**

At Support 2, FYL is the controlling limit stress. But, since it exceeds the factored stress produced by loads by a substantial margin, once again the increase of 20 percent need not be looked at.

Results of an investigation of the entire exterior girder are summarized on the girder elevation in Figure 2. (While all control sections have been checked, not all calculations are given here since they are essentially the same as those already illustrated.) Note that sections at field splices are not checked because they are not control sections for strength. The criterion for Group II loading \([1.3 \cdot (F_{DL} + F_W) \leq FY, FYE, FYL]\) and for Group III loading \([1.3 \cdot (F_{DL} + F_{L+1} + 0.3 F_W) \leq FY, FYE, FYL]\) is satisfied without lateral bracing at all sections except for two flange transition sections adjacent to each end bearing, and at the three sections near the middle of the interior span.

As stated earlier, a number of measures can be taken to lower wind stresses. Two such, are very effective solutions. Solution 1 calls for widening the bottom flange of the girder in regions where overstressing occurs. The initial measure should be to replace the overstressed flange with a wider plate of the same area as the original plate, making certain that in doing so all
ELEVATION OF EXTERIOR GIRDER
REvised BRIDGE DESIGN WITHOUT LATERAL BRACING - SOLUTION I
girder performance criteria for Group I loading are still satisfied. With the wider flange of the same area as the original flange, \( F_{DL} \) and \( F_{L+1} \) will remain approximately the same and \( F_w \) will be reduced. The girder weight will not be changed. If the limit stress cannot be satisfied with this wider, equivalent-area plate, then a wider, greater-area plate becomes necessary. A greater-area plate will reduce all three stresses—\( F_{DL} \), \( F_{L+1} \), and \( F_w \). It will add weight to the girder, but probably far less weight than that of the eliminated lateral bracing.

In solution 2, the crossframe spacing is reduced in regions where over stressing occurs. Since wind stresses are inversely proportional to the square of the crossframe spacing, they can be substantially reduced by modest adjustments in the framing. Weight will be added in the form of more crossframes but the overall weight should be lowered to a greater degree by eliminating lateral bracing.

Lateral bracing offers another solution. With lateral bracing, Article 1.7.17 specifies a different formula for the parameter \( R \) which yields lower values of \( R \) and therefore lower wind stresses. But, of the three solutions, this last is the least attractive. For this bridge, it would mean using lateral bracing in all three spans to accommodate 7 sections overstressed out of 23. And it is not even certain that lateral bracing would eliminate all over stressing. **Solution 3 should be considered only when Solutions 1 and 2 fail.**

Solutions 1 and 2 as applied to the design from Chapter 5 will now be illustrated.

**Solution I:**

The girder elevation (Fig. 2) shows that the overstressed locations are points of 18-inch bottom flange width. With other bottom flange widths of 26 and 30 inches, 22 inches would seem a good choice for a revised width of the 18-inch flange. The section 34 ft from Support 1 is investigated with a \( \frac{3}{4} \) x 22-inch bottom flange. No change is made to the top flange.

**Section 44** Rt. of Support 1 (Revised Solution 1)

Limit Stress: \( FY = 50 \text{ ksi} \)

\[
\begin{align*}
tr & = 0.875'' \\
bt & = 22'' \\
t_{tr} & = 0.6875'' \\
F_{DL} & = 16.81 \text{ ksi} \\
F_{L+1} & = 6.82 \text{ ksi} \\
F_{cb} & = \frac{(20.60)(12)}{(0.875)(22)^2/6} = 3.50 \text{ ksi} \\
F_w & = (5.997)(3.50) = 20.99 \text{ ksi} \\
Group II: & \\
1.3(F_{DL} + F_w) & = 1.3(16.81 + 20.99) \\
& = 49.14 \text{ ksi} < 50 \text{ ksi} - \text{Governs} \\
Group III: & \\
1.3(F_{DL} + F_{L+1} + 0.3F_w) & = 1.3(16.81 + 6.82 + (0.3)(20.99)) \\
& = 38.91 \text{ ksi} < 50 \text{ ksi} \\
\end{align*}
\]

It is noted that the area of the \( \frac{3}{4} \) x 22-inch revised flange is greater than the area of the original \( \frac{3}{4} \) x 18-inch flange. An equivalent-area 22-inch wide flange would not have worked, as evidenced by the closeness of the Group II stress, above, to the limit stress.

The next section, 61 ft from Support 1, is now revised.

**Section 61** Rt. of Support 1 (Revised-Solution 1)

Limit Stress: \( FY = 50 \text{ ksi} \)

\[
\begin{align*}
t_r & = 1.25'' \\
b_r & = 22'' \\
t_{tr} & = 0.6875'' \\
F_{DL} & = 21.07 \text{ ksi} \\
F_{L+1} & = 9.07 \text{ ksi} \\
F_{cb} & = \frac{(20.60)(12)}{(1.25)(22)^2/6} = 2.45 \text{ ksi} \\
F_w & = (5.997)(2.45) = 14.69 \text{ ksi} \\
Group II: & \\
1.3(F_{DL} + F_w) & = 1.3(21.07 + 14.69) \\
& = 46.49 \text{ ksi} < 50 \text{ ksi} - \text{Governs} \\
Group III: & \\
1.3(F_{DL} + F_{L+1} + 0.3F_w) & = 1.3(21.07 + 9.07 + (0.3)(14.69)) \\
& = 44.91 \text{ ksi} < 50 \text{ ksi} \\
\end{align*}
\]

Again the revised flange area is larger than the original flange area.

All other sections of the positive-moment shipping piece in Span 1 were satisfactory as originally designed. However, it would not be good practice to mix 18-inch and 22-inch bottom flange widths. The flanges of the remaining three sections are therefore changed to 22-inch width equivalent-area plates with no investigation needed.

Solution 1 is completed by making similar flange width changes to the sections 138 ft and 175 ft from Support 2. The revised Exterior Girder elevation is shown in Figure 4; and a steel weight summary and comparison with the original design is given below. **Solution 1 yields 6.2 percent savings in steel relative to the original design, gained by widening and thickening the bottom flange in certain regions to meet loading Group II and III limit stresses, and thereby eliminating lateral bracing.**

**STEEL WEIGHT SUMMARY**

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interior Girder</td>
<td>593,674</td>
</tr>
<tr>
<td>Field Splices</td>
<td>3,335</td>
</tr>
<tr>
<td>Exterior Girder</td>
<td>1,005,978</td>
</tr>
<tr>
<td>Field Splices</td>
<td>5,872</td>
</tr>
<tr>
<td>Stringer &amp; Splices</td>
<td>102,672</td>
</tr>
<tr>
<td>Lateral Bracing</td>
<td>128,207</td>
</tr>
<tr>
<td>Crossframes</td>
<td>89,364</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>1,930,102</td>
</tr>
</tbody>
</table>

Weight Per Square Foot of Slab: \( 1,930.102 \text{ lb/ft}^2 = 45.8 \# \text{ per} \text{ ft}^2 \text{ slab} \)
ELEVATION OF EXTERIOR GIRDER
REVISED BRIDGE DESIGN WITHOUT LATERAL BRACING - SOLUTION 2
STEEL WEIGHT SUMMARY
(Revised—Solution 1)

Interior Girder 593,674
Field Splices 3,335
Exterior Girders 1,013,378
Field Splices 5,872
Stringer & Splices 102,672
Lateral Bracing
Crossframes 89,364
1,808,295 #

Savings = 118,807 # (6.2%)

Weight Per Square Foot of Slab = 1,808,295 #
47'x896' = 42.9 #

Solution 2:
Solution 2 accomplishes the same thing by reducing the crossframe spacing in the region where the overstress occurs. Figure 5 shows an adjusted framing plan in which the spacing of crossframes has been decreased in the vicinity of the overstressed locations. The number of crossframe lines has increased from 37 to 43.

The section 34 ft from the end bearing is reinvestigated, this time with a crossframe spacing of 14.6 ft. The calculations below show that the stresses are now satisfactory.

Section 34 ft Rt. of Support 1 (Revised—Solution 2)

Limit stress for tension: FY = 50 ksi

\[ S_d = 14.60 \text{ ft} \]

\[ R = (0.2272(273) - 11)16.67^{1/6} = 8.545 \]

\[ M_{cb} = (0.08)(0.418)(14.60)^2 = 7.13 \text{ ft-kips} \]

\[ F_{cb} = \frac{(72)(7.13)}{(0.75)(18)^2} = 2.11 \text{ ksi} \]

\[ F_W = (8.545)(2.11) = 18.05 \text{ ksi} \]

Total factored stress =

Group II:
\[ 1.3(F_{DL} + F_W) = 1.3(20.00 + 18.05) = 49.47 \text{ ksi} < 50 \text{ ksi} \]

Group III:
\[ 1.3(F_{DL} + F_{L+1} + 0.3F_W) = 1.3[20.00 + 7.94 + (0.3)(18.05)] = 43.36 \text{ ksi} < 50 \text{ ksi} \]

A new calculation of stresses is also performed at the middle of the center span. Again the results, given below, show that the design is satisfactory.

Section 175 ft Rt. of Support 2 (Revised)

\[ S_d = 16.67 \text{ ft} \]

Limit stress for tension: FY = 50 ksi

\[ R = (0.2272(550) - 11)16.67^{1/6} = 10.5 \]

\[ M_{cb} = (0.08)(0.421)(16.67)^2 = 9.36 \text{ ft-kips} \]

\[ F_{cb} = \frac{(72)(9.36)}{(1.8125)(18)^2} = 1.15 \text{ ksi} \]

\[ F_W = (10.505)(1.15) = 12.05 \text{ ksi} \]

Total factored stress =

Group II:
\[ 1.3(F_{DL} + F_W) = 1.3(18.70 + 12.05) = 39.97 \text{ ksi} < 50 \text{ ksi} \]

Group III:
\[ 1.3(F_{DL} + F_{L+1} + 0.3F_W) = 1.3[18.70 + 11.12 + (0.3)(12.05)] = 43.47 \text{ ksi} < 50 \text{ ksi} \]

A complete investigation indicates that wind stresses in the entire structure have been reduced to allowable values by shortening the crossframe spacing in selected regions.

For comparison, weight summaries are given below for the original structure of Chapter 5 and the revised structure — Solution 2 — with lateral bracing eliminated. A 6.0 percent savings in structural steel weight is achieved.

For the particular structure in this example, with its girder-substringer construction and, of necessity, heavy crossframes, Solution 1 proved the most economical means of eliminating lateral bracing. However, in multi-girder stringer-type bridges having lighter crossframes, Solution 2 would probably be more economical. Clearly, for any given bridge, both solutions should be investigated. It should be noted that one advantage of Solution 2 is the fact that the girder design needn’t be adjusted.

STEEL WEIGHT SUMMARY
(ORIGINAL DESIGN WITH LATERAL BRACING)

Interior Girder 593,674
Field Splices 3,335
Exterior Girders 1,005,978
Field Splices 5,872
Stringer & Splices 102,672
Lateral Bracing 129,207
Crossframes 89,364
1,980,102 #

Weight Per Square Foot of Slab = 1,930,102 #
47'x896' = 45.8 #

STEEL WEIGHT SUMMARY
(REvised.—Solution 2)

Interior Girder 593,674
Field Splices 3,335
Exterior Girders 1,005,978
Field Splices 5,872
Stringer & Splices 102,672
Lateral Bracing 129,207
Crossframes 100,369
1,811,900 #

Weight Per Square Foot of Slab = 1,811,900 #
47'x896' = 43.0 #

SAVINGS = 115,202 # (6.0%)
Composite: Curved Plate Girder Load Factor Design

Introduction

This chapter discusses and illustrates design of a two-span highway bridge on horizontally curved alignment, utilizing curved I-shaped plate girders. Girders are connected to each other at regular intervals by crossframe-type diaphragms. There is no other system of lateral bracing between girders at either flange level. A reinforced concrete deck slab acts compositely with the steel girders.

Horizontally-curved plate girders are suitable for simple and continuous spans of lengths similar to those for which straight plate girders are suitable, as outlined in Chapters 4 and 4A. Curved plate girders are used for grade separation and elevated bridges, where the structure must conform to the curved roadway alignment. This condition occurs most frequently at urban crossings and interchanges but may also be found at rural intersections where the structure must follow the highway's geometric requirements.

The design example of this chapter is in accordance with the 12th Edition of the “Standard Specifications for Highway Bridges” of the American Association of State Highway and Transportation Officials (AASHTO), including the 1977 through 1982 “Interim Specifications” (all hereinafter referred to as the AASHTO Specifications), as modified by the “AASHTO Guide Specifications for Horizontally Curved Highway Bridges,” 1980 (hereinafter referred to as the Guide Specifications). The design uses ASTM A36 and ASTM A572 Grade 50 steels.

General Design Considerations

Curved I-girders are of the same general construction as straight I-girders. The curvature is achieved either by cutting the flanges to a curve or by fabricating the girder straight and heat curving it. Crossframes between girders are usually spaced more closely than the 25 ft maximum allowed for straight girders. A spacing range of 12-18 ft is very common for ordinary curved grade separation structures of moderate length; the more sharply curved the structure, the more closely spaced are the crossframes. Lateral bracing is used only when it is needed to carry wind loads, just as for straight bridges. Recently adopted provisions of the AASHTO Specifications permit elimination of lateral bracing in most bridges and this is the course that should be taken for maximum economy whether the bridge is straight or curved.
LOADS AND LOAD COMBINATIONS

Composite construction is equally applicable to straight or curved I-girder bridges. Initial dead load (DL₁), superimposed dead load (DL₂), and live load plus impact (L+I) can be computed by the rules and principles outlined in Chapters 3, 3A, 4 and 4A. Load factors for the Group loadings are those defined by the Specifications.

Several different loadings must be considered:

A) Construction Loads
   For certain structures, construction loads may govern the design of some sections.
   If this possibility exists, the load combination 1.3(Dₚ+C) should be examined,
   where Dₚ is a partial dead load and C is a load due to construction equipment.

B) Service Loads
   Service loads consist of the dead load D, plus live load with impact (L+I), plus
   centrifugal force CF. The service load D+(L+I) + CF is applied to the structure to
   determine stress range to be used in checking resistance to fatigue and to determine
   the live load deflection.

C) Maximum Design Loads
   All sections of the bridge must be proportioned for sufficient strength to resist the
   forces due to the loading, 1.3[D + ½(L+I) + CF].

D) Overload
   In addition to meeting the maximum strength requirement of (C), each girder
   section must satisfy performance criteria under the loading D + ½(L+I) + CF.

   The foregoing relationships show that centrifugal force is a primary load that must be
   accounted for in the design of any curved I-girder bridge. Centrifugal force is a horizontal
   force acting on the live load 6 ft above the deck and transmitted to the structure through the
   wheels of the vehicle. The entire horizontal force is assumed to be carried to the bridge
   bearings by way of the deck slab in transverse bending and the crossframes at each support
   in transverse shear. No particular account need be taken of the horizontal effect of cen-
   trifugal force on the bridge superstructure except that the support crossframe diagonals
   must be adequately proportioned to deliver the load to the bearings.

   Because centrifugal force acts 6 ft above the deck it also causes an overturning moment.
   Usually, part of this overturning moment is balanced by the superelevation. The remainder
   is assumed to act on the steel framing treated as a pile group supporting the slab, producing
   a downward loading on girders outside the centerline of the bridge and an upward loading
   on girders inside the centerline of the bridge.

   A derivation of the fraction of centrifugal force overturning moment balanceable by
   superelevation is given in the example below in which the superelevation is assumed to be
   0.06 ft/ft.

   If wheel reactions, P, are to be equal, moments about Point A must be zero. By inspection
   the moments due to wheel reactions cancel one another and it can be written simply:
\[ N (6 \sin \alpha) - KW (6 \cos \alpha) = 0 \]
\[ 6 \sin \alpha W = 6 \cos \alpha KW \]

\[ K = \frac{6 \sin \alpha W}{6 \cos \alpha W} = \tan \alpha = SE = 0.06 \]

It is seen that the fraction of \( W \) that can be balanced by superelevation is equal numerically to the superelevation rate.
DESIGN THEORY AND STRUCTURAL ANALYSIS

The Specifications require that the structure be analyzed as a system, taking into account the complete distribution of loads to various members. A number of very general computer programs are available which rigorously analyze large structural systems using variations of classical indeterminate structural analysis theory. These programs give excellent results. They do require large amounts of input, large computers, and considerable computer time, inasmuch as many load cases must be run in order to develop live load envelopes.

The United States Steel V-Load Method offers an alternative approach. Although it is an approximate solution, it does rationally and systematically treat the entire structure. The method was developed in 1963 and improved in 1965. A survey in 1973 indicated that 75 percent of curved bridges, designed in the United States up to that time, were designed by the V-Load Method.

In its original form, the method was limited to structures with radial supports. In 1982 another improvement was made, extending the applicability of the method to include structures with skewed supports. The derivation and background of the theory are presented in the USS Highway Structures Design Handbook, Chapter I/12.

The V-Load Method has a number of positive attributes:

1) It is rational, easily understood, and helps to improve perception of curved bridge behavior.
2) Its approach and order of accuracy are quite consistent with that currently practiced for straight girders.
3) It is readily programmed on small to medium size computers.
4) It analyzes the structure as a system and thus satisfies the specification requirement that the structure be so treated.
5) It lends itself to automatic generation of live load envelopes, eliminating hand manipulation of loads.
6) Its results have shown excellent agreement with results from rigorous solutions for a variety of structures.

The V-Load Method has been implemented on the computer in a program entitled VLOAD,* which is available on a time-sharer basis from USS Engineers and Consultants, Inc. (UEC), a division of United States Steel Corporation.

The V-Load Method provides values for dead load moments, shears, reactions and deflections, as well as live load moment and shear envelopes, and live load deflections for each girder. These values all result from normal vertical bending, which consists of two parts: The first part is vertical bending due to the applied loads acting on the girders as if they were straight. This is called primary bending moment. The second part is additional vertical bending resulting from the shears developed at the ends of the crossframes, which act as concentrated loads on the girder. These are called secondary moments and are additive to the primary moments on girders outside the centerline of the bridge, and subtractive from the primary moment on girders inside the centerline of the bridge. The stress caused by vertical bending is computed by the ordinary mechanics of composite girders as described in Chapters 3, 3A, 4 and 4A. It is designated as \( \delta_0 \).

The table below shows a comparison of the V-Load-analysis results to MSC/NASTRAN finite-element-analysis results for peak moments in the two-span structure discussed later in this chapter. The moments tabulated are from a preliminary analysis, in which the girder sections have been estimated. The standard AASHTO distribution factors for straight girders were used to compute the L+I loads on the girders for determining the primary moments in the V-Load analysis.

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*For additional information on this program contact USS Engineers and Consultants, Inc., Room 1614, 600 Grant St., Pittsburgh, PA 15230, Tel. (412) 433-7512.
<table>
<thead>
<tr>
<th>MOMENT</th>
<th>SPAN 1</th>
<th>PIER</th>
<th>SPAN 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NASTRA VLOAD</td>
<td>CUGAR NASTRA VLOAD</td>
<td>CUGAR NASTRA VLOAD</td>
</tr>
<tr>
<td>G1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DL₁</td>
<td>1333</td>
<td>1883</td>
<td>1388</td>
</tr>
<tr>
<td>DL₂</td>
<td>521</td>
<td>527</td>
<td>513</td>
</tr>
<tr>
<td>L+I</td>
<td>1535</td>
<td>1606</td>
<td>-763</td>
</tr>
<tr>
<td>TOTAL</td>
<td>3449</td>
<td>3516</td>
<td>-3537</td>
</tr>
<tr>
<td>G2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DL₁</td>
<td>1206</td>
<td>1198</td>
<td>1202</td>
</tr>
<tr>
<td>DL₂</td>
<td>459</td>
<td>445</td>
<td>446</td>
</tr>
<tr>
<td>L+I</td>
<td>1100</td>
<td>1518</td>
<td>-514</td>
</tr>
<tr>
<td>TOTAL</td>
<td>2765</td>
<td>3161</td>
<td>-3025</td>
</tr>
<tr>
<td>G3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DL₁</td>
<td>1007</td>
<td>986</td>
<td>998</td>
</tr>
<tr>
<td>DL₂</td>
<td>363</td>
<td>354</td>
<td>358</td>
</tr>
<tr>
<td>L+I</td>
<td>897</td>
<td>1271</td>
<td>-376</td>
</tr>
<tr>
<td>TOTAL</td>
<td>2267</td>
<td>2611</td>
<td>-2542</td>
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<tr>
<td>G4</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>DL₁</td>
<td>771</td>
<td>760</td>
<td>767</td>
</tr>
<tr>
<td>DL₂</td>
<td>267</td>
<td>265</td>
<td>271</td>
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<tr>
<td>L+I</td>
<td>919</td>
<td>1057</td>
<td>-391</td>
</tr>
<tr>
<td>TOTAL</td>
<td>1957</td>
<td>2082</td>
<td>-2030</td>
</tr>
</tbody>
</table>

Note: CUGAR solution not available for L+I

Inspection of the table reveals excellent correlation of all moments for the exterior girders G1 and G4, and equally good correlation of DL₁ and DL₂ moments for interior girders G2 and G3. For L+I moments in girders G2 and G3 the V-Load results are conservative, leading to the conclusion that the AASHTO live load distribution factor for interior girders is conservative. The accuracy of the V-Load Method for live load is totally dependent upon the accuracy of the distribution factors that are used to compute the primary moments. Further research is needed to develop better live load distribution factors. For lack of such development at present, this example will rely solely on the AASHTO distribution factors for straight bridges. The above table indicates that the total moments on girders G2 and G3 will be approximately 10 percent conservative on the average. Stress levels will be even less conservative.

In addition to vertical bending, lateral flange bending occurs in all curved members. As shown in Chapter I/12, the lateral flange bending moment is given by

$$M_{Lateral} = \frac{M_{Vertical}d}{12R}$$

where

- $$M_{Vertical}$$ = total vertical bending moment at the girder section in question
- $$d$$ = diaphragm or crossframe spacing along the girder
- $$R$$ = radius of girder
- $$h$$ = depth of girder, center to center of flanges

The stress caused by lateral flange bending is designated as $$f_w$$ and is obtained by

$$f_w = \frac{M_{Lateral}}{S_{Lateral}}$$

where

- $$M_{Lateral}$$ = lateral flange bending moment as computed above
- $$S_{Lateral}$$ = lateral section modulus of flange

The stresses $$f_b$$ and $$f_w$$ are additive at the tip of the flange.
These stresses, due to secondary bending moment and lateral flange bending moment, represent the effect of curvature. The more sharply curved the bridge, the greater the portion of stress attributable to curvature. The true measure of the curvature sharpness of a girder is the central angle subtended by the span length. Small central angles exist when there is little curvature. The Guide Specifications recognize the fact that for very small central angles there will be relatively little secondary bending effect. Table 1.4A of the Guide Specifications permits vertical secondary bending to be neglected under the conditions indicated.

<table>
<thead>
<tr>
<th>Number of Girders</th>
<th>Angle for 1 span</th>
<th>Angle for 2 or more spans</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2°</td>
<td>3°</td>
</tr>
<tr>
<td>3 or 4</td>
<td>3°</td>
<td>4°</td>
</tr>
<tr>
<td>5 or more</td>
<td>4°</td>
<td>5°</td>
</tr>
</tbody>
</table>

However, neglecting lateral flange bending is never permitted, because this is a function not only of central angle, but also of crossframe spacing and flange width.

DIAPHRAGMS AND CROSSFRAMES

The Guide Specifications require that crossframes or diaphragms “be provided at each support and at intermediate intervals between supports with spacings as determined by design considerations.” It has already been stated that 12-18 ft spacing is a very common range. The formula for lateral bending moments indicates that they are proportional to the square of the crossframe spacing. Thus, an increase in the crossframe spacing will require an increase in girder flange material to resist the resulting higher lateral bending moment. Conversely, if crossframe spacing is reduced, lateral bending moments will be reduced, and correspondingly less girder flange material will be needed; however, more crossframe material will be required. Selection of crossframe spacing therefore becomes a matter of judgment in choosing the most economical trade-off of girder flange material versus crossframe material.

Crossframes in curved I-girder bridges carry calculated stresses and therefore must be designed as main structural members, with each line of crossframes extending in a single plane across the width of the bridge. For crossframe members, tee sections are preferable to angles because they are symmetrical about the Y-axis and therefore carry their loads concentrically. Either “X”- or “K”-type framing may be used, depending on the geometry of the bay. The Guide Specifications require that crossframes be full depth members and that they “be framed in such a way as to transfer the horizontal and vertical forces to the flanges and web as necessary.” It is also stipulated that crossframe connection plates must be attached to both the web and the flanges of the girder.

The primary forces on curved girder crossframes are those resulting from curvature, that is, from the interaction of girders and crossframes in carrying D + L + I + CF loads. This explains why they are considered main structural elements. It also means that they must resist fatigue. Members and their end connections must be examined for strength and also for assurance that stress ranges are within allowable values.

The crossframes also carry wind and CF loads. The following assumptions are made for design of crossframes in an I-girder bridge:

1) The crossframes carry the curvature effect of all vertical loads (DL1, DL2, L + I and CFvert.) in accordance with V-Load theory.
2) The slab carries wind on the upper half of the structure, wind on live load, and centrifugal force (CF_Horiz.) back to the support crossframes, which in turn carry these forces down to the bridge bearings.
3) The wind on the lower half of the structure is carried through the intermediate crossframe diagonals up to the slab, and then to the bridge bearings through the support crossframes as described in 2) above. The support crossframes therefore transmit all wind load down to the bridge bearings.

Three distinct types of crossframes may be proportioned for the forces applicable to each type and the group loadings that govern, as indicated below:

<table>
<thead>
<tr>
<th>Crossframe Type</th>
<th>Effects to be Considered</th>
<th>Group Loadings to be Checked</th>
</tr>
</thead>
<tbody>
<tr>
<td>End Support</td>
<td>Wind, Centrifugal Force</td>
<td>Group I: 1.3 [DR+0.5 (TR)+CF_H]</td>
</tr>
<tr>
<td></td>
<td>Dead Load &amp;</td>
<td>Group II: 1.3 [DR+W_T+W_B]</td>
</tr>
<tr>
<td></td>
<td>Truck Wheel Loading</td>
<td>Group III: 1.3 [DR+TR+CF_H+0.3 (W_T+W_B)+WL]</td>
</tr>
<tr>
<td>Intermediate</td>
<td>Curvature</td>
<td>Group I: 1.3 [D+0.5 (L+I)+CF_V]</td>
</tr>
<tr>
<td></td>
<td>Wind</td>
<td>Group II: 1.3 [D+W_Bp]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Group III: 1.3 [D+(L+I)+CF_V+0.3W_Bp]</td>
</tr>
<tr>
<td>Interior Support</td>
<td>Curvature</td>
<td>Group I: 1.3 [D+0.5 (L+I)+CF_V+CF_H]</td>
</tr>
<tr>
<td>Support</td>
<td>Wind, Centrifugal Force</td>
<td>Group II: 1.3 [D+W_T+W_B]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Group III: 1.3 [D+(L+I)+CF_V+CF_H+0.3 (W_T+W_B)+WL]</td>
</tr>
</tbody>
</table>

where

\[D = \text{dead load curvature}\]
\[L+I = \text{live load plus impact, curvature}\]
\[W_T = \text{wind on upper half of structure}\]
\[W_B = \text{wind on lower half of structure}\]
\[W_L = \text{wind on live load}\]
\[CF_V = \text{vertical centrifugal force}\]
\[CF_H = \text{horizontal centrifugal force}\]
\[DR = \text{dead load on top strut}\]
\[TR = \text{truck wheel loading on top strut}\]
\[W_Bp = \text{panel wind load, lower half of structure}\]

The crossframe forces due to curvature are easily computed from V-Load theory.*

Looking at any crossframe line in the positive moment region of a four-girder system, the assumed distribution of V-Loads and torque loads to the girders is as shown below.

The corresponding distribution of end shears on the crossframes follows.

For equilibrium, the distribution of end moments on the crossframes can then be worked out.

In the above figures
\[ V = \frac{M_{1p} + M_{2p} + M_{3p} + M_{4p}}{C(RD/d)} \]

- \( M_{1p}, M_{2p}, M_{3p}, M_{4p} \) = primary moments in G1, G2, G3, G4 at crossframe line under investigation
- \( M_1, M_2, M_3, M_4 \) = total moments in G1, G2, G3, G4 at crossframe line under investigation
- \( D \) = width, G1 to G4
- \( d \) = crossframe spacing on G1
- \( R \) = radius of G1
- \( C \) = a constant (10% for a four-girder system)

Forces from curvature, wind, centrifugal force and direct wheel loading are added together for design, in accordance with the Group loadings.

GIRDER SECTION DESIGN

The Guide Specifications give allowable normal stresses that depend on whether the compression flange qualifies as compact or non-compact. Compact compression flanges may be stressed to full plasticity without local buckling; non-compact flanges are more slender and therefore subject to buckling. The expressions for allowable stress involve the geometric parameters of the girder such as unbraced length of compression flange, flange width and radius (l, b and R, respectively). They also include the ratio of the warping (lateral bending) normal stress to the vertical bending stress, \( \frac{f_w}{f_b} \). Thus, the actual stresses must be known in order to compute the allowable stress.

There are two limitations on the unbraced compression flange length

\[ l \leq 25(b) \]
\[ l \leq 0.1R \]

There are also several provisions regarding \( \frac{f_w}{f_b} \). This parameter, and the allowable stress expressions that follow below, apply to an entire unbraced length of flange. If the section is
prismatic over the unbraced length, the stress \( f_b \) is computed using the larger of the two bending moments at either end of the segment, and \( f_w \) is due to the corresponding lateral bending moment at that location. If the section changes within the unbraced length, \( f_b \) and \( f_w \) are computed for the smaller section at the location of the change in section, using the moment at that point. The ratio \( \frac{f_w}{f_b} \) is a signed quantity defined as positive for compression flanges and negative for tension flanges. In addition, its absolute value \( \frac{f_w}{f_b} \) must not exceed 0.5, except under low stress conditions not governing the design of the section.

Compactness is defined by the \( \frac{b}{t} \) ratio of the compression flange. When \( \frac{b}{t} \leq \frac{3.200}{\sqrt{F_y}} \) the flange is compact. In this case the vertical bending stress in the compression flange, \( f_b \), is limited to

\[
F_{bs} = F_{bs} \rho_B \rho_w
\]

where

\[
\rho_B = \frac{1}{1 + \frac{1}{b} \left( 1 + \frac{1}{6b} \right) \left( \frac{1}{R} - 0.01 \right)^2}
\]

\[
\rho_w = 0.95 + 18 \left[ 0.1 - \frac{1}{R} \right]^2 + \frac{f_w}{f_b} \left[ 0.3 - 0.1 \frac{1}{R} b \right] \frac{\rho_B}{\rho_{bs}/F_y}
\]

\[
\rho_B \rho_w \leq 1
\]

\[
F_{bs} = F_y (1 - 3 \lambda^2), \quad \text{with} \quad \lambda = \frac{1}{\pi} \left( \frac{1}{b} \right) \sqrt{\frac{F_y}{E}}
\]

For the tension flange the allowable stress is given by the same expressions except that

\[
F_{bs} = F_y.
\]

A non-compact compression flange falls within the range

\[
\frac{3200}{\sqrt{F_y}} \leq \frac{b}{t} \leq \sqrt{\frac{4400}{F_y}}. \quad \text{(In no case is} \quad \frac{b}{t} \quad \text{allowed to exceed} \quad \sqrt{\frac{4400}{F_y}}.)
\]

For non-compact compression flanges, \( f_b \) is limited to

\[
F_{by} = F_{bs} \rho_B \rho_w
\]

where

\[
\rho_B = \frac{1}{1 + \frac{1}{b} \frac{1}{R}}
\]

\[
\rho_w = \frac{1}{1 - \frac{f_w}{f_b} \left[ \frac{1}{1 - \frac{1}{75b}} \right]}
\]

\[
\rho_w = 0.95 + \frac{1/b}{30 + 8000 (0.1 - 1/R)^2} \quad \frac{\rho_w}{\rho_{ws}} = \frac{\rho_w}{1 + 0.6 (f_{ws}/f_b)}
\]

\[
\rho_w = \text{smaller of} \rho_w \text{ and} \rho_{ws} \text{ if} \frac{f_w}{f_b} \text{ is positive}
\]

or

\[
\rho_w = \rho_{ws} \text{ if} \frac{f_w}{f_b} \text{ is negative}
\]

\[
F_{bs} = F_y (1 - 3 \lambda^2), \quad \text{with} \quad \lambda = \frac{1}{\pi} \left( \frac{1}{b} \right) \sqrt{\frac{F_y}{E}}
\]

\[1/86\]
Again, for the tension flange the allowable stress is given by the same expressions except that

\[ F_{bs} = F_y. \]

In addition to these limitations on \( f_b \), both tension and compression flanges for the non-compact case are subject to the following limit on flange tip stress:

\[ f_b + f_w \leq F_y. \]

This limit on flange tip stress does not apply to the case in which the compression flange is compact. The reason is that the allowable stress expressions for this case are based on an ultimate strength taken as the fully plastic strength. Since all fibers are stressed to the yield point under fully plastic bending, lateral bending stress is not a meaningful parameter.

The Guide Specifications do not recognize any transition curve between the allowable stresses for compact and non-compact flanges. This means that there is an abrupt drop in allowable stress when the compression flange goes from compact to non-compact. Some experimentation by the designer may be necessary to determine what type of compression flange is most economical for a particular section. A thicker compression flange is needed for compactness than for non-compactness, but the tension flange may be considerably thinner for the former case than for the latter, because of the significant increase in allowable stress. Experience generally indicates that sections with compact compression flanges are more economical than sections with non-compact compression flanges.

**WEBS**

Provisions in the Guide Specifications for webs do not consider the post-buckling strength resulting from tension field action. Thus, there are no special rules for spacing of stiffeners adjacent to an end reaction point, and generally the criteria for web design are simple.

Transverse web stiffeners are not required if \( \frac{D}{t} \leq 150 \), and the ultimate shear capacity of unstiffened webs is the smaller of

\[ V_u = \frac{3.5Et^3}{D} \]

or

\[ V_u = 0.58 F_y D t. \]

If the maximum shear exceeds \( \frac{3.5Et^3}{D} \), transverse stiffeners are required for the web. The ultimate shear capacity of a stiffened web is

\[ V_u = 0.58 F_y D t C \]

where

\[ C = \left[ 18,088 \left( \frac{t}{D} \right) \sqrt{\frac{1 + (D/d_0)^2}{F_y}} \right] - 0.3 \leq 1.0 \]

Both stiffened and unstiffened webs are subject to the interaction equation

\[ \frac{M}{M_u} = 1.375 - 0.625 \frac{V}{V_u} \text{ when } V > 0.6V_u. \]

When transverse web stiffeners are required they must be spaced at a distance, \( d_0 \), not exceeding \( D \), the depth of the girder. Their proportions are subject to the following rules:

1. \( \frac{b}{t} \leq \frac{2,600}{F_y} \) \hspace{1cm} (AASHTO/Art.10.48.5.5)
2. \( \frac{b}{t} = 16 \)
3. \( b \geq 2 + \frac{D}{30} \) \hspace{1cm} (AASHTO/Art.10.34.5.1)
where \[ b = \text{stiffener width} \]
\[ t = \text{stiffener thickness} \]

The stiffeners must satisfy the rigidity requirement given below for moment of inertia about the mid-plane of the web
\[ I = d_o t^3 J \]

where
\[ J = \left[ 2.5 \left( \frac{D}{d_o} \right)^2 - 2 \right] X, \text{ but not less than 0.5} \]
\[ X = 1 \text{ when } d_o/D \leq 0.78 \]
\[ X = 1 + \left[ \frac{d_o/D - 0.78}{1.775} \right] Z \text{ when } 0.78 \leq d_o/D \leq 1.0 \]
\[ Z = \frac{0.95 d_o^2}{R t} \]
\[ R = \text{radius of web} \]
\[ t = \text{web thickness} \]

Longitudinal stiffeners are required if
\[ \frac{D}{t} > \frac{36,500}{\sqrt{F_y}} \left[ 1 - 8.6 \left( \frac{d_o}{R} \right) + 34 \left( \frac{d_o}{R} \right)^2 \right] \]

When one longitudinal stiffener is placed a distance \( \frac{D}{5} \) from the compression flange, the web proportions must satisfy
\[ \frac{D}{t} \leq \frac{73,000}{\sqrt{F_y}} \left[ 1 - 2.9 \sqrt{\frac{d_o}{R}} + 2.2 \left( \frac{d_o}{R} \right) \right] \]
but when another equal sized stiffener is located a distance \( \frac{D}{5} \) from the tension flange, the web need only satisfy the requirement
\[ \frac{D}{t} \leq \frac{73,000}{\sqrt{F_y}} \]

The longitudinal stiffener proportions are governed by the same AASHTO requirements as for straight girders.

**SHEAR CONNECTORS**

Design of shear connectors for fatigue is the same as that for straight girders as given in the AASHTO Specifications. For ultimate strength, the Guide Specifications require that the number of shear connectors between points of maximum positive moment and the end supports or dead-load inflection points be sufficient to satisfy:
\[ P_c \leq \phi S_u \]

where \( \phi = \text{reduction factor} = 0.85 \)
\[ S_u = \text{ultimate strength, kips, of the shear connector as given in the AASHTO Specifications for straight girders} \]
\[ P_c = \text{force, kips, on the connector} \]
\[ = \sqrt{F^2 + F^2 + 2F \sin \frac{\Theta}{2}} \]
\[ P = \frac{P}{N} \]
\[ P = 0.85 f_{c} b c + A_f F_y, \text{ whichever is smaller, at points of maximum positive moment} \]
\[ = A_f' F_y, \text{ at points of maximum negative moment as defined by the AASHTO Specifications for straight girders} \]
\[ N = \text{number of connectors between points of maximum positive moment and adjacent end supports or dead-load inflection points, or between points of maximum negative moment and adjacent dead-load inflection points} \]
\[ F = \frac{P(1 - \cos \Theta)}{4KN, \sin \Theta/2} \]
\[ \Theta = \text{angle extended between point of maximum moment (positive or negative) and adjacent point of contraflexure or support} \]
\[ f'_c = 28\text{-day compressive strength of concrete slab, ksi} \]
\[ b = \text{effective width of slab, inches} \]
\[ c = \text{thickness of slab, inches} \]
\[ A_s = \text{total area of steel section, including cover plates, sq. inches} \]
\[ A_{rs} = \text{total area of longitudinal reinforcing steel at the interior support within the effective width of flange, sq. inches} \]
\[ F'_{y} = \text{yield strength of the reinforcing steel, ksi} \]
\[ K = 0.166 \left( \frac{N}{N_s} - 1 \right) + 0.375 \]
\[ N_s = \text{number of connectors at a section} \]

**Design Example**

**Two-Span Continuous I-Girder Bridge Composite for Positive and Negative Moment**

A curved bridge with two spans and four girders is selected as the example structure for this chapter. A plan view appears below. The centerlines of bearing at the west abutment and center pier are parallel to an underpassing roadway and skewed to the centerline of bridge. The centerline of bearing at the east abutment is radial to the centerline of bridge. The radius of Girder G1 changes from 300 ft to 600 ft at a point of compound curvature in the second span. All girders are concentric.

<table>
<thead>
<tr>
<th>Girder</th>
<th>Radius</th>
<th>Arc Length</th>
<th>Central Angle (\Delta^o)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1-1</td>
<td>300.0000</td>
<td>110.0000</td>
<td>21.00844</td>
</tr>
<tr>
<td>G1-2</td>
<td>300.0000</td>
<td>60.0000</td>
<td>11.45317</td>
</tr>
<tr>
<td>G1-3</td>
<td>600.0000</td>
<td>40.0000</td>
<td>3.81972</td>
</tr>
<tr>
<td>G2-1</td>
<td>291.1667</td>
<td>110.4751</td>
<td>21.73931</td>
</tr>
<tr>
<td>G2-2</td>
<td>291.1667</td>
<td>53.1651</td>
<td>10.46183</td>
</tr>
<tr>
<td>G2-3</td>
<td>591.1667</td>
<td>39.4111</td>
<td>3.81972</td>
</tr>
<tr>
<td>G3-1</td>
<td>282.3333</td>
<td>111.0056</td>
<td>22.52711</td>
</tr>
<tr>
<td>G3-2</td>
<td>282.3333</td>
<td>46.2739</td>
<td>9.39067</td>
</tr>
<tr>
<td>G3-3</td>
<td>582.3333</td>
<td>38.8222</td>
<td>3.81972</td>
</tr>
<tr>
<td>G4-1</td>
<td>273.5000</td>
<td>111.6008</td>
<td>23.37936</td>
</tr>
<tr>
<td>G4-2</td>
<td>273.5000</td>
<td>39.3167</td>
<td>8.23650</td>
</tr>
<tr>
<td>G4-3</td>
<td>573.5000</td>
<td>38.2333</td>
<td>3.81972</td>
</tr>
</tbody>
</table>
A cross section of the bridge appears below:

![Bridge Cross Section Diagram](image)

Typical Bridge Cross Section

The following data apply to this design:


**Loading:** HS20-44.

**Structural Steel:** ASTM A36 and A572, Grade 50.

**Concrete:** $f_c = 4,000$ psi, modular ratio $n = 8$.

**Slab Reinforcing Steel:** ASTM A615, Grade 40, with $F_y = 40$ ksi.

**Loading Conditions:**

Case 1—Weight of girder and slab (DL₁) supported by the steel girder alone.

Case 2—Superimposed dead load (DL₂) (parapets and railings) supported by the composite section with the modular ratio $n=8$. (Used in design of web-to-flange fillet welds.)

Case 3—Superimposed dead load (DL₂) (parapets and railings) supported by the composite section with the increased modular ratio $3n = 3 	imes 8 = 24$.

Case 4—Live load plus impact (L+I) supported by the composite section with the modular ratio $n = 8$.

Fatigue—500,000 cycles of truck load

Redundant load-path structure.

**Loading Combinations:**

Combination A = Case 1+3+4

Combination B = Case 2+4

Combination C = Case 1+2+4
LOADS, SHEARS AND MOMENTS ON GIR德ERS

The loads on the exterior and interior girders are computed in the same manner as illustrated in Chapters 3, 3A, 4 and 4A.

Exterior Girders

**Dead Load Carried by Steel Section**

- Slab \( \frac{8}{12} \times (3.75 + 4.42) \times 0.150 = 0.817 \)
- Haunch \( 0.17 \times 1.33 \times 0.15 = 0.034 \)
- \( \frac{1}{2} \times 0.30 \times 3.08 \times 0.15 = 0.069 \)
- Girder, diaph (assumed weight) = 0.290

**DL1 for Exterior Girders** = 1.210 k/ft

**Dead Load Carried by Composite Section**

- Parapet \( 0.434 \times 2 \times \frac{1}{4} = 0.217 \)

**Future Wearing Surface** \( 0.025 \times 31.0 \times \frac{1}{4} = 0.194 \)

**DL2 for Exterior Girders** = 0.411 k/ft

**Live Load**

**Live Load Distribution**

\[
\frac{S}{4 + 0.25S} = \frac{8.83}{4 + 0.25(8.83)}
\]

= 1.423 wheels
Interior Girders

Dead Load Carried by Steel Section

<table>
<thead>
<tr>
<th>Item</th>
<th>Calculation</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slab</td>
<td>(\frac{8}{12} \times 8.83 \times 0.15)</td>
<td>0.883</td>
</tr>
<tr>
<td>Haunch</td>
<td>0.17 \times 1.33 \times 0.15</td>
<td>0.034</td>
</tr>
<tr>
<td>Girder, diaph’s (assumed weight)</td>
<td></td>
<td>0.310</td>
</tr>
<tr>
<td>DL&lt;sub&gt;1&lt;/sub&gt; for Interior Girders</td>
<td></td>
<td>1.227 k/ft</td>
</tr>
</tbody>
</table>

Dead Load Carried by Composite Section

Same as Exterior Girders

DL<sub>2</sub> for Interior Girders = 0.411 k/ft

Live Load

Live Load Distribution = \(\frac{S}{S_{5.5}} = \frac{8.83}{5.5}\)

= 1.606 wheels

Centrifugal force is defined by the Specifications as a percentage of the live load, without impact. It is a function of the design speed—taken as 30 miles per hour for this two-lane ramp on a centerline radius of slightly less than 287 ft. The superelevation rate is 0.08 ft per ft, or 8.0 percent. The AASHTO formula (Article 1.2.2d) gives a centrifugal force value of 21.0 percent, but 8.0 percent is balanceable by the superelevation,* leaving 13.0 percent as producing an overturning moment about the mid-depth of the slab. With two lanes of live load on the bridge, the CF force in terms of lanes is 0.260 lanes.

*See earlier section “Loads and Load Combinations.”
S = 30 mph
R = 286.76'

\[ C = \frac{6.68 S^2}{R} = \frac{(6.68)(30)^2}{286.76} = 21.0\% \]

Balanceable by superelevation = (100)(0.08) = \frac{8.0}{13.0\%}

CF = (0.130)(2) = 0.260 lanes

"Pile group distribution" is used to determine the vertical CF loads on the girders due to the overturning moment. The "moment of inertia" of the four girders treated as piles is 389.8 ft². Loads of 0.056 and 0.019 lanes are computed for G1 and G2. The CF vertical loads on G3 and G4 are assumed to be zero since such loads would act upward and be subtractive from the other vertical loads. Using these zero loads represents another loading case, the one corresponding to the situation in which live load is on the structure but not moving.

\[ I_{\text{Pile Group}} = 2 \left[ \left( \frac{8.83}{2} \right)^2 + \left( 1.5 \times 8.83 \right)^2 \right] = 389.8 \text{ ft}^2 \]

\[ CF_{\text{load on G1}} = \frac{(0.260)(6.352)(1.5)(8.83)}{389.8} = 0.056 \text{ lanes} \]

\[ CF_{\text{load on G2}} = \frac{(0.260)(6.352)(0.5)(8.83)}{389.8} = 0.019 \text{ lanes} \]

\[ CF_{\text{load on G3 & G4}} = 0 \text{ lanes, by inspection, assuming S = 0 mph} \]
We now have all the loads needed for an analysis of the curved bridge. At this point we check the central angle criterion to determine whether the secondary bending due to curvature need be considered. If it does not, there is no need to perform a V-Load analysis of the structure; all girders could be analyzed as if they were straight, with lateral flange bending stress computed and added in.

Turning back to the geometry table on page 12, we see that the central angles, $\Delta$, are well beyond the limits of Table 1.4A in the Guide Specifications. Thus, curvature must be considered in all its aspects.

The Computer Program VLOAD is used to perform an analysis for DL1, DL2 and L+I, on the basis of the girder sections shown below.

<table>
<thead>
<tr>
<th>G1</th>
<th>G2</th>
<th>G3</th>
<th>G4</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/8 x 34</td>
<td>7/16 x 34</td>
<td>3/8 x 34</td>
<td>15/16 x 18</td>
</tr>
<tr>
<td>1 x 18</td>
<td>3/8 x 34</td>
<td>7/16 x 34</td>
<td>15/16 x 18</td>
</tr>
<tr>
<td>1/2 x 18</td>
<td>3/8 x 34</td>
<td>7/16 x 34</td>
<td>15/16 x 18</td>
</tr>
<tr>
<td>3/8 x 34</td>
<td>7/16 x 34</td>
<td>3/8 x 34</td>
<td>15/16 x 18</td>
</tr>
</tbody>
</table>

Program VLOAD could also have been used to analyze the CF loading on G1 and G2 but there is a simpler approach. It is recalled that the V-Loads are a function of the summation of primary moment transversely across the structure at each crossframe line. Since the CF loads act downward on G1 and G2, and upward on G3 and G4, the summation of moments is essentially zero and therefore the V-Loads are zero. Thus for centrifugal force loading all that need be analyzed is the primary load effect, which can be done on an isolated straight girder basis.
In order to design Girder G1 the moment and shear curves are plotted as below:

Maximum Moment Curves for Girder G1
Although crossframe spacing has been discussed in a general way, it should be noted that the entire analysis of vertical bending in the structure has been carried out without having actually set the crossframe spacing. One of the advantages of the V-Load method is that the moments, shears, reactions and deflections are independent of the crossframe locations. We must now set the crossframe spacing so that the detailed operations of design can begin.

Recalling that the most important function of the crossframes is to control lateral flange bending stresses, we could make a preliminary estimate of required crossframe spacing by assuming a reasonable ratio of $f_w / f_b$, say 0.15. However, crossframe steel, with its high labor intensity, is expensive compared to flange steel. Current economic considerations suggest using higher ratios of $f_w / f_b$ for structures as sharply curved as this one. If we assume that $f_w$ is 0.30$f_b$, and we are designing for $f_w + f_b = F_y = 50$ ksi, then $1.30 f_b = 50$ ksi and $f_b = 38$ ksi.
An expression for lateral bending stress can then be derived as follows:

Assume \( f_b \) is equal to the flange force divided by the flange area = \( f_b = \frac{M_{\text{vertical}}}{h} + bt \)

where

\[ M_{\text{vertical}} = \text{vertical bending moment} \]
\[ h = \text{depth of girder, center to center of flanges} \]
\[ b = \text{flange width} \]
\[ t = \text{flange thickness} \]

Transposing, \( M_{\text{vertical}} = f_b h b t \).

From page 5 the lateral bending moment,

\[ M_{\text{lateral}} = \frac{M_{\text{vertical}} d^2}{12 R h} \]

Dividing by the lateral section modulus of the flange, we have for lateral bending stress

\[ f_w = \frac{6 M_{\text{vertical}} d^2}{12 R h b^2} \]

If \( M_{\text{vertical}} = f_b h b t \) is substituted, then

\[ f_w = \frac{6 f_b h b t d^2}{12 R h b^2} = \frac{f_b d^2}{2 R b} \]

With \( f_b = 38 \text{ ksi} \) and \( R = 300 \text{ ft} \) the following table giving lateral bending stresses as a function of flange width and diaphragm spacing can be generated.

### Approximate Lateral Bending Stresses, \( f_w, \text{ ksi} \)

<table>
<thead>
<tr>
<th>d, Diaphragm Spacing (ft)</th>
<th>b, Flange Width (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>10</td>
<td>8.4</td>
</tr>
<tr>
<td>12</td>
<td>12.2</td>
</tr>
<tr>
<td>15</td>
<td>19.0</td>
</tr>
<tr>
<td>18</td>
<td>27.4</td>
</tr>
<tr>
<td>21</td>
<td>37.2</td>
</tr>
<tr>
<td>25</td>
<td>52.8</td>
</tr>
</tbody>
</table>

The table shows that the diaphragms must be spaced much more closely in this curved bridge than the nominal 25 ft spacing that would be required for a straight bridge, if the lateral bending stress is to be kept within reasonable limits. We are looking for an \( f_w \) in the range of 11-12 ksi to satisfy \( f_w = 0.30 f_b \) and \( f_b + f_w = 50 \text{ ksi} \). If it is further assumed that an 18-inch bottom flange will be used, we see by scanning the table that the crossframe spacing should be about 16 ft. The framing plan below would appear to satisfy these requirements.
Framing Plan

Crossframe lines are radial except those at the west abutment and at the center pier. Field splice lines pass approximately through the dead load inflection points of the girders, with some adjustment to avoid interference with crossframes.

WEB DESIGN FOR GIRDER G1

For place girder spans of moderate length, current economics dictates a compromise between the thinnest possible web (called a fully-stiffened web) and a completely unstiffened web. Usually a web with stiffeners in the high-shear regions and without stiffeners in lower-shear regions (called a partially-stiffened web) will afford the most economical solution.

In keeping with this concept, a preliminary selection of a $\frac{3}{8}$-inch web is made for the full length of Girder G1. A diagram of maximum shears for Group I loading is plotted, and the shear capacity, $V_u$, for the $\frac{3}{8}$-inch web without stiffeners is superimposed on the diagram.
The unstiffened shear capacity is 157.4 kips, obtained as follows:

Assume \( \min t_w = \frac{7}{16} \) inches

\[
V_u = \frac{(3.5)(29000)(0.4375)^3}{54} = 157.4
\]

The diagram shows that 37 ft of Span 1 and 47 ft of Span 2 may be left unstiffened. A ¾-inch web in the positive moment regions could have been used but would have had an unstiffened shear capacity of only 99.1 kips, which was judged to provide unsatisfactorily short unstiffened regions.

The proposed stiffener arrangement is shown below. The dimensioned heavy lines represent the crossframes and the stiffeners are equally spaced within the crossframe panels.

The most critical region would appear to be the 16.87 ft crossframe panel adjacent to the interior support. With 5 equal stiffener spaces within the panel, the stiffener spacing, \( d_o \), is 40.49 inches, and the resulting shear capacity is 539.3 kips. The applied shear is less than 0.6 of the shear capacity and therefore the moment/shear interaction relationship discussed on p.14 does not have to be considered.
Capacity of $\frac{7}{16}$" web with $d_o = 40.49"$:

$$
C = 18,000 \left( \frac{t_w}{D} \right) \sqrt{\frac{1 + (D/d_o)^2}{F_y}} - 0.3 = 18,000 \left( \frac{0.4375}{54} \right) \sqrt{\frac{1 + (54/40.49)^2}{50,000}} - 0.3
$$

$$
= 0.7871
$$

$$
V_u = 0.58 F_y D t_w C = (0.58)(50)(54)(0.4375)(0.7871) = 539.3 k
$$

$$
0.6V_u = (0.6)(539.3) = 323.6 k > 298.0
$$

Only the panels immediately adjacent to the interior support carry high shear and moment. In all other panels in which stiffeners are required, they are equally spaced to satisfy $d_o \leq D$, and this spacing provides sufficient shear capacity to resist the applied shear. More than enough capacity is available as seen by a check at the left end bearing:

Capacity of $\frac{7}{16}$" web with $d_o = 47.13"$:

$$
C = 18,000 \left( \frac{0.4375}{54} \right) \sqrt{\frac{1 + (54/47.13)^2}{50,000}} - 0.3
$$

$$
= 0.6918
$$

$$
V_u = (0.58)(50)(54)(0.4375)(0.6918)
$$

$$
= 474.0 k > 251.2
$$

**DESIGN OF SECTIONS FOR GIRDER G1**

Now, we can start designing sections for Girder G1. The first section to be considered is the maximum positive moment section in Span 1. The effective width of the 7½-inch thick structural slab is 90 inches. A 2½-inch haunch is provided as illustrated below.

**Haunch Detail**

This haunch furnishes adequate clearance at the tip of the anticipated wider flange plate over the pier, and also accommodates any excess camber within the spans.
THE MAXIMUM POSITIVE SECTION—SPAN 1

The trial section has a 1" x 14" top flange, 7/16" x 5/4" web and 1 1/2" x 18" bottom flange. Section properties are computed for the steel and composite sections using the same procedures illustrated in Chapters 3, 3A, 4 and 4A.

### Steel Section

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top flg. 1 x 14</td>
<td>14.00</td>
<td>27.50</td>
<td>385</td>
<td>10,588</td>
<td>10,588</td>
<td></td>
</tr>
<tr>
<td>Web 7/16 x 5/4</td>
<td>23.63</td>
<td>27.50</td>
<td>749</td>
<td>20,792</td>
<td>20,792</td>
<td></td>
</tr>
<tr>
<td>Bot. Flg. 1 1/2 x 18</td>
<td>27.00</td>
<td>-27.75</td>
<td>749</td>
<td>20,792</td>
<td>20,792</td>
<td></td>
</tr>
</tbody>
</table>

\[
d_s = \frac{-364}{64.63} = 5.63 \text{ in}
\]

\[
d_{\text{Top of Steel}} = 28.00 + 5.63 = 33.63 \text{ in}
\]

\[
d_{\text{Bot. of Steel}} = 28.50 - 5.63 = 22.87 \text{ in}
\]

\[
S_{\text{Top of Steel}} = \frac{35.072}{33.63} = 1.043 \text{ in}^3
\]

\[
S_{\text{Bot. of Steel}} = \frac{22.87}{33.63} = 0.674 \text{ in}^3
\]

### Composite Section, 3n = 24

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>64.63</td>
<td>-364</td>
<td>935</td>
<td>31,099</td>
<td>132</td>
<td>31,231</td>
</tr>
<tr>
<td>Conc. 7 1/2 x 90</td>
<td>28.18</td>
<td>33.25</td>
<td>935</td>
<td>31,099</td>
<td>132</td>
<td>31,231</td>
</tr>
</tbody>
</table>

\[
d_s = \frac{571}{92.76} = 6.16 \text{ in}
\]

\[
d_{\text{Top of Steel}} = 28.00 - 6.16 = 21.84 \text{ in}
\]

\[
d_{\text{Bot. of Steel}} = 28.50 + 6.16 = 34.66 \text{ in}
\]

\[
S_{\text{Top of Steel}} = \frac{64.835}{21.84} = 2.969 \text{ in}^3
\]

\[
S_{\text{Bot. of Steel}} = \frac{64.835}{34.66} = 1.871 \text{ in}^3
\]
Composite Section, \( n = 8 \)

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad^2</th>
<th>I_0</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>64.63</td>
<td>-364</td>
<td>149.01</td>
<td>2,442 in^3</td>
<td>37,121</td>
<td></td>
</tr>
<tr>
<td>Conc. ( \frac{7}{8} ) x 90</td>
<td>84.38</td>
<td>33.25</td>
<td>2,806</td>
<td>93,287</td>
<td>396</td>
<td>93,683</td>
</tr>
</tbody>
</table>

\[ d_s = \frac{2.442}{149.01} = 16.39 \text{ in} \]
\[ -16.39 \times 2.442 = -40.024 \]
\[ 90,780 \text{ in}^4 \]

\[ d_{\text{Top of Steel}} = 28.00 - 16.39 = 11.61 \text{ in} \]
\[ d_{\text{Bot. of Steel}} = 28.50 + 16.39 = 44.89 \text{ in} \]

\[ S_{\text{Top of Steel}} = \frac{90,780}{11.61} = 7,819 \text{ in}^3 \]
\[ S_{\text{Bot. of Steel}} = \frac{90,780}{44.89} = 2,022 \text{ in}^3 \]

\[ S_{\text{Top Flg. Lat.}} = \frac{(1)(14)^2}{6} = 32.7 \text{ in}^3 \]
\[ S_{\text{Bot. Flg. Lat.}} = \frac{(1.5)(18)^2}{6} = 81.0 \text{ in}^3 \]

Moments are tabulated at the 0.4 point of the span, 44 ft from the end support. Lateral bending moments are computed using the formula from p. 5, and vertical and lateral flange bending stresses are calculated.

### Bending Moments 44 ft from End Support

<table>
<thead>
<tr>
<th></th>
<th>DL_1</th>
<th>DL_2</th>
<th>L + I</th>
<th>CF</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_{\text{kip-ft}} )</td>
<td>1,481</td>
<td>561</td>
<td>1,616</td>
<td>30</td>
</tr>
</tbody>
</table>

\[ M_{\text{Top Flg. Lat.}} = \frac{M_{\text{DL}}d^2}{12Rh} = \frac{(1,481)(15.71)^2}{(12)(300)\left(\frac{55.25}{12}\right)} = 22.1 \text{ k-ft} \]

\[ M_{\text{Bot. Flg. Lat.}} = 22.1 \text{ k-ft} \]

\[ M_{\text{Bot. Flg. Lat.}} = \frac{(561)(15.71)^2}{(12)(300)\left(\frac{55.25}{12}\right)} = 8.4 \text{ k-ft} \]

\[ M_{\text{Bot. Flg. Lat.}} = 8.4 \text{ k-ft} \]

\[ M_{\text{Bot. Flg. Lat.}} = \frac{(1,616)(15.71)^2}{(12)(300)\left(\frac{55.25}{12}\right)} = 24.1 \text{ k-ft} \]

\[ M_{\text{Bot. Flg. Lat.}} = 24.1 \text{ k-ft} \]

\[ M_{\text{Bot. Flg. Lat.}} = \frac{(80)(15.71)^2}{(12)(300)\left(\frac{55.25}{12}\right)} = 1.2 \text{ k-ft} \]

\[ M_{\text{Bot. Flg. Lat.}} = 1.2 \text{ k-ft} \]
Steel Stresses Due to Maximum Design Loads

### Top of Steel (Compression) vs. Bottom of Steel (Tension)

#### Vertical Bending:

For DL1: \( f_b = \frac{1481\times12}{1043} \times 1.30 = 22.2 \)  
\( f_b = \frac{1481\times12}{1534} \times 1.30 = 15.1 \)

For DL2: \( f_b = \frac{561\times12}{2969} \times 1.30 = 2.9 \)  
\( f_b = \frac{561\times12}{1871} \times 1.30 = 4.7 \)

For L+I: \( f_b = \frac{1616\times12}{7819} \times 1.30 \times \frac{5}{3} = 5.5 \)  
\( f_b = \frac{1616\times12}{2022} \times 1.30 \times \frac{5}{3} = 21.4 \)

For CF: \( f_b = \frac{80\times12}{7816} \times 1.30 = 0.2 \)  
\( f_b = \frac{80\times12}{2022} \times 1.30 = 0.6 \)

#### Lateral Bending:

For DL1: \( f_w = \frac{22.1\times12}{32.7} \times 1.30 = 10.5 \)  
\( f_w = \frac{22.1\times12}{81.0} \times 1.30 = 4.3 \)

For DL2: \( f_w = 0.0 \)  
\( f_w = \frac{8.4\times12}{81.0} \times 1.30 = 1.6 \)

For L+I: \( f_w = 0.0 \)  
\( f_w = \frac{24.1\times12}{81.0} \times 1.30 \times \frac{5}{3} = 7.7 \)

For CF: \( f_w = 0.0 \)  
\( f_w = \frac{1.2\times12}{10.5} \times 1.30 = 0.2 \)

To determine the allowable stresses, the compression flange geometry is checked and found to satisfy the requirements for compactness. The appropriate equations from the Guide Specifications yield an allowable compression stress of 45.2 ksi, considerably higher than the 30.8 ksi actual average stress. This will be discussed below.

### Allowable Steel Stresses

**Top Flange (Compression):**

\[
\frac{b}{t} = \frac{14}{1} = 14.0
\]

\[
\frac{b}{t} \text{ Max for compact flg} = \frac{3.200}{\sqrt{F_y}} = \frac{3.200}{\sqrt{50,000}} = 14.31 > 14.0
\]

\( . . . \text{ Flange is compact} \)

\[
\bar{\rho}_b = \frac{1}{1 + \frac{1}{b} \left( 1 + \frac{1}{6b} \right) \left( \frac{1}{R} - 0.01 \right)^2}
\]

\[
= \frac{1 + 15.71}{14/12} \left[ 1 + \frac{15.71}{(6)(14/12)} \right] \left( \frac{15.71}{300} - 0.01 \right)^2 = 0.92729
\]

\[
\lambda = \frac{1}{\pi} \left( \frac{1}{b} \right) \sqrt{\frac{F_y}{E}} = \frac{1}{\pi} \left( \frac{15.71}{14/12} \right) \sqrt{\frac{50}{29,000}} = 0.17798
\]

\[
F_{bs} = F_y (1 - 3 \lambda^2) = 50 \left[ 1 - (3)(0.17798)^2 \right] = 45.248
\]

\[
\bar{\rho}_w = 0.95 + 18 \left[ 0.1 - \frac{1}{R} \right]^2 + \frac{f_w}{f_b} \left[ 0.3 - 0.1 \frac{1}{R} \frac{1}{b} \right]
\]

\[
= 0.95 + 18 \left[ 0.1 - \frac{15.71}{300} \right]^2 + \frac{10.5}{30.8} \left[ 0.3 - 0.1 \left( \frac{15.71}{300} \right) \left( \frac{15.71}{14/12} \right) \right]
\]

\[
= 1.08407
\]
\[
\widetilde{\rho}_B \rho_w = (0.92729)(1.08407) = 1.00525 > 1.0, \text{ use } 1.0
\]

\[
F_{bu} = F_{bs} \rho_B \rho_w = (45.248)(1.0) = 45.2 \text{ ksi } > 30.8
\]

The allowable tensile stress in the bottom flange is computed in similar manner as 43.1 ksi. It slightly exceeds the actual average stress of 41.8 ksi.

\[
\widetilde{\rho}_B = \frac{1}{1 + \frac{18}{15.71} \left( \frac{15.71}{6(18/12)} \right) \left( \frac{15.71}{300} - 0.1 \right)^2} = 0.95092
\]

\[
F_{bs} = F_y = 50
\]

\[
\widetilde{\rho}_w = 0.95 + 18 \left[ 0.1 - \frac{15.71}{300} \right]^2 + \frac{13.8}{41.8} \left[ 0.3 - 0.1 \left( \frac{15.71}{300} \right) \left( \frac{15.71}{18/12} \right) \right] = 0.90573
\]

\[
\widetilde{\rho}_B \rho_w = (0.95092)(0.90573) = 0.86128 < 1.0
\]

\[
F_{bu} = (50)(0.86128) = 43.1 \text{ ksi } > 41.8
\]

While there appears to be inefficiency of the top flange (30.8 ksi actual stress vs 45.2 ksi allowable stress), we should consider what would happen if the top flange were made thinner and thereby become a noncompact compression flange. The following material and stresses would result:

<table>
<thead>
<tr>
<th>Material</th>
<th>Bottom Flange Stress</th>
<th>Top Flange Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Flange</td>
<td>15/16 x 16</td>
<td>Actual 24.6 ksi</td>
</tr>
<tr>
<td>Web</td>
<td>7/16 x 54</td>
<td>Allowable 24.9 ksi</td>
</tr>
<tr>
<td>Bottom Flange</td>
<td>27/8 x 18</td>
<td></td>
</tr>
</tbody>
</table>

The design with a non-compact compression flange has all its elements stressed to the maximum and has the appearance of being very efficient. However, the allowable stresses are much lower than for the section with a compact compression flange. With the compact compression flange, the total cross-sectional area is 64.63 in² and with the non-compact compression flange the cross-sectional area is 90.38 in². Thus, the seemingly inefficient section with the compact compression flange requires only 72 percent as much steel as the section with the non-compact compression flange. As is noted in the introduction, compact compression flange sections are usually (but not always) more economical.

**THE MAXIMUM NEGATIVE MOMENT SECTION**

We next design the maximum negative moment section over the interior support. The area of longitudinal reinforcing steel within the effective slab width is 7.48 inches². A steel section with a 1 1/4" x 18" top flange, 7/16" x 54" web and 1 3/8" x 18" bottom flange is investigated.
Properties for the steel section alone and the steel section plus reinforcement are computed.

### Steel Section at Interior Support

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Flg. 1½ x 18</td>
<td>22.50</td>
<td>27.63</td>
<td>622</td>
<td>17,177</td>
<td>17,177</td>
<td></td>
</tr>
<tr>
<td>Web 1/8 x 54</td>
<td>23.63</td>
<td></td>
<td></td>
<td></td>
<td>5,741</td>
<td>5,741</td>
</tr>
<tr>
<td>Bot. Flg. 1½ x 18</td>
<td>24.75</td>
<td>-27.69</td>
<td>-685</td>
<td>18,977</td>
<td>18,977</td>
<td></td>
</tr>
</tbody>
</table>

\[
d_s = \frac{-63}{70.88} = 0.89 \text{ in}
\]

\[
d_{\text{Top of Steel}} = 28.25 + 0.89 = 29.14 \text{ in}
\]

\[
S_{\text{Top of Steel}} = \frac{41,889}{29.14} = 1,436 \text{ in}^3
\]

\[
d_{\text{Bot. of Steel}} = 28.38 - 0.89 = 27.49 \text{ in}
\]

\[
S_{\text{Bot. of Steel}} = \frac{41,889}{27.49} = 1,522 \text{ in}^3
\]

\[
70.88 \text{ in}^2 - 63 \text{ in}^3 = 41,895
\]

\[
I_{NA} = 0.89 \times 63 = -56
\]

### Steel Section with Reinforcing Steel at Interior Support

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>70.88</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reinforcement</td>
<td>7.48</td>
<td>33.08</td>
<td>247</td>
<td>8,185</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
78.36 \text{ in}^2 \quad 184 \text{ in}^3 \quad 50,080
\]

\[
d_s = \frac{184}{78.36} = 2.35 \text{ in}
\]

\[
d_{\text{Top of Steel}} = 28.25 - 2.35 = 25.90 \text{ in}
\]

\[
S_{\text{Top of Steel}} = \frac{49,648}{25.90} = 1,917 \text{ in}^3
\]

\[
d_{\text{Bot. of Steel}} = 28.38 + 2.35 = 30.73 \text{ in}
\]

\[
S_{\text{Bot. of Steel}} = \frac{49,648}{30.73} = 1,616 \text{ in}^3
\]

\[
d_{\text{Reinf.}} = 33.08 - 2.35 = 30.73 \text{ in}
\]

\[
S_{\text{Reinf.}} = \frac{49,648}{30.73} = 1,616 \text{ in}^3
\]

\[
S_{\text{Top Flg. Lat.}} = \frac{(1.25)(18)^2}{6} = 67.5 \text{ in}^3
\]

\[
S_{\text{Bot. Flg. Lat.}} = \frac{(1.25)(18)^2}{6} = 74.5 \text{ in}^3
\]

The tabulation and computations below give all the vertical and lateral bending moments needed to compute stresses in the section. For lateral bending stresses at an interior support, a fraction-of-moment (FOM) parameter is used to modify the lateral bending effect. This is because the derivation of the V-Load theory is based on a smoothly curved moment diagram. The sharply peaked moment that occurs at an interior support, used as is, would unduly and irrationally penalize the design. A moment is used in lieu of the actual peak moment, which is the average of the peak moment and the moments at the crossframes immediately adjacent on either side of the support in question. The FOM value is the ratio of the average moment to the peak moment.
### Bending Moments at Interior Support

<table>
<thead>
<tr>
<th>DL₁</th>
<th>DL₂</th>
<th>+ (L + I)</th>
<th>+ CF</th>
<th>- (L + I)</th>
<th>- CF</th>
</tr>
</thead>
<tbody>
<tr>
<td>M₁</td>
<td>M₂</td>
<td>+ (L + I)</td>
<td>+ CF</td>
<td>- (L + I)</td>
<td>- CF</td>
</tr>
<tr>
<td>-2,068</td>
<td>-646</td>
<td>0</td>
<td>0</td>
<td>-1,064</td>
<td>-33</td>
</tr>
</tbody>
</table>

\[ M_{\text{Top Flg., Lat}_{DL₁}} = \frac{(2,068)(16.87)^2}{(12)(300)\left(\frac{55.31}{12}\right)} = 35.5 \text{ k-ft} \]

\[ M_{\text{Bot Flg., Lat}_{DL₁}} = \frac{(33)(16.87)^2}{(12)(300)\left(\frac{55.31}{12}\right)} = 0.6 \text{ k-ft} \]

\[ M_{\text{Bot Flg., Lat}_{DL₂}} = 35.5 \text{ k-ft} \]

\[ M_{\text{Bot Flg., Lat}_{DL₁}} = \frac{(996)(16.87)^2}{(12)(300)\left(\frac{55.31}{12}\right)} = 11.1 \text{ k-ft} \]

\[ M_{\text{Bot Flg., Lat}_{L+I}} = \frac{(1,064)(16.87)^2}{(12)(300)\left(\frac{55.31}{12}\right)} = 18.2 \text{ k-ft} \]

### Fraction of Moment (FOM) Calculation

<table>
<thead>
<tr>
<th>Crossframe Location</th>
<th>Span 1 0.8571</th>
<th>Pier</th>
<th>Span 2 0.1887</th>
</tr>
</thead>
<tbody>
<tr>
<td>M₁</td>
<td>750 k-ft</td>
<td>2,068 k-ft</td>
<td>914 k-ft</td>
</tr>
<tr>
<td>M₂</td>
<td>186</td>
<td>646</td>
<td>247</td>
</tr>
<tr>
<td>M₁+I</td>
<td>536</td>
<td>1,064</td>
<td>588</td>
</tr>
<tr>
<td>CF</td>
<td>28</td>
<td>33</td>
<td>28</td>
</tr>
<tr>
<td>M_{TOTAL}</td>
<td>1,500 k-ft</td>
<td>3,811 k-ft</td>
<td>1,777 k-ft</td>
</tr>
</tbody>
</table>

Avg. moment = \( \frac{1,500 + 3,811 + 1,777}{3} \) = 2,363 k-ft

FOM = \( \frac{2,363}{3,811} \) = 0.620

Stresses at top and bottom of steel are computed below.

### Steel Stresses Due to Maximum Design Loads

#### Top of Steel (Tension)

<table>
<thead>
<tr>
<th> </th>
<th>Top of Steel (Tension)</th>
<th>Bottom of Steel (Compression)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Vertical Bending:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>For DL₁: ( f_b = \frac{2,068 \times 12}{1,496} \times 1.30 = 22.5 )</td>
<td>( f_t = \frac{2,068 \times 12}{1,522} \times 1.30 = 21.2 )</td>
<td></td>
</tr>
<tr>
<td>For DL₂: ( f_b = \frac{646 \times 12}{1,917} \times 1.30 = 5.3 )</td>
<td>( f_t = \frac{646 \times 12}{1,616} \times 1.30 = 6.2 )</td>
<td></td>
</tr>
<tr>
<td>For L+I: ( f_b = \frac{1,064 \times 12}{1,917} \times 1.30 \times \frac{5}{3} = 14.4 )</td>
<td>( f_t = \frac{1,064 \times 12}{1,616} \times 1.30 \times \frac{5}{3} = 17.1 )</td>
<td></td>
</tr>
<tr>
<td>For CF: ( f_b = \frac{33 \times 12}{1,917} \times 1.30 = 0.3 )</td>
<td>( f_t = \frac{33 \times 12}{42.5 \text{ ksi}} \times 1.30 = 0.3 )</td>
<td>44.8 ksi</td>
</tr>
</tbody>
</table>

#### Lateral Bending:

<table>
<thead>
<tr>
<th> </th>
<th>Top of Steel (Tension)</th>
<th>Bottom of Steel (Compression)</th>
</tr>
</thead>
<tbody>
<tr>
<td>For DL₁: ( f_w = \frac{35.5 \times 12 \times 0.620}{67.5} \times 1.30 = 5.1 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>For DL₂: ( f_w = 0.0 )</td>
<td>( f_t = \frac{11.1 \times 12 \times 0.620}{74.5} \times 1.30 = 1.4 )</td>
<td></td>
</tr>
<tr>
<td>For L+I: ( f_w = 0.0 )</td>
<td>( f_t = \frac{18.2 \times 12 \times 0.620}{74.5} \times 1.30 \times \frac{5}{3} = 3.9 )</td>
<td></td>
</tr>
<tr>
<td>For CF: ( f_w = 0.0 )</td>
<td>( f_t = \frac{0.6 \times 12 \times 0.62}{74.5} \times 1.30 = 0.1 )</td>
<td>10.0 ksi</td>
</tr>
</tbody>
</table>
Allowable stresses are computed from the Guide Specification formulas and found to be satisfactory. The compression flange is again compact.

**Allowable Steel Stresses**

**Bottom Flange (Compression):**

\[
\frac{b}{t} = \frac{18}{1.375} = 13.1 < 14.31 \cdot \cdot \cdot \text{ Flange is compact}
\]

\[
\bar{\rho}_b = \frac{1}{1 + \frac{1}{b} \left(1 + \frac{1}{6b}\right) \left(\frac{1}{R} - 0.01\right)^2}
\]

\[
= \frac{1}{1 + \frac{16.87}{18/12} \left(1 + \frac{16.87}{(6)(18/12)}\right) \left(\frac{16.87}{300} - 0.01\right)^2} = 0.93536
\]

\[
\lambda = \frac{1}{\pi} \left(\frac{1}{b}\right) \sqrt{\frac{F_y}{E}} = \frac{1}{\pi} \left(\frac{16.87}{18/12}\right) \sqrt{\frac{50}{29,000}} = 0.14865
\]

\[
F_{bs} = F_y (1 - 3 \lambda^2) = 50 \left[1 - (3)(0.14865)^2\right] = 46.685
\]

\[
\bar{\rho}_w = 0.95 + 18 \left[0.1 - \frac{1}{R}\right]^2 + \frac{f_w}{f_b} \left[0.3 - 0.1 \left(\frac{1}{R}\right)\right]
\]

\[
= 0.95 + 18 \left[0.1 - \frac{16.87}{300}\right]^2 + \frac{10.0}{44.8} \left[0.3 - 0.1 \left(\frac{16.87}{300}\right) \left(\frac{16.87}{18/12}\right)\right] = 1.04499
\]

\[
\bar{\rho}_b \bar{\rho}_w = (0.93536)(1.04499) = 0.97744 < 1.0
\]

\[
F_{bu} = F_{bs} \bar{\rho}_b \bar{\rho}_w = (46.685)(0.97744) = 45.6 \text{ ksi} > 44.8
\]

**Top Flange (Tension)**

\[
\bar{\rho}_b = 0.93536
\]

\[
F_{bs} = F_y = 50
\]

\[
\bar{\rho}_w = 0.95 + 18 \left[0.1 - \frac{16.87}{300}\right]^2 + \left(\frac{5.1}{42.5}\right) \left[0.3 - 0.1 \left(\frac{16.87}{300}\right) \left(\frac{16.87}{18/12}\right)\right]
\]

\[
= 0.95411
\]

\[
\bar{\rho}_b \bar{\rho}_w = (0.93536)(0.95411) = 0.89244 < 1.0
\]

\[
F_{bu} = (50)(0.89244) = 44.6 \text{ ksi} > 42.5
\]

Finally the Stress range in the reinforcement is computed and seen to be much lower than the 20,000 psi allowable range.

**Stress Range in Reinforcement Due to Service Loads**

For \(-(L+1):\)

\[
\frac{1.064 \times 12}{1.616} = 7.9
\]

For \(-(CF):\)

\[
\frac{33 \times 12}{1.616} = \frac{0.2}{8.1} \text{ ksi} < 20.00
\]

Though not illustrated, all sections of all four girders were designed two ways: 1) with compact compression flange; 2) with non-compact compression flange. The design used was the one with the least cross-sectional area and therefore least steel. The section with
compact compression flange was the most economical in nearly all cases. This is the reason the flange material of Girders G2, G3 and G4 may not appear to follow typical patterns associated with straight girders.

In addition to the computations for maximum strength, stress ranges under service loading must be checked against allowable fatigue stress ranges for the details at hand. For any detail located on the girder flange at some distance away from the web (such as a stiffener-to-flange weld), the computed stress range must include lateral bending stress. Fatigue checks are not illustrated here because they are similar in principle to those shown in other chapters.

The calculations for Overload are not shown here either. As with straight girders, Overload does not govern the design of curved plate girders of normal proportions.

The computation of allowable stresses for curved girders by the Guide Specification formulas is a tedious and time-consuming procedure. It is virtually imperative that such operations be carried out either with a programmable calculator or computer assistance.

**TRANSVERSE WEB STIFFENER DESIGN**

Transverse web stiffeners are designed by the rules outlined in the introduction. The rigidity requirement is a function of the stiffener spacing, web thickness, radius of curvature and web depth. It is checked first at the largest stiffener spacing, \( d_0 = 49.95 \) in.

For \( d_0 = 49.95 \):

\[
\frac{d_0}{D} = \frac{49.95}{54} = 0.925 \quad 0.78 < 0.925 < 1.0
\]

\[
Z = \frac{(0.75)(49.95)^2}{(300)(12)(0.4375)} = 1.505
\]

\[
X = 1 + \left[ \frac{(49.95/54) - 0.78}{1,775} \right] (1.505)^4 = 1.00042
\]

\[
J = \left[ 2.5 \left( \frac{54}{49.95} \right)^2 - 2 \right] 1.00042 = 0.92222 > 0.5
\]

\[
I = (49.95)(0.4375)^3(0.92222) = 3.86 \text{ in.}^4
\]

Since we want to use the same size stiffeners throughout the structure, we must consider the other three girders G2, G3 and G4. We now check the smallest stiffener space, 30.36 inches, located on Girder G2.

For \( d_0 = 30.36 \):

\[
\frac{d_0}{D} = \frac{30.36}{54} = 0.562 < 0.78
\]

\[
J = \left[ 2.5 \left( \frac{54}{30.36} \right)^2 - 2 \right] (1) = 5.909
\]

\[
I = (30.36)(0.4375)^3(5.909) = 15.02 \text{ in.}^4
\]

Because the structure is to be painted, A36 steel is chosen for the stiffeners. A 3/8" x 5" stiffener is satisfactory. For the crossframe connection plates the size chosen is 3/8" x 7" to accommodate the crossframe connection.
Use A36 steel for stiffeners

Try \( \frac{3\text{"}}{8} \times 5\text{"} \) for intermediate stiffeners:

\[
\frac{b}{t} = \frac{5}{0.375} = 13.33
\]

\[
\frac{b}{t \text{ Allow}} = \frac{2,600}{\sqrt{F_y}} = \frac{2,600}{\sqrt{36,000}} = 13.70 > 13.33
\]

\[b_{\text{Min}} = \frac{1}{4} \text{ max. flg. width} = \left( \frac{1}{4} \right) (18) = 4.5" < 5\]

\[t_{\text{Min}} = \frac{b}{16} = \frac{5"}{16} < \frac{3}{8}\]

\[I_{\text{Furn'd}} = \frac{1}{3} t b^3 = \left( \frac{1}{3} \right) \left( \frac{3}{8} \right) (5)^3 = 15.63 \text{ in.}^4 > 15.02\]

Use \( \frac{3\text{"}}{8} \times 5\text{"} \) stiffeners

Try \( \frac{9\text{"}}{16} \times 7\text{"} \) for crossframe connection plates:

\[
\frac{b}{t} = \frac{7}{0.5625} = 12.44 < 13.70
\]

Use \( \frac{9\text{"}}{16} \times 7\text{"} \) connection plates

**SHEAR CONNECTOR SPACING**

Shear connector spacing is first computed for fatigue using exactly the same procedure that would be used for a straight bridge. Two \( \frac{3}{8} \text{ in.} \) diameter by 4 in. long studs per row are chosen. The fatigue strength, \( Z_r \), per connector is equal to \((10.6)(0.875)^2 = 8.12 \text{ kips.}\)

<table>
<thead>
<tr>
<th>Dist. from Live Load</th>
<th>Positive Shear (kips)</th>
<th>Negative Shear (kips)</th>
<th>Shear Range (kips)</th>
<th>( Q ) (kip)</th>
<th>( I ) (in(^4))</th>
<th>( S_r = \frac{VQ}{I} ) (kip)</th>
<th>Specg = ( \frac{2Z_r}{S_r} ) (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>End Brg.</td>
<td>Shear Load</td>
<td>Live Load</td>
<td>Shear Range</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>60.1+3.1 = 63.2</td>
<td>-7.2-0.3 = -7.5</td>
<td>70.7</td>
<td>1,215</td>
<td>74,137</td>
<td>1.16</td>
<td>14.0</td>
</tr>
<tr>
<td>0.1L</td>
<td>56.5+2.7 = 59.2</td>
<td>-5.3-0.4 = -5.7</td>
<td>64.9</td>
<td>1,215</td>
<td>74,137</td>
<td>1.06</td>
<td>15.3</td>
</tr>
<tr>
<td>0.2L</td>
<td>49.6+2.2 = 51.8</td>
<td>-7.4-0.7 = -8.1</td>
<td>59.9</td>
<td>1,215</td>
<td>74,137</td>
<td>0.98</td>
<td>16.6</td>
</tr>
<tr>
<td>0.3L</td>
<td>40.7+1.8 = 42.5</td>
<td>-13.4-0.9 = -14.3</td>
<td>56.8</td>
<td>1,508</td>
<td>90,820</td>
<td>0.94</td>
<td>17.3</td>
</tr>
<tr>
<td>0.4L</td>
<td>30.7+1.4 = 32.1</td>
<td>-23.6-1.3 = -24.9</td>
<td>57.0</td>
<td>1,508</td>
<td>90,820</td>
<td>0.95</td>
<td>17.1</td>
</tr>
<tr>
<td>0.5L</td>
<td>20.9+1.1 = 22.0</td>
<td>-34.7-1.7 = -36.4</td>
<td>58.4</td>
<td>1,508</td>
<td>90,820</td>
<td>0.97</td>
<td>16.7</td>
</tr>
<tr>
<td>0.6L</td>
<td>11.9+0.7 = 12.6</td>
<td>-44.5-2.1 = -46.6</td>
<td>59.2</td>
<td>1,508</td>
<td>90,820</td>
<td>0.98</td>
<td>16.6</td>
</tr>
<tr>
<td>0.7L</td>
<td>5.2+0.5 = 5.7</td>
<td>-56.2-2.5 = -55.1</td>
<td>60.8</td>
<td>1,215</td>
<td>74,137</td>
<td>1.00</td>
<td>16.2</td>
</tr>
<tr>
<td>0.8L</td>
<td>1.2+0.2 = 1.4</td>
<td>-58.6-3.0 = 61.6</td>
<td>63.0</td>
<td>260</td>
<td>88,998</td>
<td>0.18</td>
<td>90.1</td>
</tr>
<tr>
<td>0.9L</td>
<td>-0.1+0.1 = 0.0</td>
<td>-61.5-3.4 = 64.9</td>
<td>64.9</td>
<td>260</td>
<td>88,998</td>
<td>0.19</td>
<td>85.4</td>
</tr>
<tr>
<td>L</td>
<td>0.0+0.0 = 0.0</td>
<td>-62.1-3.8 = 65.9</td>
<td>65.9</td>
<td>237</td>
<td>89,833</td>
<td>0.17</td>
<td>95.4</td>
</tr>
</tbody>
</table>

**Diagram**

- Theoretical Spacing
- Required Spacing (fatigue)
- Required Spacing (Ult. Str.)
- Dist. from End
- End Bearing
- Pier

1/6/32 1/86
The shear connector spacing as determined for fatigue is now checked for ultimate strength. The ultimate strength of an individual connector is 37.93 kips and the governing load on the connector group (in the positive moment region of Span 1) is 2,295 kips.

**Ultimate Strength Check for Shear Connectors—Girder G1—Span 1**

\[ S_u = 0.4d^2 \sqrt{F_c E_s} = \frac{0.4(7''e)^2}{1,000} \sqrt{4,000(150)^{5/3}} \approx \frac{33}{4,000} = 37.93 \text{ k} \]

\[ P_1 = 0.85f_{lc} = 0.85(6.0)(90)(7.50) = 2,295 \text{ k} \text{ Goveurs} \]

\[ A_s = (1\times14) + (7/8\times54) + (1\frac{1}{2}\times18) = 64.625 \text{ in}^2 \]

\[ P_2 = A_s f_y = (64.625)(50) = 3,231 \text{ k} \]

\[ P' = A_{sa} f_y = (7.48)(40) = 299 \text{ k} \]

The subtended angle from the maximum positive moment section to the end bearing is 8.40° and the number of connectors needed for fatigue is 76.

The resulting connector force is just within the ultimate strength of the connector and the 76 studs are adequate.
\[ K = 0.166 \left( \frac{N}{N_s} - 1 \right) + 0.375 = 0.166 \left( \frac{76}{2} - 1 \right) + 0.375 = 6.517 \]

\[ F = \frac{P(1-\cos \theta)}{4KN_s \sin \theta/2} = \frac{(2.295)(1-\cos 8.40^\circ)}{4(6.517)(2)(\sin 8.40^\circ/2)} = 6.448 \]

\[ P = \frac{P}{N} = \frac{2.295}{76} = 30.20 \]

\[ P_e = \sqrt{P^2 + F^2 + 2P \cdot F \cdot \sin \frac{\theta}{2}} \]

\[ = \sqrt{(30.20)^2 + (6.448)^2 + (2)(30.20)(6.448) \left( \sin \frac{8.40^\circ}{2} \right)} \]

\[ = 31.34^k \]

\[ P_{ult} = (0.85)(37.93) = 32.24^k > 31.34 \]

For the 41 ft from the maximum moment section to the inflection point the subtended angle is 7.88° and the number of connectors needed for fatigue is 64. In this case, the connector force exceeds the connector ultimate strength, and more studs must be added.

\[ K = 0.166 \left( \frac{64}{2} - 1 \right) + 0.375 = 5.521 \]

\[ F = \frac{(2.295)(1-\cos 7.88^\circ)}{(4)(5.521)(2) \left( \sin \frac{7.88^\circ}{2} \right)} = 7.095 \]

\[ P = \frac{2.295}{64} = 35.86 \]

\[ P_e = \sqrt{35.86^2 + (7.095)^2 + (2)(35.86)(7.095) \left( \sin \frac{7.88^\circ}{2} \right)} = 37.03^k \]

\[ P_{ult} = 32.24^k < 37.03 \]

The number of studs furnished is increased to 76 and the connector force for ultimate strength re-calculated. The results are satisfactory.

\[ \therefore \quad \text{Let } N_s = \frac{37.03}{32.24} (64) = 73.5 \quad \text{Say 76} \]

\[ K = 6.517 \]

\[ F = (7.095) \left( \frac{5.521}{6.517} \right) = 6.011 \]

\[ P = 30.20 \]

\[ P_e = \sqrt{(30.20)^2 + (6.011)^2 + (2)(30.20) \left( \sin \frac{7.88^\circ}{2} \right)} = 30.86^k < 32.24 \]

Calculations for the remainder of the girder are similar and are not shown to avoid repetition. The ultimate strength requirement dictates an increase in the number of studs beyond that needed for fatigue in several regions. These are indicated by dashed lines and bracketed spacings on the diagrams on pages 32 and 33.

**BEARING STIFFENERS**

Bearing stiffeners are designed by the same procedures as illustrated in Chapters 4A and 5. Calculations are not shown here.
FIELD SPLICES

The high-strength bolted field splice is designed in the same manner as splices in earlier chapters, on the basis that only lateral flange bending would make a curved girder splice differ from a straight girder splice. With splices located at inflection points where the vertical bending moment is low, lateral bending moments would be correspondingly low. It is assumed that lateral flange bending can be accounted for simply by conservative proportioning of fasteners and splice plates, and does not justify the added complication of a detailed analysis in this case.

CROSSFRAME DESIGN

An intermediate crossframe at the 0.429 point of Span 1 is selected to illustrate analysis and design of this element of the curved bridge. The analysis is broken into two parts—the curvature effect and the wind effect. For the curvature effect it is convenient to tabulate the calculations. The theory is outlined in the Introduction.

<table>
<thead>
<tr>
<th>Design of Intermediate Crossframe at 0.429 Pt., Span 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Girder/Description</td>
</tr>
<tr>
<td>---------------------</td>
</tr>
<tr>
<td>G1</td>
</tr>
<tr>
<td>M_{DL1} + DL_{2}</td>
</tr>
<tr>
<td>M_{L1+I}</td>
</tr>
<tr>
<td>M_{CFy}</td>
</tr>
<tr>
<td>G2</td>
</tr>
<tr>
<td>M_{DL1} + DL_{2}</td>
</tr>
<tr>
<td>M_{L1+I}</td>
</tr>
<tr>
<td>M_{CFy}</td>
</tr>
<tr>
<td>G3</td>
</tr>
<tr>
<td>M_{DL1} + DL_{2}</td>
</tr>
<tr>
<td>M_{L1+I}</td>
</tr>
<tr>
<td>M_{CFy}</td>
</tr>
<tr>
<td>G4</td>
</tr>
<tr>
<td>M_{DL1} + DL_{2}</td>
</tr>
<tr>
<td>M_{L1+I}</td>
</tr>
<tr>
<td>M_{CFy}</td>
</tr>
<tr>
<td>Σ1.3[D+ ½(L+I)] + CFy</td>
</tr>
<tr>
<td>Σ1.3[D]</td>
</tr>
<tr>
<td>Σ1.3[D+(L+I)+CFy]</td>
</tr>
</tbody>
</table>

\[d = 15.71\text{ ft.}\]
\[R = 300\text{ ft.}\]
\[D = 26.5\text{ ft.}\]
\[C = 1\%\] (See Highway Structures Design Handbook, Vol. I, Chap. 12)

Calculation of member forces for Group I loading is as follows:

**Group I Loading: 1.3 [D + ½(L+I)+CFy]**

\[V = \frac{\Sigma M}{CRD/d} = \frac{20,082}{(1\%) (300) (26.5)/15.71} = 55.7^k\]

\[\frac{V}{3} = \frac{35.7}{3} = 11.9^k\]

\[M_1 = 6,260\text{ k-ft.} \]
\[\frac{M_{d}}{R} = \frac{(6,260)(15.71)}{300} = 327.8 \text{ k-ft.}\]

\[M_2 = 5,656\text{ k-ft.} \]
\[\frac{M_{d}}{R} = \frac{(5,656)(15.71)}{300} = 296.2 \text{ k-ft.}\]

\[M_3 = 4,569\text{ k-ft.} \]
\[\frac{M_{d}}{R} = \frac{(4,569)(15.71)}{300} = 239.3 \text{ k-ft.}\]

\[M_4 = 3,598\text{ k-ft.} \]
\[\frac{M_{d}}{R} = \frac{(3,598)(15.71)}{300} = 188.4 \text{ k-ft.}\]
Forces on Girders:

Forces on Crossframes:

Internal Forces in Crossframes:

\[
\begin{align*}
\frac{327.8}{3.58} &= 91.6^k \\
\frac{309.0}{3.58} &= 86.3^k \\
\frac{126.8}{3.58} &= 35.4^k \\
\frac{12.8}{3.58} &= 3.6^k \\
\frac{112.5}{3.58} &= 31.4^k \\
\frac{188.4}{3.58} &= 52.6^k
\end{align*}
\]

Next, member forces for Group II loading are computed.

**Group II Loading: 1.3 [D]**

\[
V = \frac{8,086{\text{(10)}}(300)(26.5)/15.71}{3} = 14.4^k
\]

\[
\frac{V}{3} = \frac{14.4}{3} = 4.8^k
\]

\[
M_1 = 2,655 \text{ k-ft.} \\
\frac{M_1 d}{R} = \frac{(2,655)(15.71)}{300} = 139.0 \text{ k-ft.}
\]

\[
M_2 = 2,289 \text{ k-ft.} \\
\frac{M_2 d}{R} = \frac{(2,289)(15.71)}{300} = 119.9 \text{ k-ft.}
\]

\[
M_3 = 1,815 \text{ k-ft.} \\
\frac{M_3 d}{R} = \frac{(1,815)(15.71)}{300} = 95.0 \text{ k-ft.}
\]

\[
M_4 = 1,327 \text{ k-ft.} \\
\frac{M_4 d}{R} = \frac{(1,327)(15.71)}{300} = 69.5 \text{ k-ft.}
\]
Forces on Girders:

Forces on Crossframes:

Internal Forces on Crossframes:

\[
\begin{align*}
\frac{139.0}{3.58} &= 38.8^k \\
\frac{131.7}{3.58} &= 36.8^k \\
\frac{57.7}{3.58} &= 16.1^k \\
\frac{11.8}{3.58} &= 3.3^k \\
\frac{37.3}{3.58} &= 10.4^k \\
\frac{69.5}{3.58} &= 19.4^k
\end{align*}
\]

Finally, forces are analyzed under Group III loading.

**Group III Loading:** \(1.3 \, [D + (L+1) + CF_j]\)

\[
V = \frac{15,340}{(10^9)(300)(26.5)/15.71} = 27.3^k
\]

\[
\frac{V}{3} = \frac{27.3}{3} = 9.1^k
\]

\[
M_1 = 4,859 \, \text{k-ft.} \quad \frac{M_1 \cdot d}{R} = \frac{(4,859)(15.71)}{300} = 254.4 \, \text{k-ft.}
\]

\[
M_2 = 4,324 \, \text{k-ft.} \quad \frac{M_2 \cdot d}{R} = \frac{(4,324)(15.71)}{300} = 226.4 \, \text{k-ft.}
\]

\[
M_3 = 3,467 \, \text{k-ft.} \quad \frac{M_3 \cdot d}{R} = \frac{(3,467)(15.71)}{300} = 181.6 \, \text{k-ft.}
\]

\[
M_4 = 2,690 \, \text{k-ft.} \quad \frac{M_4 \cdot d}{R} = \frac{(2,690)(15.71)}{300} = 140.9 \, \text{k-ft.}
\]
Forces on Girders:

Forces on Crossframes:

Internal Forces in Crossframes:

\[
\frac{254.4}{3.58} = 71.1^k \quad \frac{239.7}{3.58} = 67.0^k \quad \frac{100.2}{3.58} = 28.0^k \\
\frac{13.3}{3.58} = 3.7^k \quad \frac{81.4}{3.58} = 22.7^k \quad \frac{140.7}{3.58} = 39.4^k
\]

In addition to the forces from curvature, the intermediate crossframes must transfer the panel wind force from the lower half of the structure up to the slab. The resulting member forces are relatively small but nonetheless serve to complete the Group II and III loading cases. For computational purposes they are shown to three decimal places, then rounded to one decimal place for consistency with the forces due to curvature.

Wind Loading, \( W_{Bp} \)
Internal Forces in Crossframes:

Crossframe member design loads are summarized in the table below.

<table>
<thead>
<tr>
<th>Member</th>
<th>Group I</th>
<th>Group II</th>
<th>Group III</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$1.3(D+%(L+I)+CF_v)$</td>
<td>$1.3(D+W_{BP})$</td>
<td>$1.3(D+(L+I)+CF_v)+0.3W_{BP}$</td>
</tr>
<tr>
<td>Top Strut</td>
<td>T 91.6 k</td>
<td>38.8+(1.3)(0.567) = 39.5 k</td>
<td>71.1+(1.3)(0.3)(0.567) = 71.3 k</td>
</tr>
<tr>
<td></td>
<td>C -52.6 k</td>
<td>-19.4+(1.3)(-0.567) = -20.1 k</td>
<td>-39.4+(1.3)(0.3)(-0.567) = -39.6 k</td>
</tr>
<tr>
<td>Diagonal</td>
<td>T 75.7 k</td>
<td>30.4+(1.3)(0.729) = 31.3 k</td>
<td>57.7+(1.3)(0.3)(0.729) = 58.0 k</td>
</tr>
<tr>
<td></td>
<td>C -75.7 k</td>
<td>-30.4+(1.3)(-0.729) = -31.3 k</td>
<td>-57.7+(1.3)(0.3)(-0.729) = -58.0 k</td>
</tr>
<tr>
<td>Bot. Strut</td>
<td>T 8.6 k</td>
<td>1.6+(1.3)(-0.567) = 0.9 k</td>
<td>57.7+(1.3)(0.3)(0.567) = 5.5 k</td>
</tr>
<tr>
<td></td>
<td>C -47.6 k</td>
<td>-21.0+(1.3)(-2.833) = -24.7 k</td>
<td>-37.4+(1.3)(0.3)(-2.833) = -38.5 k</td>
</tr>
</tbody>
</table>

Inspection of the table reveals that the Group I loading combination governs the design of all members. The top strut must be proportioned for a compressive force of 52.6 kips and a tensile force of 91.6 kips. A WT5x15 section is checked first as a compression member. Gross and net areas are computed; the net area being the area of the section with half of the flange coped at the end connection. The critical $\frac{K_l}{r}$ is that about the y-axis of the member, for which K1 is taken as the full length. The capacity of the member is calculated as the smaller value of

$$P_u = 0.85 A_{gross} F_{cr}$$

or

$$P_u = 0.85 A_{net} F_y$$

The critical buckling stress, $F_{cr}$, is that defined in Article 1.7.69(A) of the Specifications. To guard against local buckling, the width to thickness ratio of the relatively slender stem of the WT section is checked against

$$\frac{b'}{t} \leq \frac{2.200}{\sqrt{F_y}} \sqrt{\frac{P_u}{P}}.$$
Top Strut Design

Try WT5x15  ASTM A572 Grade 50

As compression member:

\[ p = 52.6^k \]

\[ A_{g\text{gross}} = 4.42 \text{ in}^2 \]

\[ A_{\text{snet}} = 4.42 - \left( \frac{5.81 - 0.30}{2} \right) (0.51) = 3.01 \text{ in}^2 \]

\[ \frac{Kl}{r_x} = \frac{(4.42)(12)}{1.45} = 36.6 < 120 \]

\[ \frac{Kl}{r_y} = \frac{(8.88)(12)}{1.37} = 77.3 < 120 \]

\[ \frac{2\pi^2 E}{r_y} = \sqrt{\frac{(2)(\pi)^2(29,000)}{50}} = 107.0 > 77.3 \]

\[ F_{cr} = F_y \left[ 1 - \frac{F_y}{4\pi^2 E} \left( \frac{Kl}{r} \right)^2 \right] = 50 \left[ 1 - \frac{50}{4\pi^2(29,000)} (77.3)^2 \right] = 37.0 \text{ ksi} \]

\[ P_u = 0.85A_{g\text{gross}} F_{cr} = (0.85)(4.42)(37.0) = 139.0^k \]

or

\[ P_u = 0.85A_{\text{snet}} F_y = (0.85)(3.01)(50) = 127.9^k > 52.6 \text{ Governs} \]

\[ \frac{b'}{t} = \frac{5.235-0.51}{0.300} = 15.8 \]

\[ \frac{b'}{t_{\text{allow}}} = \frac{2.200}{\sqrt{P_u}} = \frac{2.200}{\sqrt{50,000}} \sqrt{139.0 \sqrt{52.6}} = 16.0 > 15.8 \]

As a tension member, the strut must have a capacity of 91.6 kips. The capacity is assumed to be

\[ P_u = A_{\text{snet}} F_y \]

where \( A_{\text{snet}} \) is the net area with half the flange coped for the end connection and half of the remaining flange discounted as specified in Article 1.7.8 of the Specifications.

As tension member:

\[ P = 91.6^k \]

\[ A_{\text{snet}} = 4.42 - (1.5) \left( \frac{5.81-0.30}{2} \right) (0.51) = 2.31 \text{ in}^2 \]

\[ P_u = A_{\text{snet}} F_y = (2.31)(50) = 115.5^k > 91.6 \]

The WT5x15 section has adequate strength for the top strut.

The diagonal member of the crossframe is designed in similar fashion. Maximum strength requirements are satisfied by an ST4x11.5 section.
Diagonal Design

Try ST4 x 11.5    ASTM A572 Grade 50

As compression member:

\[ P = 75.7^k \]

\[ A_{\text{gross}} = 3.38 - \left( \frac{4.171 - 0.441}{2} \right) (0.425) = 2.59 \text{ in}^2 \]

\[ A_{\text{snet}} = 3.38 - \frac{50}{4\pi^2(29,000)(85.6)^2} = 3.38 \text{ in}^2 \]

\[ K_I \]
\[ r_x = \frac{(5.69)(12)}{1.22} = 56.0 < 120 \]

\[ K_I \]
\[ r_y = \frac{(5.69)(12)}{0.798} = 85.6 < 120 \]

\[ \sqrt{\frac{2\pi^2E}{F_y}} = 107.0 > 85.6 \]

\[ F_{er} = 50 \left[ 1 - \frac{50}{4\pi^2(29,000)(85.6)^2} \right] = 34.0 \text{ ksi} \]

\[ P_u = 0.85A_{\text{gross}} F_{er} = (0.85)(3.38)(34.0) = 97.7^k \]

or

\[ P_u = 0.85A_{\text{snet}} F_y = (0.85)(2.59)(50) = 110.1^k \]

\[ b' = \frac{4.000-0.425}{0.441} = 8.1 \]

\[ \frac{b'}{t} = \frac{2.200}{\sqrt{F_y}} \sqrt{\frac{P_u}{P}} = \frac{2.200}{\sqrt{50,000}} \sqrt{\frac{97.7}{75.7}} = 11.2 > 8.1 \]

As tension member:

\[ P = 75.7^k \]

\[ A_{\text{snet}} = 3.38 - \frac{50}{4\pi^2(29,000)(85.6)^2} (0.425) = 2.19 \text{ in}^2 \]

\[ P_u = A_{\text{snet}} F_y = (2.19)(50) = 109.5^k > 75.7 \]

For the bottom strut the same WT5x15 as used for the top strut proves to be satisfactory.

Bottom Strut Design

Try WT 5x15    ASTM A572 Grade 50

As compression member:

\[ P = 47.6^k \]

\[ A_{\text{gross}} = 4.42 \text{ in}^2 \]

\[ A_{\text{snet}} = 3.01 \text{ in}^2 \]

\[ K_I \]
\[ r_x = 36.6 \]

\[ K_I \]
\[ r_y = 77.3 \]

\[ F_{er} = 37.0 \text{ ksi} \]

\[ P_{u\text{gross sect}} = 139.0^k \]

\[ P_{u\text{net sect}} = 127.9^k > 47.6 \text{ Governs} \]

\[ \frac{b'}{t} = 15.8 \]

\[ \frac{b'}{t}_{\text{allow}} = \frac{2.200}{\sqrt{50,000}} \sqrt{\frac{139.0}{47.6}} = 16.8 > 15.8 \]
As tension member:

\[ P = 8.6^k \]
\[ A_{\text{net}} = 2.31 \text{ in}^2 \]
\[ P_{u\text{net sect.}} = 115.5^k > 8.6 \]

Strength design of the crossframe is completed by design of the end connections. Welds and connection material are proportioned for a design load, \( P_D \), defined as follows:

\[ P_D = 0.75 P_u \]

or

\[ P_D = \frac{P + P_u}{2} \text{, whichever is greater,} \]

and where \( P_u \) = capacity of section
\( P \) = applied load in member

The fasteners in high-strength bolted friction joints are designed for the above load \( P_D \), divided by 1.3. For the sake of brevity, design calculations for the end connections are not illustrated.

Details of the intermediate crossframe are illustrated below.

Typical Intermediate Crossframe
However, work on this crossframe is still not complete until fatigue has been accounted for. Stress ranges in all members with welded end connections should be checked against allowable stress ranges for Category E. Additionally, stress ranges in the weld metal should be compared to allowable stress ranges for Category F. For fully bolted connections, only Category B need be considered.

Load ranges in the crossframe members are calculated by the same procedures used for the maximum design loads. All loads are service loads. Pp. 43–45 show tabulated computations and a summary for fatigue load ranges. However, stress range calculations are not shown.

This completes the design example for a curved I-girder bridge. The total weight of fabricated structural steel is 191,041 lb, giving a weight per square foot of deck area of 28.2 lb. Of the total weight, 171,850 lb is girder material and 9,191 lb is crossframe material, a breakdown of approximately 90 percent–10 percent. Plans for this structure are shown on pages 46, 47 and 48.

### Fatigue Check for Intermediate Crossframe

**at 0.429 Pt., Span 1**

<table>
<thead>
<tr>
<th>Girder/Description</th>
<th>Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td></td>
</tr>
<tr>
<td>$M_{(L+I+CFv)}$</td>
<td>1,616 + 80 = 1,696</td>
</tr>
<tr>
<td>$M_{-(L+I+CFv)}$</td>
<td>-313</td>
</tr>
<tr>
<td>G2</td>
<td></td>
</tr>
<tr>
<td>$M_{(L+I+CFv)}$</td>
<td>1,537 + 28 = 1,565</td>
</tr>
<tr>
<td>$M_{-(L+I+CFv)}$</td>
<td>-238</td>
</tr>
<tr>
<td>G3</td>
<td></td>
</tr>
<tr>
<td>$M_{(L+I+CFv)}$</td>
<td>1,271</td>
</tr>
<tr>
<td>$M_{-(L+I+CFv)}$</td>
<td>-140*</td>
</tr>
<tr>
<td>G4</td>
<td></td>
</tr>
<tr>
<td>$M_{(L+I+CFv)}$</td>
<td>1,048</td>
</tr>
<tr>
<td>$M_{-(L+I+CFv)}$</td>
<td>-83*</td>
</tr>
<tr>
<td>$\Sigma M_{(L+I+CFv)}$</td>
<td>5,580</td>
</tr>
<tr>
<td>$\Sigma M_{-(L+I+CFv)}$</td>
<td>-774</td>
</tr>
</tbody>
</table>

*Negative CFv neglected

Positive $L + I + CFv$ Moments:

\[
V = \frac{\Sigma M}{GRD/d} = \frac{5,580}{(10^6)(300)(28.2)/15.71} = 9.92^k
\]

\[
\frac{V}{3} = \frac{9.92}{3} = 3.31^k
\]

- $M_1 = 1,696$ k-ft  
  \[
  \frac{M_1}{R} = \frac{(1,696)(15.71)}{300} = 88.81$ k-ft
  
- $M_2 = 1,565$ k-ft  
  \[
  \frac{M_2}{R} = \frac{(1,565)(15.71)}{300} = 81.95$ k-ft
  
- $M_3 = 1,271$ k-ft  
  \[
  \frac{M_3}{R} = \frac{(1,271)(15.71)}{300} = 66.56$ k-ft
  
- $M_4 = 1,048$ k-ft  
  \[
  \frac{M_4}{R} = \frac{(1,048)(15.71)}{300} = 54.88$ k-ft
Forces on Girders:

Forces on Crossframes:

Internal Forces in Crossframes:

\[
\begin{align*}
\frac{88.81}{3.58} &= 24.81k \\
\frac{83.17}{3.58} &= 23.23k \\
\frac{32.71}{3.58} &= 9.14k \\
\frac{1.22}{3.58} &= 0.34k \\
\frac{33.85}{3.58} &= 9.46k \\
\frac{54.88}{3.58} &= 15.33k
\end{align*}
\]

Negative L + I + CFV Moments:

\[
V = \frac{774}{(190) (300) (26.5)/15.71} = 1.38k
\]

\[
\frac{V}{3} = 0.46k
\]

\[
\begin{align*}
M_1 &= 313 \text{ k-ft} \\
M_2 &= 238 \text{ k-ft} \\
M_3 &= 140 \text{ k-ft} \\
M_4 &= 83 \text{ k-ft}
\end{align*}
\]

\[
\begin{align*}
M_d = \frac{(313)(15.71)}{300} = 16.39 \text{ k-ft} \\
M_d = \frac{(238)(15.71)}{300} = 12.46 \text{ k-ft} \\
M_d = \frac{(140)(15.71)}{300} = 7.33 \text{ k-ft} \\
M_d = \frac{(83)(15.71)}{300} = 4.35 \text{ k-ft}
\end{align*}
\]

II/6.44
Forces on Girders:

Forces on Crossframes:

Internal Forces in Crossframes:

\[
\frac{16.39}{3.58} = 4.57^k \\
\frac{16.66}{3.58} = 4.65^k \\
\frac{7.83}{3.58} = 2.19^k \\
\frac{4.20}{3.58} = 1.17^k \\
\frac{0.50}{3.58} = 0.14^k \\
\frac{4.35}{3.58} = 1.22^k
\]

Summary of Fatigue Design Loads for Intermediate Crossframes

<table>
<thead>
<tr>
<th>Member</th>
<th>L + I + CF_V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Strut</td>
<td>T 24.81</td>
</tr>
<tr>
<td></td>
<td>C 4.57</td>
</tr>
<tr>
<td></td>
<td>Range 29.38^k</td>
</tr>
<tr>
<td>Diagonal</td>
<td>T 21.03</td>
</tr>
<tr>
<td></td>
<td>C 2.90</td>
</tr>
<tr>
<td></td>
<td>Range 23.93^k</td>
</tr>
<tr>
<td>Bottom Strut</td>
<td>T 2.87</td>
</tr>
<tr>
<td></td>
<td>C 12.58</td>
</tr>
<tr>
<td></td>
<td>Range 15.45^k</td>
</tr>
</tbody>
</table>
Framing Plan

- Design Live Load: AASHTO H20 with impact
- Future Working Load: 20 ksi per sq. ft.
- Structural steel: ASTM A572 Grade 50 and ASTM A36
- Total Steel Weight: 32,041 lbs
- Steel Weight per Sq. ft. of Dead area

Curved Plate Girder Design Example
Two Span Bridge
Framing Plan and Cross Sections
Composite: Box Girder Load Factor Design

Introduction
Box girders are an alternative form of welded steel plate girders. Each of the plate girders discussed in Chapters 4 and 4A has a single vertical web and is I-shaped in cross section. A box girder, however, has two or more webs, which may be vertical or inclined, and its vertical cross section is a hollow rectangle or trapezoid.

The bottom flange of a box girder usually is a continuous, horizontal plate extending between and connected to the bottom of the webs. The top flange may be a similar steel plate between the tops of the webs or a combination of a narrow steel plate on each web and a composite reinforced concrete deck.

CROSS SECTION OF TYPICAL BOX-GIRDER BRIDGE

The box-shaped cross section of the girders has many advantages for bridge construction. As a result, box-girder highway bridges are economical for simple spans of 75 ft or more and continuous spans of 100 ft or more. Such bridges are often used as grade-separation and elevated structures in urban areas where aesthetics is important. They have also proven advantageous in rural applications, such as stream crossings.

A pleasing appearance is often one of the most important reasons for selection of box-girder construction. They look good because they have a smooth, uninterrupted profile and because they can be given attractive shapes compatible with high structural efficiency.

Maintenance is easier and less costly for box-girder bridges than for plate-girder structures. If the box girders are sealed, their interior need not be painted. When exterior painting is desired, box girders present a smaller exposed surface than do plate girders. And the uninterrupted, continuous exterior surface of a box girder makes this area easier to paint and also less subject to corrosion.

From a structural viewpoint, box girders offer the advantage of a more efficient cross section for resisting torsion than that of plate girders. The high torsional resis-
tance makes box sections particularly advantageous for curved bridges.

In addition, box girders generally can compete favorably in construction cost with plate girders. While the fabrication cost per pound of steel for a box girder may be larger than that for a plate girder, box girders generally require less steel. So the total cost of fabrication of box girders may be about the same as or less than that of plate girders. Erection costs of box girders also may be less. For example, box girders may be advantageous where erection must be performed under difficult conditions or on a limited time schedule. Because one box girder is equivalent to two or more plate girders, placement of a box girder in a single erection lift accomplishes the equivalent of lifting and connecting two plate girders. Also, when laterally braced, a box girder is much stabler during handling and erection than a plate girder of the same length.

This chapter illustrates the design of a two-span, composite, box-girder highway bridge with geometry and general arrangement conforming to Interstate System requirements. The example bridge is typical of a structure that carries one roadway of an Interstate Highway over both roadways of another Interstate Highway. Both spans of the bridge are 120 ft. ASTM A36, A588, and A572, Grade 50, steels are used for the steel portions of the superstructure.

At mid-length of the bridge, the box girders are supported on and rigidly connected to a single center steel pier. Consequently, the girders and pier act as a rigid frame. (The girders, if desired, could be simply supported at the center pier and analyzed as ordinary continuous beams.)

The chapter also presents an alternate design with a reinforced concrete pier at mid-length of the bridge. This pier also is rigidly connected to the girders.

Criteria for load factor design used in the example are in accordance with the American Association of State Highway and Transportation Officials "Standard Specifications for Highway Bridges," 1973, and 1974, 1975, and 1976 "Interim Specifications." These specifications are referred to for brevity in this chapter as AASHTO followed by an article and section reference.

**General Design Considerations**

As shown in the cross section of a typical box-girder bridge, a composite box girder usually consists of:

1. Two steel web plates.
2. A steel bottom-flange plate joining the two webs and forming the underside of the box.
3. Two steel top-flange plates.
4. The reinforced concrete bridge deck made composite with the steel by embedment in the deck concrete of shear connectors welded to the steel top flanges.

Because the bottom flange of a box girder is usually wide, compressive stresses in it could cause it to buckle. To prevent this, the bottom flange, where compression may occur, is stiffened by longitudinal stiffeners welded at equal intervals across the width of the flange.

The steel top flanges for composite box girders need be no wider than necessary to provide adequate bearing for the concrete deck that they support and to allow sufficient space for welding of shear connectors to the flanges.

Webs of box girders are similar to webs of plate girders. Like such webs, box-girder webs may be stiffened transversely or both transversely and longitudinally. They are, however, often sloped rather than vertical. This is done not only to improve the appearance of the bridge but also to reduce the width of the bottom flange.

Box girders also differ from plate girders in that cross bracing between webs is concealed within box girders. Internal diaphragms or cross frames are required within a box girder at each support to resist transverse rotation, displacement and excessive distortion of the girder cross section. Diaphragms or cross frames are also occasionally positioned at other locations to stabilize box girders during handling and erection.
In addition, in continuous spans with field splices, cross frames are usually installed on each side of the splice. When box girders are curved, internal cross frames should be provided at regular intervals along the span, with lateral bracing between the cross frames at the top-flange level. For a tangent alignment, diaphragms or cross frames along the span are unnecessary.

In design of a composite box girder with a vertical axis of symmetry, each half of the cross section may be considered equivalent to a plate-girder section. Principles of composite design presented in detail in Chapters 3 and 4 for wide-flange beams and plate girders therefore may be applied to the box-girder sections.

Box girders that are unsymmetrical with respect to the vertical centroidal axis are a special case that requires a more vigorous analysis. Such girders are not treated in this chapter.

CONCRETE DECK

In a composite box-girder bridge, the roadway slab spans from web to web of each box girder and between the webs of adjacent box girders. Often, the slab cantilevers beyond the outer webs of exterior box girders. The slab may be designed in the same manner as for a series of plate girders with composite construction throughout the full length of the girders.

LATERAL DISTRIBUTION OF DEAD LOAD

When the bridge deck is supported by a single box girder, the dead load on the girder equals its own weight plus the weight of the deck, which consists of the weights of slab, haunches, parapets, wearing surface and railings. When two box girders support the deck, each girder carries its own weight and the weight of half the bridge deck.

In unshored composite design, the dead load on a box girder is divided into two parts, the initial loads, or loads applied before the deck concrete has hardened, and superimposed loads, those applied after the concrete has hardened. The initial dead load is made up of the weights of the girder and slab and is assumed to be carried by the steel portions of the girder alone. The superimposed dead load is made up of the weights of parapets, wearing surface and railings and is assumed to be carried by the steel portions of the girder acting compositely with the concrete slab.

When three or more box girders support the deck, each girder carries as initial dead load its own weight plus the weight of the part of the slab immediately above it. In addition, each girder is assumed to support the portion of the slab extending halfway to the nearest web of an adjacent girder and, for exterior girders, slab cantilevers. The superimposed dead load may be distributed equally to all the girders.

For shear design, the total dead load acting on a girder may be distributed equally to each web of the box.

LATERAL DISTRIBUTION OF LIVE LOAD

For computation of live-load bending moments, a box girder may be assumed to carry the fraction of a wheel load \( W_L \) computed from

\[
W_L = 0.1 + 1.7R + \frac{0.85}{N_w}
\]

where \( N_w = W_c / 12 \), reduced to the nearest whole number

\( W_c = \) roadway width, ft, between curbs

\( R = N_w \) divided by the number of box girders, but not less than 0.5 nor more than 1.5

The reduction in load intensity for multiple lane loading required by Art. 1.2.9 of the AASHTO Specifications is not applicable to box girders, because it has been taken into account in the development of the preceding equation. The reduction
should be applied, however, for design of other bridge components, such as the substructure.

The wheel-load distribution determined by the equation is applicable to bridges for which the distance center to center of adjacent top flanges is within the approximate range of 80 to 120% of the width of the box girders and for which the deck cantilever does not exceed either 6 ft or 60% of the distance center to center of adjacent top flanges, and to continuous bridges for which the box girders are composite throughout their entire length.

One-half the distribution factor for moment should be used, in general, in calculation of the live-load vertical shear in each box-girder web. In calculation of shears at points of support of the girders, however, the wheel load immediately adjacent to the support should be distributed as if the deck acted as a simple beam between the webs.

The distribution factor for live-load deflection may be obtained by dividing the number of lanes by the number of girders.

**STRUCTURAL ANALYSIS**

The longitudinal variations of moments, shears and deflections are calculated from an analysis of the structure as a rigid frame, simply supported at the end bearings of the girders and fixed at the base of the center pier. In the analysis, the variation in moments of inertia of the cross sections of girders and pier along their lengths should be taken into account in determination of the stiffness of frame members. The analysis should also provide for the change in stiffness after the deck concrete hardens.

For the initial dead load, the stiffness is that of the steel section alone. For the superimposed dead load and the live load, the stiffness is that of the composite section, based, respectively, on the modular ratios $3n$ and $n$, where $n$ is the ratio of the modulus of elasticity of the steel $E_s$ to the modulus of elasticity of the concrete $E_c$. For the stiffness calculations, the concrete slab may be considered effective over the full length of the structure.

As for all statically indeterminate structures, the stiffness of the members of the box-girder bridge is not known accurately until the structure has been designed and therefore has to be assumed initially. As a result, after member sizes have been determined from stresses calculated from the initial analysis, the stiffness of the frame members should be calculated and the structure analyzed and designed again with the new values of stiffness. The procedure should be repeated until the stiffness on which an analysis is based agrees reasonably with the stiffness of the designed members.

The example bridge was analyzed by computer. Final member sizes were selected after three cycles of analysis and design. The results of the final cycle are presented in this chapter.

**DESIGN LOADS**

Members designed by the Load Factor method are required to meet certain criteria for three theoretical load levels: Maximum Design Load, Overload and Service Load.

Service Loads are the design loads used in working-stress design. They are applied in design calculations to keep live-load deflections and fatigue life (for assumed fatigue loading) of structural members within acceptable limits.

The Maximum Design Load and the Overload are computed from the service loads by multiplying by a factor of unity or larger the dead, live and impact service loads. Maximum Design Load is applied in design calculations to insure that the structure can withstand in emergencies (simultaneously in more than one lane) a few passages of very heavy vehicles that may induce significant permanent deformations. An Overload is applied to limit permanent deformations that may be caused by occasional overweight vehicles and that would impair riding quality of the deck. The weight of these vehicles is taken as 5/3 the live and impact service loads (simultaneously in more than one lane).

In determination of moments, shears and other forces, the structure is assumed
to act elastically under the three loading levels. The loads for these levels are defined as follows:

Service Load: $D + (L + I)$
Overload: $D + \frac{5}{3}(L + I)$

Maximum Design Load: $1.30 \left[ D + \frac{5}{3}(L + I) \right]$

where $D =$ dead load
$L =$ live load
$I =$ impact load

Effects of uncertainties in strength, theory, loading, analysis, material properties and dimensions are included in the factor 1.30. The factor $5/3$ is incorporated to allow for Overloads. Factors for other loading combinations are given in AASHTO Art. 1.2.22.

**DESIGN FOR MAXIMUM DESIGN LOADS**

Box girders usually do not have as much bending strength as a compact section, because the webs do not normally meet compactness criteria. The maximum-moment capacity at any section therefore should be computed for positive bending from

$$M_u = F_v S$$

where $F_v =$ specified minimum yield stress, psi, of the steel
S = elastic section modulus

The capacity usually need not be reduced to allow for overall buckling. Both flanges of a box girder may be considered braced against lateral torsional buckling. The top flange is braced by the concrete deck, and the bottom flange normally is too wide to buckle in its plane.

For positive bending, with the bottom flange in tension, the effective width of that flange for calculation of the section modulus may not be taken as more than one-fifth the girder span. This limitation accounts for the phenomenon of shear lag in the box section.

Hence, for positive bending, a box girder should be so proportioned that

$$F_v S \geq 1.30 \left[ D + \frac{5}{3}(L + I) \right]$$

Here, $D$, $L$ and $I$ represent moments induced by the Service Loads.

In negative bending, the moment capacity at any section is governed by the critical local buckling stress $F_{cr}$ of the bottom flange, which is in compression. The section, therefore, should be proportioned so that

$$F_{cr} S \geq 1.30 \left[ D + \frac{5}{3}(L + I) \right]$$

The critical bottom-flange buckling stress $F_{cr}$ is a function of the width-thickness ratio $w/t$ for the flange plate and a buckling coefficient $k$:

When $w/t \leq 3.070 \sqrt{k}/\sqrt{F_v}$, $F_{cr} = F_v$

When $3.070 \sqrt{k}/\sqrt{F_v} < w/t \leq 6.650 \sqrt{k}/\sqrt{F_v}$, $F_{cr} = 0.592 F_v (1 + 0.687 \sin \frac{\pi}{2})$

where $c = \left( 6.650 \sqrt{k} - \frac{w}{t} \sqrt{F_v} \right) / 3.580 \sqrt{k}$
When \( w/t > 6,650 \sqrt{k/\sqrt{F_y}} \),

\[ F_{cr} = 26.2 \times 10^4 k \left( \frac{t}{w} \right)^2 \]

In the preceding equations, \( w \) is the spacing of the longitudinal stiffeners on the flange, and \( t \) is the plate thickness.

When there are no longitudinal stiffeners on the bottom flange \( b \), the spacing of the girder webs, is substituted for \( w \) and \( k \) should be taken as 4. For a bottom flange with \( n \) longitudinal stiffeners with equal spacing \( w \), \( k \) may be computed from the following:

When \( n = 1 \),

\[ k = \frac{3}{4} 8 I_s / w \overline{t}^3 \]

where \( I_s \) = moment of inertia, in.\(^4\), of a longitudinal stiffener about an axis parallel to the bottom flange and at the base of the stiffener

When \( n = 2, 3, 4 \) or \( 5 \),

\[ k = \frac{3}{4} 14.3 I_s / w \overline{t} \overline{n} \]

The value of \( k \), however, should not exceed 4.

**CHANGES IN FLANGE-PLATE THICKNESS**

The same principles that govern design for changes in flange-plate thickness of plate girders also apply to box girders. Because the bottom flange of a box girder is very wide and the steel stop flanges usually are narrow, changes in thickness of the bottom-flange plate will be smaller than for the top-flange plates.

**FLANGE WIDTH-THICKNESS RATIOS**

To prevent local buckling of compression flanges, AASHTO Specifications require that the width-thickness ratio of projecting compression flanges not exceed

\[ \frac{b'}{t} = \frac{2,200}{\sqrt{F_y}} \]

where \( b' \) = width of projecting flange element

\( t \) = flange thickness

When the bending moment \( M \) on a section is less than the moment capacity \( M_u \) of the section, \( b'/t \) may be increased in the ratio \( \sqrt{M_u/M} \).

The \( b'/t \) requirement need not be satisfied for compression flanges of composite girders in positive-bending regions after the deck concrete has hardened. Before the deck concrete has hardened, however, the steel top flange is subject to local buckling under the initial dead load. For this condition, the \( b'/t \) limit of \( 2,200/\sqrt{F_y} \) may be increased in the ratio \( \sqrt{F_y/f_{ud}} \). Here, \( f_{ud} \) is the actual stress in the flange due to the initial, factored dead-load moment and \( F_y \) is the yield stress or the top-flange stress used in calculation of the moment capacity of the section.

**WEBS**

For a box girder to qualify as a braced noncompact section with a moment capacity under maximum design load of \( M_u = F_y S \) or \( M_u = F_{cr} S \), each web must satisfy the following requirements:

1. The web depth-thickness ratio should not exceed

\[ \frac{D}{t_w} = 150 \]

where \( D \) = web depth measured in the plane of the web (along the slope of inclined webs)

\( t_w \) = thickness of web measured normal to the plane of the web
2. At any section, shear due to maximum design load should not exceed
\[ V_p = 0.58F_pDt_w \]
When the web is slopped, \( V_p \) is the shear along the slope. The permissible vertical shear then is the vertical component of \( V_p \).

3. If there are no transverse stiffeners on the web, the design shear in the plane of the web also must not exceed the buckling capacity of an unstiffened web:
\[ V_b = \frac{3.5E_I}{D} \]
where \( E \) = steel modulus of elasticity
When the shear exceeds \( V_b \), the web should be stiffened transversely. In that case, the web depth-thickness ratio should not be greater than
\[ \frac{D}{t_w} = \frac{36,500}{\sqrt{F_y}} \]
or, when \( D_o \), the clear distance between the neutral axis and the compression flange, exceeds \( D/2 \),
\[ \frac{D_o}{t_w} \leq \frac{18,250}{\sqrt{F_y}} \]
The shear capacity \( V_u \) of a transversely stiffened web fulfilling the preceding requirements may be computed from
\[ V_u = V_p \left[ C + \frac{0.87(1-C)}{\sqrt{1+\left(d_o/D\right)^2}} \right] \]
where \( d_o \) = spacing of transverse stiffeners and
\[ C = 18,000 \frac{t_w}{D} \left[ \frac{1+(D/d_o)^2}{F_y} \right] - 0.3 \leq 1 \]

If a section is subjected to simultaneous shear \( V \) and bending moment \( M \) and \( V \) exceeds \( 0.6V_u \), then the concurrent bending moment for the maximum design load is limited to
\[ M = M_u \left( 1.375 - 0.625 \frac{V}{V_u} \right) \]
where \( M_u \) = moment capacity of the section when not subject to design shears exceeding \( 0.6V_u \).

(Many designers conservatively use the maximum shear on the section in computation of \( M \), although that shear may not occur for the same loading condition as for maximum moment on the section. This procedure is used in the example in this chapter.)

When the web depth-thickness ratio is larger than that permitted for a web with transverse stiffeners, then a longitudinal stiffener is required in addition to transverse stiffeners. The longitudinal stiffener should be placed at an averaged clear distance equal to approximately \( 2D_o/5 \) from the compression flange. This distance should be adjusted to accommodate welding. Under the preceding conditions, the web depth-thickness ratio may be as large as
\[ \frac{D}{t_w} = \frac{73,000}{\sqrt{F_y}} \]
or, when \( D_o \) exceeds \( D/2 \),
\[ \frac{D_o}{t_w} \leq \frac{36,500}{\sqrt{F_y}} \]
The shear capacity of a web with a longitudinal stiffener may be calculated in the same way as for a web with transverse stiffeners only. Similarly, if the shear on the section exceeds $0.6V_w$, the maximum simultaneous moment is limited by the same equation as for a web with transverse stiffeners only.

**WEB STIFFENERS**

Bearing and intermediate transverse stiffeners are designed by the procedures given in Chapter 4A.

When longitudinal stiffeners are used in combination with transverse stiffeners, the criteria given in Chapter 4A for stiffeners still apply, except that the depth of the subpanel, $0.8D$, rather than $D$, should be used in all equations. In addition, the section modulus of each transverse stiffener should be at least

$$S_t = \frac{1}{3}(D/d_o)S_l$$

where $D =$ clear unsupported depth between flange components measured in the plane of the web

$d_o =$ spacing of transverse stiffeners

$S_l =$ section modulus of the longitudinal stiffener

Longitudinal stiffeners must satisfy the same requirement for width-thickness ratio as transverse stiffeners. For rigidity, the moment of inertia of each longitudinal stiffener should be at least

$$I = Dt_o[2.4(d_o/D)^4 - 0.13]$$

Also, the radius of gyration should not be less than

$$r = \frac{d_o \sqrt{I}}{23,000}$$

$I$ and $r$ should be computed for an axis through the mid-plane of the web, and the section should include both the longitudinal stiffener and a strip of web $18t_w$ wide centrally located with respect to the stiffener.

**HYBRID SECTIONS**

In a hybrid girder, the steel in one or both flanges has a higher yield strength than the web plate. The bending strength of a girder section is based on the properties of the flange steel, which are modified by a reduction factor, $R$.

In positive-moment regions of a composite box girder, the area of the steel compression flange should be equal to or smaller than the area of the tension flange. In negative-moment regions, the area of the compression flange should be equal to the area of the steel tension flange or larger by an amount not exceeding 25%. Also, the minimum specified yield strength of the web should not be less than 35% of the minimum specified yield strength of the tension flange.

The moment capacity $M_u$ at any section of a noncompact, hybrid box girder is given by

$$M_u = F_y/SR$$

where $F_y =$ the minimum specified yield strength of a flange steel

$S =$ elastic section modulus

$$R = 1 - \frac{\beta \psi(1 - \rho)^2(3 - \psi + \rho \psi)}{6 + \beta \psi(3 - \psi)}$$

$$\rho = \frac{\text{yield strength of web}}{\text{yield strength of tension flange}}$$
\[ \beta = \frac{\text{area of web}}{\text{area of tension flange}} \]
\[ \psi = \text{distance from the outer fiber of the tension flange to the neutral axis of the composite section divided by the depth of the steel section} \]

The expression for \( M_u \) shall be applied to both flanges.

Tension-field action is not taken into account in design of hybrid sections with stiffened webs. (See Commentary, "Tentative Criteria for Load Factor Design of Steel Highway Bridges," American Iron and Steel Institute Bulletin No. 15, March, 1969.) The shear capacity of a stiffened web is therefore

\[ V_u = V_p C \]

where \( V_p \) and \( C \) are defined as for sections with flanges and webs made of the same steel. Also, the area requirement given in Chapter 4A for transverse stiffeners is not applicable to hybrid girders.

**DESIGN FOR OVERLOAD**

To guard against objectionable deformation under occasional Overload, the following moment relationship must be observed for noncomposite sections and negative bending of composite sections of a homogeneous girder.

\[ 0.8F_p S \geq \left[ D + \frac{5}{3}(L+I) \right] \]

For the same purpose, composite sections of a homogeneous girder in positive bending must satisfy the relationship

\[ 0.95F_p S \geq \left[ D + \frac{5}{3}(L+I) \right] \]

Objectional deformations in a hybrid girder, under the Overload, will occur at a lower moment level than in a homogeneous girder because of premature yielding in the web. To account for this, the above moment relationships must be modified by the reduction factor, \( R \).

For noncomposite sections and negative bending of composite sections of a hybrid girder,

\[ 0.8F_p S R \geq \left[ D + \frac{5}{3}(L+I) \right] \]

For composite sections in positive bending of a hybrid girder,

\[ 0.95F_p S R \geq \left[ D + \frac{5}{3}(L+I) \right] \]

For an unsymmetrical section, stresses in both flanges shall be checked.

**DESIGN FOR SERVICE LOADS**

Fatigue should be investigated in the same manner as for working-stress design, with Service loads, to satisfy the provisions of AASHTO Art. 1.7.3. The strength of longitudinal reinforcing steel of the concrete deck, in tension in negative-moment regions, should be taken into account in computation of section properties for sections in those regions. For fatigue computations, the stress range in the reinforcing steel is limited to 20,000 psi.

Fatigue becomes critical under tension or stress reversal at the following locations in box girders with groove-welded flange transitions, stud shear connectors, fillet-welded transverse web stiffeners and fillet-welded bottom-flange longitudinal stiffeners:

1. Base metal adjacent to a fillet weld at the end of a longitudinal flange or web stiffener (AASHTO Category E).
2. Base metal adjacent to stud shear connectors (AASHTO Category C).
3. Base metal in the girder web at the toe of a transverse-stiffener fillet weld or at the toe of a fillet weld for a cross-frame connection plate (AASHTO Category C).
4. Base metal adjacent to full-penetration groove-welded flange transitions (AASHTO Category B).

Groove-welded splices at transitions in width or thickness of flanges may be assigned to AASHTO fatigue Category B if transition slopes not exceeding 1 to 2½ are used and the welds are finished smooth and flush.

SHEAR CONNECTORS
Shear connectors should be designed in the same way as for working-stress design.

DEFLECTIONS
Dead-load and live-load deflections should be calculated in the same way as for working-stress design.

DESIGN OF PIER
The girders and pier of the following design example are designed for vertical dead, live and impact loads, wind and longitudinal force from braking and traction. AASHTO Specifications call for a transverse wind load of 50 psf and a simultaneous longitudinal wind load of 12 psf acting on the surface of the bridge as seen in elevation. Also, simultaneous transverse and longitudinal wind forces of 100 lb per lin ft and 40 lb per lin ft, respectively, are specified for wind on live load. In addition, a longitudinal braking and traction force equal to 5% of the live load should be considered applied to the bridge 6 ft above the deck.

For transverse loads, the structure is analyzed as a grid; that is, as a structure loaded normal to its plane. For vertical and longitudinal loads, the structure is analyzed as a rigid frame.

In design of the pier, loads are combined in accordance with the following groupings:

Group I: \(1.30 \left[ D + \frac{5}{3} (L + J) \right] \)
Group II: \(1.30 (D + W) \)
Group III: \(1.30 (D + L + I + 0.3W + WL + LF) \)

where \(W = \) wind on the structure
\(WL = \) wind on the live load
\(LF = \) longitudinal force

The pier is initially designed with a steel, rectangular, hollow-box cross section. Unit stresses in the section are calculated from

\[ f = \frac{P}{A} + \frac{M_x y}{I_x} + \frac{M_y x}{I_y} \]

where \(P = \) vertical load on the pier
\(A = \) cross-sectional area of the pier
\(M_x = \) bending moment about principal axis \(XX\) of the section
\(M_y = \) bending moment about principal axis \(YY\) of the section
\(x = \) distance from point where stress is to be computed to the \(YY\) axis
\(y = \) distance from the point to the \(XX\) axis
\(I_x = \) moment of inertia of the section about the \(XX\) axis
\(I_y = \) moment of inertia of the section about the \(YY\) axis

Each plate of the box section is assumed to be analogous to the bottom compression plate of a box girder and is designed for a critical buckling stress \(F_{cr}\).
Design Example—Two-Span Rigid-Frame Box Girder (120-120 Ft) Composite for Positive and Negative Bending

The following data apply to this design:

Roadway Section: See typical bridge cross section.

TYPICAL CROSS SECTION OF EXAMPLE BRIDGE


Loading: HS20-44.

Structural Steel: ASTM A36, A588 and A572, Grade 50.
Concrete: $f'_c = 4,000$ psi, modular ratio $n = 8$.
Slab Reinforcing Steel: ASTM A615, Grade 40, with $F_y = 40,000$ psi.

Loading Conditions:
Case 1—Weight of girder and slab ($DL_1$) supported by the steel girder alone.
Case 2—Superimposed dead load ($DL_2$) (parapets and railings) supported by the composite section with the modular ratio $n = 8$. (Used in design of web-to-flange fillet welds.)
Case 3—Superimposed dead load ($DL_3$) (parapets and railings) supported by the composite section with the increased modular ratio $3n = 3 \times 8 = 24$.
Case 4—Live load plus impact ($L + I$) supported by the composite section with the modular ratio $n = 8$.

Fatigue—500,000 cycles of truck loading
100,000 cycles of lane loading

Loading Combinations:
Combination A = Case 1 + 3 + 4
Combination B = Case 2 + 4
Combination C = Case 1 + 2 + 4
GEOMETRY OF BRIDGE
The geometric layout of the example structure is shown in an elevation view.

ELEVATION OF TWO-LANE OVERPASS STRUCTURE

LOADS, SHEARS AND MOMENTS FOR BOX GIRDERS
The initial dead load \( (DL_1) \) consists of an assumed weight of 475 lb per lin ft for the box girder, plus the weight of a 7½-in. concrete slab and haunches. An average depth and width is assumed for the haunch, because the actual depth and width varies along the girder.

The superimposed dead load \( (DL_2) \) carried by the composite section consists of the weights of the parapet, 2-in. future wearing surface and single-tube steel railings.

Dead Load on Steel Box Girder
- Slab = 0.63 \times 20.9 \times 0.150 = 1.976
- 0.12 \times 4.83 \times 0.150 = 0.087
- Haunches = 0.19 \times 1.67 \times 0.150 \times 2 = 0.095
- Girder (assumed weight) = 0.475

\[ DL_1 \text{ per girder} = 2.633 \text{ k/ft} \]

Dead Load Carried by Composite Section
- Parapet = 1.50 \times 0.92 \times 0.150 = 0.207
- 0.37 \times 0.50 \times 0.150 = 0.028
- 0.17 \times 1.42 \times 0.150 = 0.036
- Wearing surface = 0.020 \times 19.5 = 0.390
- Railing = 0.020

\[ DL_2 \text{ per girder} = 0.681 \text{ k/ft} \]

Live Load on Box Girder
The live load distribution factor is calculated from the AASHTO Specification formula previously discussed. For a roadway width \( W_r = 40 \) ft,

\[ N_w = \frac{W_r}{12} = 3.33 \]

Reduced to the integer, 3. Because there are two box girders,

\[ R = \frac{N_w}{2} = \frac{3}{2} \]

The distribution factor for live load per girder then is

\[ W_L = 0.1 + 1.7R + \frac{0.85}{N_w} = 0.1 + 1.7 \times \frac{3}{2} + \frac{0.85}{3} = 2.933 \text{ wheels} = 1.467 \text{ axles} \]

\[ \text{Impact} = \frac{50}{100 + 120} = 0.227 \]
Maximum moment and maximum shear may be calculated by any convenient method. The following curves were obtained by including the effect of the center pier, which is rigidly connected to the box girder, on girder shears and moments.

**MAXIMUM VERTICAL SHEAR PER WEB**

**MAXIMUM-MOMENT CURVES FOR BOX GIRDERS**
DESIGN OF GIRDER SECTIONS

In determination of the effective width of the concrete slab for the composite section, each half of the box girder is considered equivalent to a plate girder and the usual AASHTO criteria for effective slab width are applied. Hence, the effective slab width for the box girder equals the sum of the effective slab widths for each flange.

For negative bending, the longitudinal slab reinforcement is considered part of the composite section. This steel consists of the normal distribution reinforcement and the additional bars for crack control. The area of the reinforcement and location of its center of gravity with respect to the bottom of the slab are calculated from data shown on the slab half section.

SLAB HALF SECTION

Effective Slab Width

1. One-fourth the span: $\frac{1}{4} \times \frac{1}{4} \times 120 \times 12 \times 2 = 540$ in.
2. Center to center of girders: $12 \left( 9.83 + \frac{1}{2} \left( 9.83 + 11 \right) \right) = 243$ in.
3. 12 x slab thickness: $12 \times 7.5 \times 2 = 180$ in. (governs)

Area of Slab Reinforcement for Negative-Moment Section

<table>
<thead>
<tr>
<th>Bar Location</th>
<th>No. of Bars</th>
<th>Area per Bar</th>
<th>Total Area</th>
<th>$d$</th>
<th>$Ad$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top row</td>
<td>31</td>
<td>0.31</td>
<td>9.61</td>
<td>4.313</td>
<td>41.45</td>
</tr>
<tr>
<td>Bottom row</td>
<td>18</td>
<td>0.31</td>
<td>5.58</td>
<td>2.188</td>
<td>12.21</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>15.19 in.²</td>
<td></td>
<td>53.66 in.²</td>
</tr>
</tbody>
</table>

$d_{Reinf.} = \frac{53.66}{15.19} = 3.63$ in.

Dimensions of Bottom Flange

A check of the effective width of the bottom flange of the box girder in positive-moment regions indicates that the maximum effective width is greater than the full width of the flange plate. Hence, the full width is used.

Max. Effective Width = $\frac{1}{4}$ Span = $\frac{1}{4} \times \frac{1}{4} \times 120 \times 12 = 216 > 92$ in.

Therefore, use the actual plate width.

In design of the bottom flange in negative-moment regions, where the flange is in compression, it is convenient to read values of the critical buckling stress $F_c$, directly from a graph rather than to calculate $F_c$, from the equations given previously. The following sets of curves, which show the variation of $F_c$, with $w/t$ for $k$ values of 1 to 4, may be used to obtain $F_c$. One set of curves is based on $F_c = 36$ ksi, and the second set on $F_c = 50$ ksi. Linear interpolation may be used to determine $F_c$ for values of $k$ between the values plotted.
Critical Buckling Stress $F_{cr}$, ksi

For $w \leq \frac{3.070}{{F_y}^2}$ : $F_{cr} = F_y$

For $\frac{3.070}{{F_y}^2} < w \leq \frac{6.650}{{F_y}^2}$ : $F_{cr} = 0.592 F_y \left(1 + 0.587 \sin \frac{\pi}{2} \right)$

where $c = \frac{6.650 \sqrt{c} - (w/1) \sqrt{F_y}}{3.580 \sqrt{k}}$

For $w > \frac{6.650}{{F_y}^2}$ : $F_{cr} = 26.2 k \left(\frac{k}{w}\right)^{10}$

$F_y = 36$ ksi

$F_y = 50$ ksi

BUCKLING STRESS FOR BOTTOM PLATE OF LONGITU DINALLY STIFFENED BOX GIRDER
GIRDER DEPTH AND WEB DESIGN

Because of limitations in AASHTO Specifications on depth-span ratios of girders, the box girder should be at least 36 in. deep. For greater economy, however, a 57-in. depth is selected.

AASHTO requires that the depth-span ratio \( H/L \) not exceed \( 1/30 \) for the box girder alone nor \( 1.25 \) for the box girder plus the slab. The span \( L \) is determined by the distance from the girder support at the abutment to the point of contraflexure.

**Minimum Depth of Structure**

\[
\frac{H_{\text{min}}}{90} = \frac{1}{30} \quad H_{\text{min}} = \frac{90}{30} = 3 \text{ ft} = 36 \text{ in.}
\]

\[
\frac{H_{\text{min}} + 10.5/12}{90} = \frac{1}{25} \quad H_{\text{min}} = \frac{90}{25} \frac{10.5}{12} = 2.725 \text{ ft} = 32.7 \text{ in.}
\]

The minimum permissible depth, therefore, is 36 in. A deeper section, however, will be more economical. Costs will increase, though, if the depth exceeds that at which the thickness of the flange is governed by minimum-thickness requirements rather than by stress.

Another consideration affecting economy is fabrication costs. The best current design practice prefers minimization of detail material, such as stiffeners, despite increase in main material.

Accordingly, the web for the example girders is arbitrarily designed as unstiffened in the positive-moment region. The web thickness required for this condition is maintained through the negative-moment region. In this region, however, the web is transversely stiffened where it is subject to high shear. No longitudinal stiffeners are used.

On this basis then, a \( \frac{3}{4} \)-in.-thick web with a depth when projected on the vertical of 57 in. is selected. The web is sloped at 57 in. on 14 in., or 4.071:1. The 57-in. vertical depth is about the maximum at which, in this structure, design of most of the flange material is controlled by stress rather than minimum permissible thickness. Studies have shown that such a depth is most economical for spans of this range.

In all calculations for the web, the shear is computed for the maximum design load \( 1.30[D + (5/3)(L + I)] \) and resolved in the direction of the slope. The web depth \( D \) is measured along the slope.

\[
D = \sqrt{57^2 + 14^2} = 58.69
\]

\[
D = \frac{58.69}{\frac{3}{2}} = 117 < 150
\]

Hence, the depth-thickness requirements for an unstiffened web are satisfied.

**Unstiffened Web—Positive-Moment Region**

At the end bearing, the maximum design shear along the slope is

\[
V' = \frac{58.69}{57} \times 1.30 \left[ 56.7 + 14.5 + \frac{5}{3} \times (88.3) \right] = 248 \text{ kips}
\]

Maximum shear strength of the web is

\[
V_s = 0.58F_vD_{tu} = 0.58 \times 36 \times 58.69 \times \frac{3}{4} = 613 > 248 \text{ kips}
\]

Maximum capacity of the unstiffened web for buckling is

\[
V_b = \frac{3.5Et^3}{D} = \frac{3.5 \times 29,000 \times (\frac{3}{4})^3}{58.69} = 216 < 248 \text{ kips}
\]

While the \( \frac{3}{4} \)-in. web satisfies ultimate strength and \( D/t_w \) requirements, the buckling capacity of the unstiffened web is less than the end shear. Rather than use a thicker web, it is more economical to add one or two stiffeners adjacent to the end
bearing. Superposition of the 216-kip buckling capacity on the shear diagram indicates that the web should be transversely stiffened for a distance of 4.5 ft from the end bearing and of 33 ft from the pier. (Web-stiffener design is presented after design of the girder sections for bending stresses.)

![Shear Diagram]

**FATIGUE REQUIREMENTS**

Before design of the girder sections for positive and negative-bending moment is begun, it will be helpful to summarize the fatigue checks that should be made.

At locations of maximum negative moment and at flange transitions in negative-bending regions, the top flange is in tension. If stud shear connectors are welded to this flange, as they are in this example, AASHTO fatigue Category C determines the maximum stress range permitted at those locations.

At locations of maximum positive moment and at flange transitions in positive-bending regions, the bottom flange is in tension. If a transverse stiffener or cross-frame connection plate is nearby, the stress range in the web is determined by fatigue Category C. Also, if the section being investigated is at a groove-welded flange splice, the maximum stress range in the bottom flange may not exceed that for fatigue Category B.

Box-girder sections near points of contraflexure, where stress reversals are likely to occur, should be checked for the stress range for fatigue Category C, at the top flange where shear connectors are likely to be attached and at the bottom of the web where a transverse stiffener or cross-frame connection plate is attached. Also, the bottom flange should be checked for fatigue Category B if the section is close to a transition groove weld. If a longitudinal flange stiffener is terminated in this region, the flange base metal at the end of the stiffener-to-flange fillet weld should be checked for the stress range for fatigue Category E.

AASHTO Specifications assign the following allowable ranges of stress to Categories B, C and E:

<table>
<thead>
<tr>
<th>Category</th>
<th>500,000 Cycles (Truck Loading)</th>
<th>100,000 Cycles (Lane Loading)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>27,500</td>
<td>45,000</td>
</tr>
<tr>
<td>C</td>
<td>19,000</td>
<td>32,000</td>
</tr>
<tr>
<td>E</td>
<td>12,500</td>
<td>21,000</td>
</tr>
</tbody>
</table>
CRITICAL BUCKLING STRESSES AT NEGATIVE-MOMENT SECTIONS

A single, longitudinal, structural tee (ST shape) is used to stiffen the bottom flange in the negative-moment region. For the box girders in this example, the single stiffener is more economical than several stiffeners.

A structural tee is an efficient shape for a longitudinal stiffener for the flange, because the tee provides a high ratio of stiffness to cross-sectional area. Other shapes, such as plates, angles or channels, however, may also be used.

The following ST shapes are chosen as possible longitudinal stiffeners, and the moment of inertia \( I \), about the base of the stem of each stiffener is calculated.

<table>
<thead>
<tr>
<th>ST Shape</th>
<th>Moment of Inertia, In. (^4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 × 35</td>
<td>84.7 + 10.3(6.06)(^2) = 463.0</td>
</tr>
<tr>
<td>7.5 × 25</td>
<td>40.6 + 7.35(5.25)(^2) = 243.2</td>
</tr>
<tr>
<td>6 × 25</td>
<td>25.2 + 7.35(4.16)(^2) = 152.4</td>
</tr>
<tr>
<td>6 × 20.4</td>
<td>18.9 + 6.00(4.42)(^2) = 136.1</td>
</tr>
<tr>
<td>5 × 17.5</td>
<td>12.5 + 5.15(3.44)(^2) = 73.4</td>
</tr>
<tr>
<td>4 × 11.5</td>
<td>5.03 + 3.38(2.85)(^2) = 32.5</td>
</tr>
<tr>
<td>3.5 × 10</td>
<td>3.36 + 2.94(2.46)(^2) = 21.2</td>
</tr>
</tbody>
</table>

With \( I \) known and the stiffener spacing chosen as \( w = 90/2 = 45 \) in., the value of the flange buckling coefficient \( k \) furnished by the stiffener is calculated for various plate thicknesses from the equation previously given for a flange with a single stiffener.

\[
k = \sqrt{\frac{8T}{wt}} = \sqrt{\frac{8T}{45t^3}} = \frac{0.562}{t} \sqrt{\frac{I}{t}}
\]

With \( k \) known, the critical buckling stress \( F_{cr} \) is obtained from the curves previously presented and listed in the following table for several plate thicknesses. Stiffeners listed in the table provide a \( k \) value as near to 4 as practicable.

### Critical Flange Buckling Stress with One Longitudinal Stiffener, Ksi

<table>
<thead>
<tr>
<th>( t )</th>
<th>( w/t )</th>
<th>ST Stiffener</th>
<th>( I )</th>
<th>( k )</th>
<th>( F_{cr} ) with ( F_y = 36 )</th>
<th>( F_{cr} ) with ( F_y = 50 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{3}{16} )</td>
<td>55.4</td>
<td>6 × 25</td>
<td>152.4</td>
<td>3.70</td>
<td>28.5</td>
<td>31.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6 × 20.4</td>
<td>136.1</td>
<td>3.56</td>
<td>27.6</td>
<td>30.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5 × 17.5</td>
<td>73.4</td>
<td>2.90</td>
<td>24.2</td>
<td>24.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4 × 11.5</td>
<td>32.5</td>
<td>2.20</td>
<td>18.6</td>
<td>18.7</td>
</tr>
<tr>
<td>( \frac{3}{8} )</td>
<td>51.4</td>
<td>7.5 × 25</td>
<td>243.2</td>
<td>4.00</td>
<td>31.6</td>
<td>37.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6 × 25</td>
<td>152.4</td>
<td>3.43</td>
<td>29.3</td>
<td>33.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6 × 20.4</td>
<td>136.1</td>
<td>3.30</td>
<td>28.8</td>
<td>32.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5 × 17.5</td>
<td>73.4</td>
<td>2.69</td>
<td>25.2</td>
<td>26.7</td>
</tr>
<tr>
<td>( \frac{5}{16} )</td>
<td>48.0</td>
<td>7.5 × 25</td>
<td>243.2</td>
<td>3.74</td>
<td>32.1</td>
<td>38.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6 × 25</td>
<td>152.4</td>
<td>3.20</td>
<td>30.3</td>
<td>34.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6 × 20.4</td>
<td>136.1</td>
<td>3.08</td>
<td>30.0</td>
<td>34.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5 × 17.5</td>
<td>73.4</td>
<td>2.51</td>
<td>26.3</td>
<td>28.4</td>
</tr>
<tr>
<td>1</td>
<td>45.0</td>
<td>7.5 × 25</td>
<td>243.2</td>
<td>3.50</td>
<td>32.6</td>
<td>39.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6 × 25</td>
<td>152.4</td>
<td>3.00</td>
<td>31.3</td>
<td>36.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6 × 20.4</td>
<td>136.1</td>
<td>2.27</td>
<td>30.6</td>
<td>35.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5 × 17.5</td>
<td>73.4</td>
<td>2.35</td>
<td>27.3</td>
<td>29.8</td>
</tr>
</tbody>
</table>
FLANGE TRANSITION 2 FT FROM INTERIOR SUPPORT

Adjacent to the center pier, the section chosen is hybrid, with the yield stress $F_{y/}=50$ ksi for the top and bottom flange plates and $F_{y/}=36$ ksi for the web plates. In this case, $F_{y/}$ is much larger than the minimum of 35% of $F_{y/}$ required by AASHTO for a hybrid section.

![Diagram of flange transition](image)

**NEGATIVE-MOMENT SECTION 2 FT FROM THE INTERIOR SUPPORT**

Properties are calculated for the steel section alone where a flange transition occurs 2 ft from the center of the interior support and for that steel section plus the slab reinforcement. The moment of inertia of each inclined web $I_{sw}$ with respect to a horizontal axis at mid-depth of the web is computed from

$$I_{sw} = \frac{S^2}{S^2 + 1} I_u$$

where $S$ = web slope with respect to the horizontal $= 57/14 = 4.071$

$I_u$ = moment of inertia with respect to an axis normal to the web. In the calculation of section properties, $d$ is measured vertically from a horizontal axis through the mid-depth of the web to the centroid of each element of the box girder.

At the interior support, the bottom flange of the box girder should be designed for a biaxial state of stress at the connection to the cross girder over the pier. For this reason, design of the maximum-moment section is included with the design of the section at the negative-moment flange transition 2 ft from the interior support. This section is investigated first. The biaxially stressed flange is investigated later in this chapter.

**Steel Section at Transition 2 Ft from Center of Interior Support**

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_u$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 T. Flg. Pl. 2x15</td>
<td>60.00</td>
<td>29.50</td>
<td>1,770</td>
<td>52,215</td>
<td>20</td>
<td>52,235</td>
</tr>
<tr>
<td>2 Web Pl. 1/2x58.69</td>
<td>58.69</td>
<td>-28.94</td>
<td>-2,330</td>
<td>67,421</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stiff. ST 7.5x25</td>
<td>7.35</td>
<td>-23.25</td>
<td>-171</td>
<td>3,973</td>
<td></td>
<td>40,143</td>
</tr>
</tbody>
</table>

$d = \frac{-731}{206.54} = -3.54$ in.$^2$

$-731$ in.$^3$

$-3.54 \times 731 = -2,588$

$I_{NA} = 136,973$ in.$^4$

$d_{Top \ of \ steel} = 30.50 + 3.54 = 34.04$ in.

$d_{Bot \ of \ steel} = 29.38 - 3.54 = 25.84$ in.

$S_{Top \ of \ steel} = \frac{136,973}{34.04} = 4,024$ in.$^3$

$S_{Bot \ of \ steel} = \frac{136,973}{25.84} = 5,301$ in.$^3$
Steel Section, with Reinforcing Steel, 2 Ft from Interior Support

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_o$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>206.54</td>
<td>35.03</td>
<td>-731</td>
<td>18.640</td>
<td>139,561</td>
<td></td>
</tr>
<tr>
<td>Reinforcement</td>
<td>15.19</td>
<td>35.03</td>
<td>532</td>
<td>18.640</td>
<td></td>
<td>18,640</td>
</tr>
</tbody>
</table>

\[
d = \frac{-199}{221.73} = -0.90 \text{ in.} \quad \text{in.}^3\]

\[
I_{NA} = \frac{-0.90 \times 199}{158,022 \text{ in.}^4}
\]

\[
d_{\text{Top of steel}} = 30.50 + 0.90 = 31.40 \text{ in.} \quad d_{\text{Bot of steel}} = 29.38 - 0.90 = 28.48 \text{ in.}
\]

\[
S_{\text{Top of steel}} = \frac{158,022}{31.40} = 5,033 \text{ in.}^3 \quad S_{\text{Bot of steel}} = \frac{158,022}{28.48} = 5,549 \text{ in.}^3
\]

\[
d_{\text{Reinf.}} = 35.03 + 0.90 = 35.93 \text{ in.}
\]

\[
S_{\text{Reinf.}} = \frac{158,022}{35.93} = 4,398 \text{ in.}^3
\]

As discussed previously, a hybrid section is designed for the higher strength of the flange steel but reduced by a factor $R$.

**Investigation of Hybrid Section**

In negative-moment regions, the area of the compression flange may not exceed the area of the tension flanges by more than 25%. The area of the tension flanges 2 ft from the interior support, including the area of the slab reinforcing steel, is

\[
A_{f,2} = 2 \times 15 \times 2 + 15.19 = 75.19 \text{ in.}^2
\]

The area of the compression flange, including the area of the longitudinal stiffener, is

\[
A_{f,0} = 92 \times \frac{3}{8} + 7.35 = 87.85 \text{ in.}^2
\]

The ratio of the compression-flange area to the tension-flange area is

\[
\frac{87.85}{75.19} = 1.168 < 1.25
\]

For determination of $R$ for the section 2 ft from the interior support, the parameters $\rho$, $\psi$, and $\beta$ are calculated. For yield strength of web $F_{yw} = 36 \text{ ksi}$ and yield strength of flanges $F_{yf} = 50 \text{ ksi},$

\[
\rho = \frac{F_{yw}}{F_{yf}} = \frac{36}{50} = 0.72
\]

\[
\psi = \frac{31.40}{31.40 + 28.28} = 0.524
\]

\[
\beta = \frac{A_{w}}{A_{f}} = \frac{58.69 \times \frac{3}{8}}{15 \times 2} = 0.978
\]

The reduction factor for the hybrid section then is

\[
R = 1 - \frac{\beta \psi (1 - \rho)^2 (3 - \psi + \rho \psi)}{6 + \beta \psi (3 - \psi)}
\]

\[
= 1 - \frac{(0.978)(0.524)(1 - 0.72)^2(3 - 0.524 + (0.72)(0.524))}{6 + (0.978)(0.524)(3 - 0.524)} = 0.984
\]

The design relationship for Maximum Design Load on a hybrid section is

\[
RF_{yw}S \geq 1.30 \left[ D + \frac{5}{3}(L + I) \right]
\]
When the bottom flange is in compression, as it is 2 ft from the interior support, the flange yield stress in the preceding relationship should be replaced by the critical buckling stress $F_{cr}$. Thus, the maximum allowable bending stress becomes:

Top flange: $RF_{cr} = 0.984 \times 50 = 49.2$ ksi (tension)

Bot. flange: $RF_{cr} = 0.984 \times 37.4 = 36.8$ ksi (compression)

The value of $F_{cr}$ is obtained from the table of critical buckling stresses previously presented.

**Maximum Service-Load Moments 2 Ft from Interior Support**

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$(L+I)$</th>
<th>$-(L+I)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$, kip-ft</td>
<td>5,850</td>
<td>-1,320</td>
<td>2,980</td>
<td>+50</td>
</tr>
</tbody>
</table>

**Steel Stresses 2 Ft from Interior Support Due to Maximum Design Loads**

<table>
<thead>
<tr>
<th></th>
<th>Top of Steel (Tension)</th>
<th>Bottom of Steel (Compression)</th>
</tr>
</thead>
<tbody>
<tr>
<td>For $DL_1$: $F_b$ =</td>
<td>$\frac{5,850 \times 12}{4,024} \times 1.30$ = 22.7</td>
<td>$\frac{5,850 \times 12}{5,301} \times 1.30$ = 17.2</td>
</tr>
<tr>
<td>For $DL_2$: $F_b$ =</td>
<td>$\frac{1,320 \times 12}{5,033} \times 1.30$ = 4.1</td>
<td>$\frac{1,320 \times 12}{5,549} \times 1.30$ = 3.7</td>
</tr>
<tr>
<td>For $L+I$: $F_b$ =</td>
<td>$\frac{2,980 \times 12 \times \frac{5}{3}}{5,033} \times 1.30 \times \frac{5}{3}$ = 14.0</td>
<td>$\frac{2,980 \times 12 \times \frac{5}{3}}{5,549} \times 1.30 \times \frac{5}{3}$ = 14.0</td>
</tr>
<tr>
<td></td>
<td>42.2 &lt; 49.2 ksi</td>
<td>36.8 &gt; 34.9 ksi</td>
</tr>
</tbody>
</table>

**Reinforcing Steel Stress (Tension) 2 Ft from Interior Support**

$$f_r = \frac{1.3 \times 12 \left(1,320 + \frac{5}{3} \times 2,980\right)}{4,398} = 22.3 < 40 \text{ ksi}$$

**Check of Fatigue-Stress Range**

The fatigue-stress range in the reinforcing steel due to Service Loads is limited to 20 ksi. The allowable stress range at the interior support is computed from the Service-Load moments with a section modulus in tension of 4,398 in.$^3$.

$$f_r = \frac{12(2,980 + 50)}{4,398} = 8.27 < 20 \text{ ksi}$$

In addition to the check of the Maximum Design Load, the transition section should also be investigated for fatigue at the weld of the stud shear connector. On the assumption that a row of connectors will be placed on the top flange near the transition, the live-load stress range for the top of the steel girder at this location is determined to be

$$f_r = \frac{12(2,980 + 50)}{5,033} = 7.22 < 32 \text{ ksi (lane load controls)}$$

The section is satisfactory for fatigue near the interior support.

Although not presented in this chapter, calculations indicate that the following arrangements could have been used as alternates for the bottom flange of the hybrid girder:

- $\frac{3}{16}$-in. plate with two ST$7.5 \times 25$ stiffeners
- $\frac{3}{8}$-in. plate with three ST$7.5 \times 25$ stiffeners

But the amount of steel saved with each $\frac{3}{16}$-in. reduction in flange thickness does not offset the additional amount of steel added by another flange stiffener. Hence, use of
a single flange stiffener is the most economical.

If the section is designed entirely of A36 steel, about 10% more material is required. But A36 steel is less expensive than the higher-strength steel required by the hybrid design. Current data indicate that A572 steel costs about 10% more than A36 steel. One-quarter of the hybrid-girder section (the webs), however, is A36 steel. Consequently, the hybrid girder costs slightly less than the A36 girder. Therefore, the hybrid section is used with a single, longitudinal, bottom-flange stiffener for the region near the center pier.

**POSITIVE-MOMENT SECTION**

The section for maximum positive-bending moment, which is located 48 ft from the end bearing (0.4L) is fabricated entirely of A36 steel and is designed for composite action with the concrete slab. The bottom flange of this section does not require a longitudinal stiffener.

**SECTION FOR MAXIMUM POSITIVE MOMENT**

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>Io</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 T. Fll. Pl. $\frac{3}{16} \times 12$</td>
<td>13.50</td>
<td>28.78</td>
<td>389</td>
<td>11,182</td>
<td>11,182</td>
<td></td>
</tr>
<tr>
<td>2 Web Pl. $\frac{3}{4} \times 58.69$</td>
<td>58.69</td>
<td>-28.72</td>
<td>-1,156</td>
<td>33,200</td>
<td>15,891</td>
<td>15,891</td>
</tr>
<tr>
<td>Bot. Fll. $\frac{3}{16} \times 92$</td>
<td>40.25</td>
<td>-28.72</td>
<td>-1,156</td>
<td>33,200</td>
<td>15,891</td>
<td>15,891</td>
</tr>
</tbody>
</table>

$A_s = -767$ in.$^2$  
$112.44$ in.$^2$  
$-767$  
$60,273$ in.$^4$

$d_{Top \ of \ steel} = 29.06 + 6.82 = 35.88$ in.  
$d_{Bot \ of \ steel} = 28.94 - 6.82 = 22.12$ in.

$S_{Top \ of \ steel} = \frac{55,042}{35.88} = 1,534$ in.$^3$  
$S_{Bot \ of \ steel} = \frac{55,042}{22.12} = 2,488$ in.$^3$
Composite Section, $3n=24$

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_z$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>112.44</td>
<td>56.25</td>
<td>35.25</td>
<td>1,983</td>
<td>69,894</td>
<td>267</td>
</tr>
<tr>
<td>Conc. 180 × 7.5/24</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$168.69 \text{ in.}^2$   $1,216 \text{ in.}^3$   $130,434$

$d_{4x} = \frac{1.216}{168.69} = 7.21 \text{ in.}$

$d_{\text{Top of steel}} = 29.06 - 7.21 = 21.85 \text{ in.}$

$d_{\text{Bot of steel}} = 28.94 + 7.21 = 36.15 \text{ in.}$

$S_{\text{Top of steel}} = \frac{121,667}{21.85} = 5,568 \text{ in.}^3$

$S_{\text{Bot of steel}} = \frac{121,667}{36.15} = 3,366 \text{ in.}^3$

Composite Section, $n=8$

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_o$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>112.44</td>
<td>168.75</td>
<td>35.25</td>
<td>5,948</td>
<td>209,682</td>
<td>791</td>
</tr>
<tr>
<td>Conc. 180 × 7.5/8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$281.19 \text{ in.}^2$   $5,181 \text{ in.}^3$   $270,746$

$d_6 = \frac{5,181}{281.19} = 18.43 \text{ in.}$

$d_{\text{Top of steel}} = 29.06 - 18.43 = 10.63 \text{ in.}$

$d_{\text{Bot of steel}} = 28.94 + 18.43 = 47.37 \text{ in.}$

$S_{\text{Top of steel}} = \frac{175,260}{10.63} = 16,487 \text{ in.}^3$

$S_{\text{Bot of steel}} = \frac{175,260}{47.37} = 3,700 \text{ in.}^3$

$d_{\text{Top of conc.}} = 39.00 - 18.43 = 20.57 \text{ in.}$

$S_{\text{Top of conc.}} = \frac{175,260}{20.57} = 8,520 \text{ in.}^3$

The relationship for Maximum Design Load

$$F_s S \geq 1.30 \left[ D + \frac{5}{3} (L+I) \right]$$

governs the design of the maximum-positive-moment section.

**Bending Moments 48 Ft from End Support**

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$-(L+I)$</th>
<th>$+(L+I)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$, kip-ft</td>
<td>2,261</td>
<td>606</td>
<td>-607</td>
<td>2,541</td>
</tr>
</tbody>
</table>

**Steel Stresses—Combination A**

<table>
<thead>
<tr>
<th>Top of Steel (Compression)</th>
<th>Bottom of Steel (Tension)</th>
</tr>
</thead>
<tbody>
<tr>
<td>For $DL_1$: $F_s = \frac{2,261 \times 12}{1,534} \times 1.30 = 23.0$</td>
<td>$F_s = \frac{2,261 \times 12}{2,488} \times 1.30 = 14.2$</td>
</tr>
<tr>
<td>For $DL_2$: $F_s = \frac{606 \times 12}{5,568} \times 1.30 = 1.7$</td>
<td>$F_s = \frac{606 \times 12}{3,366} \times 1.30 = 2.8$</td>
</tr>
<tr>
<td>For $L+I$: $F_s = \frac{2,541 \times 12}{16,487} \times 1.30 \times \frac{5}{3} = 4.0$</td>
<td>$F_s = \frac{2,541 \times 12}{3,700} \times 1.30 \times \frac{5}{3} = 17.9$</td>
</tr>
</tbody>
</table>

$28.7 < 36 \text{ ksi}$   $36 > 34.9 \text{ ksi}$

6/78 II/7.23
Stress at Top of Concrete—Combination B

\[ f_c = \frac{1.3 \times 12 \left( 606 + \frac{5}{3} \times 2.541 \right)}{8,520 \times 8} = 1.11 < (0.85 \times 4.0 = 3.4 \text{ ksi}) \]

**Check for Fatigue at Web Fillet Welds**

As pointed out previously, box girders often are braced by cross frames at intervals throughout the span, to stabilize the sections during handling. In this example, cross frames are placed at about the one-third points of each span. At each cross frame, a connection plate is fillet welded to the box-girder webs like a transverse-stiffener connection. The webs, therefore, should be investigated for fatigue at the toe of the connection-plate fillet weld.

**SECTION AT WEB STIFFENER**

It is recommended practice to terminate the fillet weld that connects a transverse stiffener to the web at a distance of four to six times the web thickness \( t_w \) from the inner face of the tension flange. With the end of the weld at a distance of \( 4t_w \), the maximum bending stress at the toe of the stiffener fillet weld is

\[ f_s = \frac{M(y-4t_w-t_f)}{I} \]

where \( y \) = distance from centroidal axis of girder to bottom of steel section  
\( t_f \) = flange thickness

The cross-frame connection plate in the positive-moment region is located 41 ft from the end support. The range of the live-load moments at this location is

\[ M_L = \text{Range} = 2,470 + 520 = 2,990 \text{ kip-ft} \]

The range of tensile stress at the connection-plate fillet weld is then calculated as

\[ f_s = \frac{2,990 \times 12}{175,260} \left( 47.37 - 4 \times \frac{44.37}{0.5} = 9.2 < 19 \text{ ksi} \right) \]

The positive-moment section therefore is satisfactory.

**FLANGE-PLATE TRANSITION 25 FT FROM END SUPPORT**

The thickness of the bottom flange is reduced from \( \frac{7}{8} \) in. to \( \frac{5}{8} \) in. at a distance of 25 ft from the end support. The thickness of the steel top flanges is maintained at \( \frac{7}{8} \) in. The section at the transition is investigated.

### Steel Section

<table>
<thead>
<tr>
<th>Material</th>
<th>( A )</th>
<th>( d )</th>
<th>( Ad )</th>
<th>( Ad^2 )</th>
<th>( I_s )</th>
<th>( I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 T. Flg. Pl. ( \frac{7}{8} \times 12 )</td>
<td>13.50</td>
<td>28.78</td>
<td>389</td>
<td>11,182</td>
<td>11,182</td>
<td></td>
</tr>
<tr>
<td>2 Web Pl. ( \frac{7}{8} \times 58.69 )</td>
<td>58.69</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bot. Flg. ( \frac{7}{8} \times 92 )</td>
<td>28.75</td>
<td>-28.66</td>
<td>-824</td>
<td>23,615</td>
<td>23,615</td>
<td></td>
</tr>
</tbody>
</table>

\[ d_s = \frac{-435}{100.94} = -4.31 \text{ in.} \]

\[ -4.31 \times 435 = -1,875 \]

\[ I_{NA} = 48,813 \text{ in.}^4 \]

\[ d_{\text{Top of steel}} = 29.06 + 4.31 = 33.37 \text{ in.} \]

\[ d_{\text{Bot. of steel}} = 28.81 - 4.31 = 24.50 \text{ in.} \]

\[ S_{\text{Top of steel}} = \frac{48,813}{33.37} = 1,463 \text{ in.}^3 \]

\[ S_{\text{Bot. of steel}} = \frac{48,813}{24.50} = 1,992 \text{ in.}^3 \]
Composite Section, 3n = 24

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad^2</th>
<th>I_s</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>100.94</td>
<td>35.25</td>
<td>-435</td>
<td>69,894</td>
<td>267</td>
<td>50,688</td>
</tr>
<tr>
<td>Conc. 180 x 7.5/24</td>
<td>56.25</td>
<td>35.25</td>
<td>1,983</td>
<td>69,894</td>
<td>267</td>
<td>70,161</td>
</tr>
</tbody>
</table>

\[ d_{24} = \frac{1.548}{157.19} = 0.0985 \text{ in} \]
\[ I_{NA} = \frac{-9.85 \times 1.548}{105,601} = 15,248 \text{ in}^4 \]
\[ d_{\text{top of steel}} = 29.06 - 9.85 = 19.21 \text{ in} \]
\[ S_{\text{top of steel}} = \frac{105,601}{19.21} = 5,497 \text{ in}^3 \]

Composite Section, n = 8

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad^2</th>
<th>I_s</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>100.94</td>
<td>35.25</td>
<td>-435</td>
<td>209,682</td>
<td>791</td>
<td>50,688</td>
</tr>
<tr>
<td>Conc. 180 x 7.5/8</td>
<td>168.75</td>
<td>35.25</td>
<td>5,948</td>
<td>209,682</td>
<td>791</td>
<td>210,473</td>
</tr>
</tbody>
</table>

\[ d_8 = \frac{5,513}{269.69} = 0.2044 \text{ in} \]
\[ I_{NA} = \frac{-20.44 \times 5,513}{148,475} = 112,686 \text{ in}^4 \]
\[ d_{\text{top of steel}} = 29.06 - 20.44 = 8.62 \text{ in} \]
\[ S_{\text{top of steel}} = \frac{148,475}{8.62} = 17,224 \text{ in}^3 \]

\[ d_{\text{top of conc.}} = 39.00 - 20.44 = 18.56 \text{ in} \]
\[ S_{\text{top of conc.}} = \frac{148,475}{18.56} = 7,999 \text{ in}^3 \]

As with the maximum-positive-moment section, the relationship for Maximum Design Load governs the design of the section 25 ft from the end support. Fatigue in the base metal adjacent to the butt-welded flange transition should be checked. Fatigue in the girder webs at the toe of the transverse stiffener fillet welds need not be investigated, because stiffeners are used only in the vicinity of the end bearing, where the live-load stress range is small.

**Bending Moments 25 Ft from End Support**

<table>
<thead>
<tr>
<th></th>
<th>DL_1</th>
<th>DL_2</th>
<th>-(L+I)</th>
<th>+(L+I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M, kip-ft</td>
<td>1,970</td>
<td>510</td>
<td>-310</td>
<td>1,960</td>
</tr>
</tbody>
</table>

**Steel Stresses 25 Ft from End Support Due to Maximum Design Loads**

**Top of Steel (Compression)**

For \( DL_1 \): \( F_b = \frac{1,970 \times 12}{1,463} \times 1.30 = 21.0 \)

For \( DL_2 \): \( F_b = \frac{510 \times 12}{5,497} \times 1.30 = 1.4 \)

For \( L+I \): \( F_b = \frac{1,960 \times 12}{17,224} \times 1.30 \times 5 = 3.0 \)

25.4 < 36 ksi

**Bottom of Steel (Tension)**

For \( DL_1 \): \( F_b = \frac{1,970 \times 12}{1,992} \times 1.30 = 15.4 \)

For \( DL_2 \): \( F_b = \frac{510 \times 12}{2,732} \times 1.30 = 2.9 \)

For \( L+I \): \( F_b = \frac{1,960 \times 12}{3,015} \times 1.30 \times 5 = 16.9 \)

36 > 35.2 ksi
Stress at Top of Concrete (Compression)

\[
 f_c = \frac{1.3 \times 12 \left( \frac{510 + \frac{5}{3} \times 1,960}{7,999 \times 8} \right)}{\text{ksi}} = 0.92 < (0.85 \times 4.0 = 3.4 \text{ ksi})
\]

The section therefore is satisfactory for Maximum Design Load.

Check for Fatigue at Flange Transition

The range of live-load stress in the bottom flange at the transition is

\[
f_{v*} = \frac{12(1,960 + 310)}{3,015} = 9.0 < 27.5 \text{ ksi}
\]

Resistance to fatigue therefore is satisfactory at the transition 25 ft from the end bearing.

SECTION TRANSITION 17 FT FROM INTERIOR SUPPORT

A hybrid section is used for the box girder from the interior support to a point on the girder 17 ft away. There, the top and bottom flanges are changed to A36 steel. There are two reasons for fabricating the section entirely of A36 steel rather than hybrid. One reason is that the bending moment decreases rapidly with distance from the pier. As a result, the strength of a hybrid section more than 17 ft from the interior support would be excessive. The second reason is that a compression flange of suitable thickness of steel with \( F_v = 50,000 \text{ psi} \) would require tension flanges larger than necessary to satisfy the criteria:

\[
\frac{\text{Compression-flange area}}{\text{Tension-flange area}} \leq 1.25
\]

The transition section is investigated in the same manner as for the transition section 2 ft from the pier, except that the former section is not hybrid. At the transition, the top flanges are made 1 in. thick, and the bottom flange is made \( \frac{11}{16} \) in. thick. The ST7.5 \times 25 bottom-flange longitudinal stiffener, continued from the pier to the inflection region, is stiff enough to provide the maximum \( k \) value of 4.

Steel Section 17 Ft from Interior Support

<table>
<thead>
<tr>
<th>Material</th>
<th>Area</th>
<th>( d )</th>
<th>( Ad )</th>
<th>( Ad^2 )</th>
<th>( I_v )</th>
<th>( I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 T. Flg. Pl. 1\times 15</td>
<td>30.00</td>
<td>29.00</td>
<td>870</td>
<td>25,230</td>
<td>15,891</td>
<td>15,891</td>
</tr>
<tr>
<td>2 Web Pl. ( \frac{1}{2} \times 58.69 )</td>
<td>58.69</td>
<td>-28.84</td>
<td>-1,824</td>
<td>52,608</td>
<td>41</td>
<td>41</td>
</tr>
<tr>
<td>Bot. Flg. Pl. ( \frac{11}{16} \times 92 )</td>
<td>63.25</td>
<td>-23.25</td>
<td>-171</td>
<td>3,973</td>
<td>41</td>
<td>41</td>
</tr>
<tr>
<td>Stiff. ST7.5 \times 25</td>
<td>7.35</td>
<td>-23.25</td>
<td>-171</td>
<td>3,973</td>
<td>41</td>
<td>41</td>
</tr>
</tbody>
</table>

\[
d_r = \frac{-1,125}{159.29} = -7.06 \text{ in.}
\]

\[
d_{\text{Top of steel}} = 29.50 + 7.06 = 36.56 \text{ in.}
\]

\[
d_{\text{Bot. of steel}} = 29.19 - 7.06 = 22.13 \text{ in.}
\]

\[
S_{\text{Top of steel}} = \frac{89,801}{36.56} = 2,456 \text{ in.}^2
\]

\[
S_{\text{Bot. of steel}} = \frac{89,801}{22.13} = 4,058 \text{ in.}^2
\]

II/7.26
Steel Section, with Reinforcing Steel, 17 Ft from Interior Support

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad^2</th>
<th>I_s</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>159.29</td>
<td>35.03</td>
<td>-1,125</td>
<td>532</td>
<td>18,640</td>
<td>97,743</td>
</tr>
<tr>
<td>Reinforcement</td>
<td>15.19</td>
<td>35.03</td>
<td>532</td>
<td>18,640</td>
<td>18,640</td>
<td></td>
</tr>
</tbody>
</table>

\[ d_e = \frac{-593}{174.48} = -3.40 \text{ in.} \]
\[ I_{NA} = \frac{-3.40 \times 593}{114,367} = 23.9 \text{ in.}^4 \]

\[ d_{Top \text{ of steel}} = 29.50 + 3.40 = 32.90 \text{ in.} \]
\[ d_{Bot \text{ of steel}} = 29.19 - 3.40 = 25.79 \text{ in.} \]

\[ S_{Top \text{ of steel}} = \frac{114,367}{32.90} = 3,476 \text{ in.}^3 \]
\[ S_{Bot \text{ of steel}} = \frac{114,367}{25.79} = 4,435 \text{ in.}^3 \]

\[ d_{\text{Reinf.}} = 35.03 + 3.40 = 38.43 \text{ in.} \]
\[ S_{\text{Reinf.}} = \frac{114,367}{38.43} = 2,976 \text{ in.}^3 \]

**Service-Load Moments 17 Ft from Interior Support**

<table>
<thead>
<tr>
<th></th>
<th>( DL_1 )</th>
<th>( DL_2 )</th>
<th>Lane Load</th>
<th>Truck Load</th>
<th>Truck Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_s ), kip-ft</td>
<td>-3,000</td>
<td>-630</td>
<td>-1,650</td>
<td>-1,290</td>
<td>580</td>
</tr>
</tbody>
</table>

**Steel Stresses 17 Ft from Interior Support Due to Maximum Design Loads**

**Top of Steel (Tension)**

For \( DL_1 \): \( F_t = \frac{3,000 \times 630 \times 1.30}{2,456} = 19.1 \)

For \( DL_2 \): \( F_t = \frac{630 \times 12}{3,476} \times 1.30 = 2.8 \)

For \( L + I \): \( F_t = \frac{1,650 \times 12 \times \frac{5}{3}}{3,476} \times 1.30 \times \frac{5}{3} = 12.3 \)

\( 34.2 < 36 \text{ ksi} \)

**Bottom of Steel (Compression)**

For \( DL_1 \): \( F_b = \frac{3,000 \times 12}{4,058} \times 1.30 = 11.5 \)

For \( DL_2 \): \( F_b = \frac{630 \times 12}{4,435} \times 1.30 = 2.2 \)

For \( L + I \): \( F_b = \frac{1,650 \times 12 \times \frac{5}{3}}{4,435} \times 1.30 \times \frac{5}{3} = 9.7 \)

\( 23.4 < 36 \text{ ksi} \)

**Reinforcing Steel Stress (Tension) 17 Ft from Interior Support**

\[ f_r = \frac{1.3 \times 12 \left( 630 + \frac{5}{3} \times 1,650 \right)}{2,2976} = -17.7 < 40 \text{ ksi} \]

**Check for Buckling**

\[ k = \sqrt{\frac{8I_s}{\pi^2 t^3}} = \sqrt{\frac{8 \times 243.2}{45(\frac{1}{16})^3}} = 5.1 > 4 \]

Use \( k = 4 \). The ratio of stiffener spacing to flange thickness \( w/t = 45/(\frac{1}{16}) = 65.5 \). From the curves for allowable buckling stress, \( F_{cr} = 24.0 > 23.4 \). Resistance of the bottom flange to buckling therefore is satisfactory.

**Check for Fatigue**

Because shear connectors are welded to the top flange, a fatigue check should be made at the transition section to insure that the tensile-stress range in the top flange is within the allowable. The range of live-load stress in the top flange is

\[ f_{cr} = \frac{12 \left( 1,290 + 580 \right)}{3,476} = 6.46 < 19.0 \text{ ksi} \]
The fatigue-stress range in the reinforcing steel is investigated next.

\[ f_r = \frac{12(1,290 + 580)}{2,976} = 7.54 < 20 \text{ ksi} \]

Resistance of the reinforcement to fatigue is therefore satisfactory.

**SECTION NEAR FIELD SPlice**

A field splice is placed 37 ft from the interior support. Here, a transition is made from the negative-moment section made of A36 steel to the positive-moment section used through the maximum-positive-moment region.

For some distance into the dead-load positive-moment region, the bending moment resulting from the sum of the dead-load moment and the negative live-load moment is negative and produces compression in the bottom flange. The \( \frac{3}{8} \)-in. bottom flange used for the maximum-positive-moment section would not have sufficient buckling resistance under this condition unless it is stiffened longitudinally. Therefore, the ST7.5×25 longitudinal stiffener used in the negative-moment region is extended through the field splice and into the dead-load positive-moment region. The stresses on the gross section at the field splice are checked as follows:

### Steel Section 37 Ft from Interior Support

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad^2</th>
<th>I_0</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 T. Flg. Pl. 3/8×12</td>
<td>13.5</td>
<td>28.78</td>
<td>389</td>
<td>11,182</td>
<td>11,182</td>
<td></td>
</tr>
<tr>
<td>2 Web Pl. 3/8×58.69</td>
<td>58.69</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bot. Flg. Pl. 3/8×92</td>
<td>40.25</td>
<td>-28.72</td>
<td>-1,156</td>
<td>33,200</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stiff. ST7.5×25</td>
<td>7.35</td>
<td>-23.25</td>
<td>-171</td>
<td>3,973</td>
<td>41</td>
<td>4,014</td>
</tr>
</tbody>
</table>

\[ 119.79 \text{ in.}^2 \quad -938 \text{ in.}^3 \]

\[ d_s = \frac{-938}{119.79} = -7.83 \quad -7.83 \times 938 = -7,345 \]

\[ I_{NA} = \frac{56,942}{21.11} = 2,697 \text{ in.}^3 \]

\[ d_{Top\ of\ steel} = 29.06 + 7.83 = 36.89 \text{ in.} \]

\[ d_{Bot.\ of\ steel} = 28.94 - 7.83 = 21.11 \text{ in.} \]

\[ S_{Top\ of\ steel} = \frac{56,942}{36.89} = 1,544 \text{ in.}^3 \]

\[ S_{Bot.\ of\ steel} = \frac{56,942}{21.11} = 2,697 \text{ in.}^3 \]

### Steel Section, with Reinforcing Steel, 37 Ft from Interior Support

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad^2</th>
<th>I_0</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>119.79</td>
<td>-938</td>
<td>532</td>
<td>18,640</td>
<td>64,287</td>
<td></td>
</tr>
<tr>
<td>Reinforcement</td>
<td>15.19</td>
<td>35.03</td>
<td></td>
<td></td>
<td>18,640</td>
<td></td>
</tr>
</tbody>
</table>

\[ 134.98 \text{ in.}^2 \quad -406 \text{ in.}^3 \]

\[ d_s = \frac{-406}{134.98} = -3.01 \text{ in.} \quad -3.01 \times 406 = -1,222 \]

\[ I_{NA} = \frac{81,705}{25.93} = 3,151 \text{ in.}^3 \]

\[ d_{Top\ of\ steel} = 29.06 + 3.01 = 32.07 \text{ in.} \]

\[ d_{Bot.\ of\ steel} = 28.94 - 3.01 = 25.93 \text{ in.} \]

\[ S_{Top\ of\ steel} = \frac{81,705}{32.07} = 2,548 \text{ in.}^3 \]

\[ S_{Bot.\ of\ steel} = \frac{81,705}{25.93} = 3,151 \text{ in.}^3 \]

\[ d_{Reinf.} = 35.03 + 3.01 = 38.04 \text{ in.} \]

\[ S_{Reinf.} = \frac{81,705}{38.04} = 2,148 \text{ in.}^3 \]

### Service-Load Moments at Field Splice
**37 Ft from Interior Support**

<table>
<thead>
<tr>
<th>( DL_1 )</th>
<th>( DL_2 )</th>
<th>( (L+I) )</th>
<th>( -(L+I) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>M, kip-ft</td>
<td>-100</td>
<td>50</td>
<td>1,690</td>
</tr>
</tbody>
</table>
Steel Stresses 37 Ft from Interior Support Due to Maximum Design Loads

Top of Steel (Tension)

For $DL_1$: $F_t = \frac{100 \times 12}{1,544} \times 1.30 = 1.0$

For $DL_2$: $F_t = \frac{-50 \times 12}{2,548} \times 1.30 = 0.3$

For $L+I$: $F_t = \frac{1,050 \times 12}{2,548} \times 1.30 \times \frac{5}{3} = 10.7$

Bottom of Steel (Compression)

$F_c = \frac{100 \times 12}{2,697} \times 1.30 = 0.6$

$F_c = \frac{-50 \times 12}{3,151} \times 1.30 = 0.2$

$F_c = \frac{1,050 \times 12}{3,151} \times 1.30 \times \frac{5}{3} = 8.7$

Reinforcing Steel Stress (Tension) 37 Ft from Interior Support

$$f_t = \frac{1.3 \times 12 (-50 + \frac{5}{3} \times 1,050)}{2,148} = 12.3 < 40 \text{ ksi}$$

Check for Buckling

$$k = \frac{8I_t}{wt^3} = \frac{8 \times 243.2}{45(7/6)^2} = 8.02 > 4$$

Use $k = 4$. The ratio of stiffener spacing to flange thickness $w/t = 45/(7/6) = 102.9$. The curves for critical buckling stress presented previously do not extend to a value of $w/t$ this large. Hence, $F_{cr}$ must be calculated.

$$\frac{w}{t} = 6,650 \sqrt{k} = 6,650 \sqrt{4} = 70.1 < 102.9$$

$$F_{cr} = 26.2 \times 10^3 \left( \frac{t}{w} \right)^2 = 26,200 \times 4 \left( \frac{1}{102.9} \right)^2 = 9.9 > 9.1 \text{ ksi}$$

Check for Fatigue

Because the fillet weld of the longitudinal stiffener to the bottom flange is interrupted at the field splice, fatigue of the base metal adjacent to the weld should be checked for tension at the bottom of the stiffener stem, where the weld passes across the thickness of the stem. On the assumption that the longitudinal stiffener is made continuous at the field splice by some sort of splice arrangement, the section properties of the girder with the longitudinal stiffener may be used for calculation of stresses. Section properties therefore are computed for the composite section, including the longitudinal flange stiffener.

**Composite Section, $n = 8$**

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_x$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel section</td>
<td>119.79</td>
<td>35.25</td>
<td>938</td>
<td>359,682</td>
<td>791</td>
<td>210,473</td>
</tr>
<tr>
<td>Conc. 180 x 7.5/8</td>
<td>168.75</td>
<td>35.25</td>
<td>5,948</td>
<td>209,682</td>
<td>791</td>
<td>210,473</td>
</tr>
</tbody>
</table>

$$288.54 \text{ in.}^2 \quad 5,010 \text{ in.}^3 \quad 274,760$$

$$d_s = \frac{5,010}{288.54} = 17.36 \text{ in.}$$

$$d_{Top \ of \ steel} = 29.06 - 17.36 = 11.70 \text{ in.}$$

$$d_{Bottom \ of \ steel} = 29.06 + 17.36 = 46.30 \text{ in.}$$

$$S_{Top \ of \ steel} = \frac{187,786}{11.70} = 16,050 \text{ in.}^3$$

$$S_{Bottom \ of \ steel} = \frac{187,786}{46.30} = 4,056 \text{ in.}^3$$
The maximum range of live-load stress in the bottom flange at the end of the stiffener-to-flange fillet weld is

\[ f_{st} = \frac{1,050 \times 12 \times 25.49}{81,705} + \frac{1,690 \times 12 \times 45.86}{187,786} = 3.9 + 5.0 = 8.9 < 12.5 \text{ ksi} \]

An additional fatigue check is made for tension in the top flange of the girder. On the assumption that shear connectors are welded to the top flange near the splice, the stress range at that section may not exceed 19 ksi. The stress range is

\[ f_{st} = \frac{1,050 \times 12}{2,548} + \frac{1,690 \times 12}{16,050} = 4.9 + 1.3 = 6.2 < 19 \text{ ksi} \]

**TERMINATION OF LONGITUDINAL STIFFENER**

Next, an investigation is made to determine the location of the section at which the ST7.5 x 25 may be terminated in the positive-bending region. The following calculations indicate that at a distance of 10 ft from the field splice, or 47 ft from the interior support, the compressive stress in the bottom flange is less than the critical buckling stress for the bottom-flange plate, so that the longitudinal stiffener is no longer needed.

### Service-Load Moments 47 Ft from Interior Support

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$(L+I)$</th>
<th>$-(L+I)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$, kip-ft</td>
<td>930</td>
<td>300</td>
<td>2,110</td>
<td>925</td>
</tr>
</tbody>
</table>

### Steel Stresses 47 Ft from Interior Support Due to Maximum Design Loads

**Top of Steel (Tension)**

For $DL_1$: \[ F_b = \frac{-930 \times 12}{1,534} \times 1.30 = -9.5 \]

For $DL_2$: \[ F_b = \frac{-300 \times 12}{1,534} \times 1.30 = -3.1 \]

For $-(L+I)$: \[ F_b = \frac{925 \times 12}{1,534} \times 1.30 \times \frac{5}{3} = 15.7 \]

**Bottom of Steel (Compression)**

For $DL_1$: \[ F_b = \frac{-930 \times 12}{2,488} \times 1.30 = -5.8 \]

For $DL_2$: \[ F_b = \frac{-300 \times 12}{2,488} \times 1.30 = -1.9 \]

For $-(L+I)$: \[ F_b = \frac{925 \times 12}{2,488} \times 1.30 \times \frac{5}{3} = 9.7 \]

### Check for Buckling

For the bottom flange without a stiffener, $k = 4$. The ratio of flange width to thickness $w/t = 90/\beta(\beta) = 205.7$. The curves for allowable buckling stress presented previously do not extend to a value of $w/t$ this large. Consequently, $F_{cr}$ must be calculated.

\[ \frac{w}{t} = \frac{6,650 \sqrt{K}}{\sqrt{F_v}} = \frac{6,650 \sqrt{4}}{\sqrt{36,000}} = 70.1 < 205.7 \]

\[ F_{cr} = 26.2 \times 11^4 \left( \frac{\beta}{\beta} \right)^2 = 26,200 \times 4 \left( \frac{76}{90} \right)^2 = 2.5 > 2.0 \text{ ksi} \]

### Fatigue Check at End of Longitudinal Stiffener

The maximum range of stress in the bottom flange at the end of the longitudinal stiffener is

\[ f_{st} = \frac{12(2,110 + 925)46.93}{175,260} = 9.8 < 12.5 \text{ ksi} \]

### LATERAL FLANGE BENDING

The change along the girder span, kips per ft, in the horizontal component of the web shear acts as a lateral horizontal force on the flange of the box girder. Under
the initial dead load $DL_1$, the lateral force due to shear is assumed to be equally distributed to the top and bottom flanges. The lateral force on the top flange causes lateral bending of that flange. The change in vertical shear, and therefore the lateral load on both flanges, is constant. It is equal to the difference between the shears at the girder supports divided by the span.

Let $\Delta V_V$ be the change in $DL_1$ vertical shear, kips per ft, along the girder. With the shear at the end bearing equal to 56.7 kip-ft and the shear at the interior support equal to −108.9 kip-ft,

$$\Delta V_V = \frac{56.7 + 108.9}{120} = 1.38 \text{ kips per ft}$$

The horizontal component of the web shear then is

$$\Delta V_H = \frac{14}{57} \times 1.38 = 0.34 \text{ kips per ft}$$

One-half of $\Delta V_H$, or 0.17 kips per ft, is applied to the top flange as a uniformly distributed lateral force.

To support the top flange under this loading, a strut is placed at appropriate intervals between the webs of the box girder, just below the top flange. The spacing of the struts, the forces acting on them and the deflection of the top flange midway between struts are determined.

**Lateral Bracing in Positive-Moment Region**

The top flange is checked first in the positive-moment region for the combination of lateral and vertical bending. Previous investigations of vertical bending indicated that this flange is understressed. The stress in the flange at the section of maximum positive moment is 28.7 ksi. The capacity available at the section for resisting lateral bending is

$$f_L = 36.0 - 28.7 = 7.3 \text{ ksi}$$

Factored $\Delta V_H = 1.30 \times 0.17 = 0.22 \text{ kips per ft}.$

Assume that the lateral bending moment at a strut is

$$M = \frac{\Delta V_H d^2}{12}$$

where $d =$ spacing, ft, of struts.

The section modulus of the $\frac{9}{16} \times 12$-in. top flange is

$$S_f = \frac{tw^3}{6} = \frac{\frac{9}{16} \times 12}{6} = 13.5 \text{ in.}^3$$

The lateral bending stress then is

$$f_L = \frac{12M}{S_f} = \frac{12 \times 0.889M}{13.5} = 0.889 \times \frac{\Delta V_H d^2}{12} = 0.074 \times 0.22d^2 = 0.0163d^2$$

With $f_L = 7.3$ ksi, solving for $d$ yields

$$d \leq \sqrt{\frac{7.3}{0.0163}} = 21.2 \text{ ft}$$

Bracing of the top flange is provided by a strut incorporated in the cross frames, which are placed at about the third points of the girder span. In addition, a strut is placed midway between the end bearing and the cross frame in the positive-moment region. Spacing of the struts then is 20.5 ft.

The force in the struts $= wd = \Delta V_H d = 0.22 \times 20.5 = 4.5 \text{ kips.}$
The deflection midway between struts is
\[ \Delta = \frac{wd^4}{384EI} = \frac{(0.22/12)(20.5 \times 12)^4}{384 \times 28,000(1/12)(9/16)(12)^3} = 0.07 \text{ in.} \]

**Lateral Bracing in Negative-Moment Region**

The top flange is checked next in the negative-moment region. A strut is placed about midway between the cross frame 36 ft from the interior support, near the field splice, and the cross girder 1.5 ft from the interior support. The total stress in the top flange is computed first for the unbraced span between the strut and the cross frame 36 ft from the interior support and then for the unbraced span between the strut and the cross girder. The unbraced spans are \( \frac{1}{2} (36 - 1.5) = 17.25 \) ft long.

\[ M = \frac{0.22(17.25)^2}{12} = 5.5 \text{ kip-ft} \]

To simplify calculations for the stress at the strut, the vertical bending stress is taken as that at the flange transition 17 ft from the interior support. The resulting stress is on the conservative side, because the strut is actually 18 ft 3 in. from the interior support, where the stress is smaller.

The section modulus of the section at the transition is
\[ S_f = \frac{1(15)^2}{6} = 37.5 \text{ in}^3 \]
\[ f_L = \frac{5.5 \times 12 \times 6}{37.5} = 1.8 \text{ ksi} \]

Stress from vertical bending = \( \frac{34.2}{36.0} = F_v \)

Similarly, to simplify calculations for stress at the cross girder, the vertical bending stress is taken as that at the flange transition 2 ft from the interior support. The lateral bending stress is on the conservative side, because the cross girder is actually 1.5 ft from the interior support. Hence, the unbraced span is 18.25 - 1.50 = 16.75 < 17.25 ft. The section modulus is
\[ S_f = \frac{2(15)^2}{6} = 75 \text{ in}^3 \]
\[ f_L = \frac{5.5 \times 12}{75} = 0.9 \text{ ksi} \]

Stress from vertical bending = \( \frac{42.2}{43.1} < (0.984 \times 50 = 49.2 \text{ ksi}) \)

**Design of Struts**

The struts are all the same size and designed for the maximum lateral force, \( R = 4.5 \) kips. Considered to be secondary members, the struts are always in tension.
The area required for a strut is

\[ A = \frac{R}{F_v} = \frac{4.5}{36} = 0.125 \text{ in.}^2 \]

For the struts as secondary tension members, the largest permissible slenderness ratio \( L/r = 240 \).

Try a \( 3 \times 2\frac{1}{2} \times \frac{1}{4} \)-in. angle.

\[ A = 1.31 > 0.125 \text{ in.}^2 \]

\[ \frac{L}{r} = \frac{114}{0.528} = 216 < 240 \]

The angle is satisfactory.

**FLANGE-TO-WEB WELDS**

The flanges are fillet welded to the webs on both sides of each web. Each pair of welds must resist the horizontal shear at the interface of the web and flange. The welds are checked at the end bearing and near the interior support, where the vertical shear is largest. The horizontal shear flow \( S \), kips per lin. in., may be computed from

\[ S = \frac{VQ}{I} \]

where \( V \) = vertical shear, kips

\( I \) = moment of inertia of girder, in.\(^4\)

\( Q \) = moment of flange area about centroidal axis of girder, in.\(^3\)

Load factor design specifies a maximum strength \( F_v \) of 0.45\( F_u \) for fillet welds, where \( F_u \) is the specified minimum tensile strength of the welding rod. But \( F_u \) may not exceed the tensile strength of the connected parts. Fillet welds are designed for \( F_v \) under the Maximum Design Load. Calculations indicate that the size of the weld is governed by the thickness of the flange, rather than by stress.

With \( F_v = 58 \text{ ksi} \) for A36 steel, the design relationship for strength of a fillet weld is

\[ f_v \text{ due to } 1.30 \left[D + \frac{5}{3}(L+I)\right] \leq (0.45 \times 58 \times 0.707 = 18.5 \text{ ksi}) \]

Here, \( D \), \( L \) and \( I \) are the shear stresses due to dead, live and impact loads.

Investigation of the welds at the end bearing begins with a tabulation of the vertical shears and calculation of horizontal shear flow, kips per lin. in. of weld width.

**Service-Load Shears at End Support**

<table>
<thead>
<tr>
<th></th>
<th>( DL_1 )</th>
<th>( DL_2 )</th>
<th>((L+I))</th>
<th>(-(L+I))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V ), kips</td>
<td>56.7</td>
<td>14.5</td>
<td>68.3</td>
<td>-6.8</td>
</tr>
</tbody>
</table>

**Section Properties at End Support**

Steel Section Only

\[ I = 48,813 \text{ in.}^4 \]

Top Flg.: \( Q = \frac{3}{16} \times 12 \times 33.09 = 223 \text{ in.}^3 \)

Bot. Flg.: \( Q = \frac{1}{8} \times \frac{92}{16} \times 24.34 = 350 \text{ in.}^3 \)

Composite Section, \( n = 8 \)

\[ I = 148,475 \text{ in.}^4 \]

Top Flg.: \( Q = \frac{3}{16} \times 12 \times 8.34 = 56 \)

Conc.: \( Q = \frac{1}{8} \times \frac{180}{8} \times 7.5 \times 14.81 = 1,250 \text{ in.}^3 \)

Bot. Flg.: \( Q = \frac{1}{8} \times \frac{92}{16} \times 49.07 = 705 \text{ in.}^3 \)
Shear Flow $S = \frac{VQ}{I}$ Due to Maximum Design Loads

Top Weld

For $DL_1: S = \frac{56.7 \times 223}{48,813} \times 1.30 = 0.337$

For $DL_2: S = \frac{14.5 \times 1,306}{148,475} \times 1.30 = 0.165$

For $L+I: S = \frac{68.3 \times 1,306}{148,475} \times 1.30 \times \frac{5}{3} = 1.302$

Bottom Weld

$S = \frac{56.7 \times 350}{48,813} \times 1.30 = 0.529$

$S = \frac{14.5 \times 705}{148,475} \times 1.30 = 0.090$

$S = \frac{68.3 \times 705}{148,475} \times 1.30 \times \frac{5}{3} = 0.702$

1.804 kips per in. 1.321 kips per in.

The AASHTO Specifications require that the web be fully developed by the flange-to-web weld, to insure adequate fatigue resistance with respect to transverse distortional stresses. The provisions therefore state that the total effective thickness (based on the throat dimension in the case of fillet welds) must be at least equal to the web thickness.

Shear in the top weld governs. For two welds, the shear flow in each weld is $1.804/2 = 0.902$ kips per in.

Weld size required $= \frac{0.902}{18.5} = 0.049$ in.

This, however, is less than the minimum weld size required by either the thickness of flange or thickness of web. The weld size required by thickness of flange, AASHTO 1.7.21(B)

$= \frac{1}{4}$ in.

The weld size required by thickness of web, AASHTO 1.7.49(E)

$= \frac{0.707 \times 2}{1.414} = 0.5$ in. — governs

Next, the flange-to-web welds are designed at the transition 2 ft from the interior support in the same manner as at the end bearing.

<table>
<thead>
<tr>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$L+I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>V, kips</td>
<td>106</td>
<td>26</td>
</tr>
</tbody>
</table>

Section Properties 2 Ft from Interior Support

Steel Section Only

Top Flg.: $Q = 2 \times 15 \times 33.04 = 991$ in.$^3$

Bot. Pl.: $Q = \frac{1}{2} \times \frac{1}{4} \times 92 \times 25.40 = 1,022$

Stiff.: $Q = \frac{1}{2} \times 7.35 \times 19.71 = 72$

$= 1,094$ in.$^3$

Steel Plus Reinforcing

Top Flg.: $Q = 2 \times 15 \times 30.40 = 912$

Reinf.: $Q = \frac{1}{2} \times 15.19 \times 35.93 = 273$

$= 1,185$ in.$^3$

Bot. Pl.: $Q = \frac{1}{2} \times \frac{1}{4} \times 92 \times 28.04 = 1,129$

Stiff.: $Q = \frac{1}{2} \times 7.35 \times 22.35 = 82$

$= 1,211$ in.$^3$
Shear Flow \( S = \frac{VQ}{I} \) Due to Maximum Design Loads

**Top Weld**

For \( DL_1 \):
\[
S = \frac{106 \times 991}{136,973} \times 1.30 = 0.997
\]

For \( DL_2 \):
\[
S = \frac{26 \times 1,185}{158,022} \times 1.30 = 0.253
\]

For \( L+I \):
\[
S = \frac{72 \times 1,185}{158,022} \times 1.30 \times \frac{5}{3} = 1.170
\]

2.420 kips per in.

**Bottom Weld**

For \( DL_1 \):
\[
S = \frac{106 \times 1,094}{136,973} \times 1.30 = 1.101
\]

For \( DL_2 \):
\[
S = \frac{26 \times 1,211}{158,022} \times 1.30 = 0.259
\]

For \( L+I \):
\[
S = \frac{72 \times 1,211}{158,022} \times 1.30 \times \frac{5}{3} = 1.195
\]

2.555 kips per in.

Shear in the bottom weld governs. For two welds, the shear flow in each weld is \( 2.555/2 = 1.278 \) kips per in.

Weld size required \( = \frac{1.278}{18.5} \approx 0.069 \) in.

Again the weld size is governed by the \( \frac{1}{4} \)-in. web thickness rather than by stress or by flange thickness. A \( \frac{3}{16} \)-in. web-to-flange weld is required throughout the length of the box girder.

**Shear-Stress Range in Bottom Weld Due to Service Loads**

The maximum shear range at the transition 2 ft from the interior support equals

\[
S_r = \frac{72 \times 1,211}{158,022} = 0.552 \text{ kips per in.}
\]

The actual stress range in the \( \frac{3}{16} \)-in. fillet weld equals

\[
f_{wr} = \frac{0.552}{2 \times 0.707 \times \frac{3}{16}} = 1.04 < 12 \text{ ksi}
\]

**END DIAPHRAGM AND EXPANSION DAM**

At the end support, each box girder is supported at the center of the bottom flange on a single bearing. An end diaphragm is required at this support to retain the shape of the cross section and to transfer the loads on the girder into the bearing. A \( \frac{1}{2} \)-in. plate is used for the web and a \( \frac{1}{2} \times 10 \)-in. plate as the top flange of the diaphragm. The bottom flange of the box girder serves as the bottom flange of the diaphragm.

---

**Sections at End Diaphragm**
Design of Bearing Stiffeners at End Support

For access to the bridge-seat area between the diaphragm and the backwall of the abutment, a 14 × 26-in. screen-covered manhole is provided in the center of the diaphragm. The manhole is flanked by two bearing stiffeners. These stiffeners are designed as columns in accordance with load-factor-design provisions for compression members.

Assume that each stiffener consists of two 5-in.-wide plates welded to opposite sides of the diaphragm web. The minimum thickness required by width-thickness-ratio limitations for a stiffener is

\[ t = \frac{b'}{12} \sqrt{ \frac{F_v}{33,000} } = \frac{5}{12} \sqrt{ \frac{36,000}{33,000} } = 0.435 \text{ in.} \]

### End Reactions

<table>
<thead>
<tr>
<th></th>
<th>DL₁</th>
<th>DL₂</th>
<th>L + I</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V ), kips</td>
<td>56.7</td>
<td>14.5</td>
<td>68.3</td>
<td>139.5</td>
</tr>
</tbody>
</table>

The allowable bearing stress for a stiffener under service loads is 29 ksi. The minimum stiffener thickness required for bearing then is

\[ t = \frac{139.5/2}{29(5 - 0.25 - 0.25)} = 0.534 > 0.435 \text{ in.} \]

Try two \( \frac{3}{8} \times 5\)-in. plates for each bearing stiffener. (See Section A-A of Section at End Diaphragm.)

The stiffener column consists of the two \( \frac{3}{8} \times 5\)-in. plates plus a length of web equal to

\[ L_w = 9t_w = 9 \times \frac{5}{8} = 4.5 \text{ in.} \]

Area of the equivalent column is

\[ A_e = 2 \times \frac{3}{8} \times 5 + \frac{5}{8} \times 4.5 = 8.5 \text{ in.}^2 \]

Moment of inertia of the equivalent column is

\[ I_e = \frac{(5/8)(5 + 0.5 + 5)^3}{12} + 60.3 \text{ in.}^4 \]

and the radius of gyration is

\[ r = \sqrt{ \frac{I}{A} } = \sqrt{ \frac{60.3}{8.5} } = 2.66 \text{ in.} \]

Consequently, the slenderness ratio of the stiffener equals

\[ \frac{KL_e}{r} = \frac{D}{r} = \frac{57}{2.66} = 21.4 \]

\[ \sqrt{ \frac{2\pi^2E}{F_v} } = \sqrt{ \frac{2\pi^2 	imes 29,000}{36} } = 126 > 21.4 \]

The allowable stress then is

\[ F_{cv} = F_v \left[ 1 - \frac{F_v}{4\pi^2(\frac{D}{r})^2} \right] = 36 \left[ 1 - \frac{36}{4\pi^2 \times 29,000 \left( \frac{57}{2.66} \right)^2} \right] = 35.5 \text{ ksi} \]

The Maximum Design Load on the columns is

\[ V_w = 1.3 \left( 56.7 + 14.5 + \frac{5}{3} \times 68.3 \right) = 241 \text{ kips} \]
 Bracket Design

The maximum bending stress in the bracket, which is connected to the outer web of the box girder, occurs at Section A-A through the bracket where the bracket stem is 21.75 in. long.

In calculation of section properties for design of the bracket, neglect the area of web holes and deduct the area of flange holes exceeding 15% of the top-flange area.

Flange area $A_f = 0.585 \times 8.96 = 5.21$ in.$^2$

15% $A_f = 0.15 \times 5.24 = 0.79$ in.$^2$

Area of two flange holes $= 2 \times 1 \times 0.585 = 1.170$ in.$^2$

Areas of holes over 15% $A_f = 1.17 - 0.79 = 0.38$ in.$^2$

Properties of Section A-A Through Bracket

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_x$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Flg. 0.585 x 8.96</td>
<td>5.24</td>
<td>11.17</td>
<td>58.5</td>
<td>654</td>
<td>654</td>
<td>654</td>
</tr>
<tr>
<td>Flg. Holes</td>
<td>-0.38</td>
<td>11.17</td>
<td>-4.2</td>
<td>-47</td>
<td>-47</td>
<td>-47</td>
</tr>
<tr>
<td>Stem 0.415 x 21.75</td>
<td>9.03</td>
<td></td>
<td></td>
<td>356</td>
<td>356</td>
<td>356</td>
</tr>
</tbody>
</table>

$d_c = \frac{54.0}{13.88} = 3.89$ in.

$-3.91 \times 54.3 = \frac{-212}{2}$

$I_{NA} = \frac{751}{14.79} = 50.8$ in.$^3$

If dead load on the bracket is neglected, the maximum-design-load moment is

$M = 1.3 \times \frac{5}{3} \times 65.9 = 142.8$ kip-ft

The bending stress in the stem under maximum design load is

$f_s = \frac{142.8 \times 12}{50.8} = 33.7 < 36$ ksi

The W18 x 35 bracket is satisfactory for bending.

Bracket Web Connection

Try two vertical rows of seven $\frac{3}{8}$-in.-dia A325 bolts in the bracket web, 3 in. c. to c., and two horizontal rows of four $\frac{3}{8}$-in.-dia bolts in the bracket flange. Thus, there are a total of 22 bolts. The location $y$ of the horizontal axis of the 22 bolts with respect to the horizontal axis of the web bolts is calculated as follows, noting that the sum of the moments of the web bolts about their axis equals zero:

$y = \frac{8 \times 11.875}{22} = 4.32$ in.
The 14 bolts in the bracket-web connection must carry the vertical shear. All 22 bolts are subjected to the bracket maximum bending moment. The forces imposed on the bolts are produced by the Overload \( 5/3(L+I) \), DL being ignored.

Shear = \( \frac{5}{3} \times 20.8 = 34.7 \) kips

Moment = \( \frac{5}{3} \times 65.9 = 109.8 \) kip-ft

**Moments of Inertia of Bolts**

\[
I_r = 14(1.5)^2 + 4(2)^2 + (6)^2 = 192
\]

The polar moment of inertia of the bracket bolts is

\[
I = I_r + I_z = 1222 + 192 = 1,414
\]

The distance from the centroid to the outermost bolt is

\[
d = \sqrt{(12 + 4.32)^2 + (2)^2} = 16.44 \text{ in.}
\]

The load per bolt due to shear is

\[
P_s = \frac{34.7}{14} = 2.5 \text{ kips}
\]

The load on the outermost bolt due to moment is

\[
P_m = \frac{109.8 \times 12 \times 16.44}{1,414} = 15.32 \text{ kips}
\]

The vertical component of this load is

\[
P_v = \frac{15.32 \times 1.5}{16.44} = 1.4 \text{ kips}
\]

The horizontal component is

\[
P_h = \frac{15.32(9 + 4.32)}{16.44} = 12.4 \text{ kips}
\]

Therefore, the total load on the outermost bolt is the resultant

\[
P = \sqrt{(2.5 + 1.4)^2 + (12.4)^2} = 13.0 \approx 12.6 \text{ kips}
\]

Use fourteen \( \frac{3}{8} \)-in.-dia bolts in the web in two rows and 8 bolts in the flange in two rows.

**Bracket Flange Connection**

The force due to bending on the flange bolts is

\[
P = \frac{109.8 \times 12 \times 7.56}{1,414} = 7.04 \text{ kips per bolt} < 12.6 \text{ kips}
\]
Top Splice Plate on Bracket

The average top-flange bending stress under maximum design load is

\[ F_b = \frac{142.8 \times 12(11.17 - 3.89)}{750} = 16.6 \text{ ksi} \]

The force in the top flange is

\[ P = F_b A_j = 16.6 \times 5.21 = 86.5 \text{ kips} \]

The required area of splice plate therefore is

\[ A = \frac{86.5}{36} = 2.40 \text{ in.}^2 \]

Use Pl. ½ × 9 in. Net area = 0.5(9 - 2 × 1) = 3.50 > 2.40 in.²

**BOX-GIRDER ACCESS MANHOLE**

A manhole is provided near the end bearing for access to the box-girder interior. Because the stress is small in the bottom flange, the manhole is placed in that flange rather than in the web, which is subject to high shearing stress. With the manhole close to the end bearing, the bottom flange needs no reinforcement around the hole. Details of the manhole are shown in horizontal and vertical box-girder sections.

---

**WEB STIFFENERS**

The distance, \( d_s \), of the first stiffener from the end bearing is governed either by \( D \), the depth of the web, or by the formula

\[ d_s = 14,500 \sqrt{\frac{D t_s^3}{V}} \]

For this girder the web depth, \( D \), is 58.69 inches and

\[ d_s = 14,500 \sqrt{\frac{58.69(\frac{1}{2})^3}{248,000}} = 78.8 \text{ in.} > 58.69 \text{ in.} \]
Therefore, one stiffener is placed 58 in. from the end bearing.

Calculations indicate that stiffeners are required over almost all of the negative-moment region. If the actual shear exceeds 60% of the shear capacity, the actual moment is limited by

\[ \frac{M}{M_u} \leq 1.375 - 0.625 \frac{V}{V_u} \]

A reasonable way of spacing the web stiffeners in the negative-moment region is to use the maximum possible spacing that does not reduce the moment capacity of the box girder.

The first stiffener space adjacent to the pier is measured from the cross-girder web, which is 18 in. from the centerline of the pier. The bending moment in the box girder at the cross girder is much smaller than the moment capacity of the box girder, because the bottom-flange thickness is increased for a bi-axial state of stress.

At the flange transition of the hybrid section 2 ft from the pier, however, the moment capacity of the section is much closer to the design moment. A 57-in. stiffener spacing is selected and checked to insure that the ratio of design moment \( M \) to the moment capacity \( M_u \) is satisfactory.

At the flange transition, the Maximum-Design-Load shear along the sloped web is 335 kips. For calculation of the web shear capacity \( V_u \) with the stiffener spacing \( d_o = 57 \) in.,

\[ C = 18,000 \frac{V_u}{D} \left( \frac{1 + (D/d_o)^2}{F_v} \right) - 0.3 = 18,000 \times \frac{\frac{\frac{1}{2}}{58.69}}{36,000} - 0.3 = 0.860 \]

\[ V_v = 0.58F_vD_v = 0.58 \times 36 \times 58.69 \times \frac{1}{2} = 613 \text{ kips} \]

For a hybrid girder,

\[ V_u = V_v C = 613 \times 0.860 = 527 \text{ kips} \]

\[ \frac{V}{V_u} = \frac{335}{527} = 0.636 > 0.6 \]

Therefore, the maximum permissible value of \( M/M_u \) is

\[ \frac{M}{M_u} = 1.375 - 0.625 \frac{V}{V_u} = 1.375 - 0.625 \times 0.636 = 0.978 \]

Previous calculations of the flange stress at the transition determined that under the Maximum Design Load the stress in the top flange is 42.2 ksi and in the bottom flange 34.9 ksi. The design strength in tension \( F_v \) is 49.2 ksi and the critical buckling stress \( F_v^* \) is 36.8 ksi.

The actual value of \( M/M_u \) is

**Top Flange**

\[ \frac{M}{M_u} = \frac{F_v}{F_v^*} = \frac{42.2}{49.2} = 0.858 < 0.978 \]

**Bottom Flange**

\[ \frac{F_v}{F_v^*} = \frac{34.9}{36.8} = 0.948 < 0.978 \]

The 57-in. stiffener spacing is satisfactory at the flange transition 2 ft from the interior support.

Stiffeners are placed on opposite sides of the field splice at a distance of 12 in. from the centerline of the splice. Try four stiffeners spaced at 71.4 in. c. to c. between the stiffener adjacent to the field splice and the stiffener 57 in. from the cross-girder web, or 6.25 ft from the pier. A check of the stiffener spacing starts with the second stiffener space from the pier.

At 6.25 ft from the pier, the Maximum Design Shear is 311 kips.

**Bending Moments 6.25 Ft from Interior Support**

<table>
<thead>
<tr>
<th>( M ), kip-ft</th>
<th>( DL_1 )</th>
<th>( DL_2 )</th>
<th>(- (L+I))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5,000</td>
<td>-1,120</td>
<td>-2,570</td>
<td></td>
</tr>
</tbody>
</table>
Steel stresses 6.25 ft from the interior support are computed with the section moduli calculated for the section 2 ft from that support.

### Steel Stresses 6.25 Ft from Interior Support

Due to Maximum Design Loads

<table>
<thead>
<tr>
<th>Section</th>
<th>Top of Steel</th>
<th>Bottom of Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>For $DL_1$: $F_b = \frac{5,000 \times 12}{4,024} \times 1.30 = 19.4$</td>
<td>$F_b = \frac{5,000 \times 12}{5,301} \times 1.30 = 14.7$</td>
<td></td>
</tr>
<tr>
<td>For $DL_2$: $F_b = \frac{1,120 \times 12}{5,033} \times 1.30 = 3.5$</td>
<td>$F_b = \frac{1,120 \times 12}{5,549} \times 1.30 = 3.1$</td>
<td></td>
</tr>
<tr>
<td>For $L+I$: $F_b = \frac{2,570 \times 12}{5,033} \times 1.30 \times \frac{5}{3} = 13.3$</td>
<td>$F_b = \frac{2,570 \times 12}{5,549} \times 1.30 \times \frac{5}{3} = 12.0$</td>
<td></td>
</tr>
</tbody>
</table>

For calculation of the shear capacity $V_u$ of the web, with $d = 71.4$ in. and $V_r = 613$ kips.

$$C = 18,000 \times \frac{1}{58.69} \left(1 + \frac{(58.69)(71.4)}{36,000}\right) - 0.3 = 0.746$$

$$V_u = V_r C = 613 \times 0.746 = 457 \text{ kips}$$

$$\frac{V}{V_u} = \frac{311}{457} = 0.681 > 0.6$$

Therefore, the maximum permissible value of $M/M_u$ is

$$\frac{M}{M_u} = 1.375 - 0.625 \times 0.681 = 0.949$$

The actual value of $M/M_u$ is

**Top Flange**

$$\frac{M}{M_u} = \frac{F_b}{F_u} = 36.2 \div 49.2 = 0.736 < 0.949$$

**Bottom Flange**

$$\frac{F_b}{F_u} = \frac{29.8}{36.8} = 0.810 < 0.949$$

For the second stiffener space from the pier, 71.4 in. is satisfactory.

Another check is made of the stiffener space measured from the transition section 17 ft from the interior support. Calculations show that with the 71.4-in. spacing, the shear is less than 60% of the shear capacity. Hence, no reduction in moment capacity is required. Thus, the spacing of 71.4 in. is satisfactory at the transition.

From previous design calculations for this transition section, the maximum design shear is 268 kips and $V_u = 457$ kips.

$$\frac{V}{V_u} = \frac{268}{457} = 0.586 < 0.6$$

Therefore, moment capacity need not be reduced.

On the basis of the preceding calculations, the stiffener spacing in the negative-moment region is set as follows:

1. First stiffener: 75 in. from interior support.
2. Next four stiffeners: Equally spaced at 71.4 in.
3. Sixth and seventh stiffeners: 12 in. on either side of the field splice centerline.

### Design of Intermediate Stiffeners

The web stiffeners are attached to the inside face of the web. They must satisfy requirements for minimum area and moment of inertia, as described in the Introduction of this chapter.

A plate $\frac{3}{8} \times 5$ in. is selected for all the stiffeners. The stiffener adjacent to the end bearing is located at a section that is subjected to larger shears than the sections
at the other stiffeners. Therefore, it is checked, and because it is satisfactory, the other stiffeners also are.

For calculation of the shear capacity of the web 58 in. from the end bearing, with \( V_p = 613 \) kips,

\[
C = 18,000 \times \frac{1}{58.69} \left[ 1 + \frac{(58.69/58)^2}{36,000} \right] - 0.3 = 0.850
\]

\[
V_u = V_p \left[ C + \frac{0.87(1-C)}{\sqrt{1 + (d_w/D)^2}} \right] = 613 \left[ \frac{0.850 + \frac{0.87(1-0.850)}{\sqrt{1 + (58/58.69)^2}}} \right] = 578 \text{ kips}
\]

The area of the stiffener should be at least

\[
A = \frac{Y}{V_u} \left[ 0.15BDt_w(1-C) \frac{V}{V_u} - 18t_w^2 \right]
\]

where \( B = 2.4 \) for a single-plate stiffener

\[ Y = \text{ratio of yield strength of web to that of stiffener} \]

\[
A = \frac{36}{36} \left[ 0.15 \times 2.4 \times 58.69 \times \frac{1}{2} (1-0.850) \frac{248}{578} - 18 \left( \frac{1}{2} \right)^2 \right] = -3.82
\]

The negative result indicates that the web contribution, \( 18t_w^2 \), is more than enough in itself to satisfy the area requirement.

The width-thickness ratio \( b'/t \) of the stiffener plate is \( 5/(3\frac{1}{2}) = 13.3 \). The permissible maximum ratio is

\[
b' = \frac{2.600}{13.7} = 2.600 \text{ in.} < 13.3
\]

The moment of inertia of the stiffener plate about the edge connected to the web is

\[
I = \frac{td^3}{3} = \frac{36}{3} (3\frac{1}{2}) (5)^3 = 15.6 \text{ in.}^4
\]

The minimum moment of inertia required is computed as follows:

\[
J = 2.5(D/d_w)^2 - 2 = 2.5(58.69/58.0)^2 - 2 = 0.56
\]

\[
I = d_w^2 J = 58(3\frac{1}{2}) (0.56) = 4.06 < 15.6 \text{ in.}^4
\]

The \( 3\frac{1}{2} \times 5 \)-in. plate also satisfies width-to-thickness and moment of inertia requirements.

**SHEAR CONNECTORS**

A \( 3\frac{1}{2} \)-in.-dia., 5-in.-high, welded stud is placed on each side of the web at intervals along both top flanges of the box girder, to serve as a shear connector between the flanges and the concrete slab. The shear-connector spacing is calculated in exactly the same manner as for the composite wide-flange beam of Chapter 3A. The spacing of the connectors is governed by fatigue under service loads in the positive-moment regions. Maximum spacing is 24 in. in the negative-moment region.

Allowable stud loads are determined for a ratio of stud height \( H \) to diameter \( d \) greater than 4.

\[
\frac{H}{d} = \frac{5}{0.875} = 5.7 > 4
\]

For \( H/d > 4 \), AASHTO Specifications give the ultimate strength of a shear connector as

\[
S_u = 0.4d^2 \sqrt{f_{c} E_c}
\]

where \( E_c = \) modulus of elasticity of concrete = 57,000 \( \sqrt{f_{c}} \)

\( f_{c} = 28 \text{-day strength of concrete, psi} = 4,000 \text{ psi} \)

\[
S_u = 0.4d^2 \sqrt{57,000 (f_{c})^{3/4}} = 0.4(3\frac{1}{2})^2 \sqrt{57,000 (4,000)^{3/4}} = 36,800 \text{ psi}
\]
With $\alpha$ given in AASHTO Specifications as 10.6 for 500,000 cycles of load, the load range per shear connector is

$$Z_r = \alpha d^2 = 10.6 \left(\frac{3}{4}\right)^2 = 8.11 \text{ kips per stud}$$

**Shear-Connector Spacing for Service Behavior (Fatigue)**

Shear-connector spacing is computed initially at the section at the end bearing. A similar computation is made for each tenth point of the span, and a curve of theoretical spacing for service behavior is plotted. (This curve is relatively flat. As a result, it is not necessary to calculate the spacing at every point.) The actual, stepped, shear-connector spacing diagram is drawn below the theoretical curve.

At the end support, the shear range $V_r = 68.3 + 6.8 = 75.1 \text{ kips per web}$. Section properties for computation of the shear per inch $S_r$ between the concrete slab and the top flanges of the girder at the end support are the same as the section properties determined previously for the transition section 25 ft from the end support, with $n = 8$. The area of the effective concrete slab is 168.75 in.² The distance from the centroidal axis of the girder to the centroidal axis of the slab is 35.25 – 20.44 = 14.81 in. The moment of inertia of the girder $I = 148,475 \text{ in.}^4$. Hence,

$$S_r = \frac{V_r Q}{I} = \frac{75.1 \times 168.75 \times 14.81}{148,475} = 1.26 \text{ kips per in.}$$

Spacing required (4 studs, 2 webs) = \frac{4 \times 8.11}{2 \times 1.26} = 12.9 \text{ in. Use 12 in.}

**Shear-Connector Spacing**

**Shear Connectors—Strength Requirements**

The number of connectors provided for fatigue is checked to insure that adequate connectors are provided for ultimate strength. The number of connectors between the point of maximum positive moment and the end support must equal or exceed

$$N_i = \frac{P}{0.85S_e}$$
where $P$ is the smaller of the following two forces computed at the point of maximum moment, 48 ft from the end support:

$$P_1 = A_Fv = 112.44 \times 36 = 4,048 \text{ kips}$$

$$P_2 = 0.85f'bc = 0.85 \times 4 \times 180 \times 7.5 = 4,590 > 4,048 \text{ kips}$$

$P_1$ governs. Thus, the number of connectors required is

$$N_1 = \frac{4,048}{0.85 \times 36.8} = 129.4 \text{ or } \frac{129.4}{4} = 33 \text{ rows}$$

Service-Load design provides 42 rows of shear connectors between the end support and the maximum-positive-moment section. The ultimate-strength requirement for this region therefore is satisfied.

The number of connectors required between the point of maximum positive moment and the interior support is determined for the section at the interior support from

$$N_2 = \frac{P_1 + P_3}{0.85S_u}$$

where $P_3 = A_Fv = 15.19 \times 40 = 608 \text{ kips}$

Therefore, the number of connectors required is

$$N_2 = \frac{4,048 + 608}{0.85 \times 36.8} = 148.8 \text{ or } \frac{148.8}{4} = 38 \text{ rows}$$

The number of rows furnished for Service Loads between the maximum-positive-moment section and the interior support is 47. This number satisfies strength requirements.

**WELDED FIELD SPLICE**

A field splice is placed at the inflection point of each span, 37 ft from the interior support. The splice is made with full-penetration butt welds. Because there is a thickness change in both the bottom and top flanges, fatigue restrictions for base metal adjacent to a butt weld must be satisfied. The condition of fatigue in base metal adjacent to a fillet weld, previously investigated at the cut-off of the longitudinal bottom-flange stiffener, however, is more severe and the section was found to be satisfactory for that condition. Hence, no further investigation for fatigue is necessary at the section 37 ft from the interior support.

The change in width of the top-flange is made in accordance with the taper required by Art. 1.7.10 of the AASHTO Specifications. Details of the welded splice are illustrated.
BOLTED FIELD SPLICE

For Load-Factor design of a bolted field splice, AASHTO Specifications require that the splice material be proportioned for the Maximum Design Load and resistance to fatigue under Service Loads. Because friction connections must resist slip under Overload, fastener size must be selected for an allowable stress \( F_s = 21 \text{ ksi} \) under the Overload of \( D + 5.3(L + I) \).

The allowable load for a \( \frac{7}{8} \text{-in.-dia.}, \ A325 \) bolt in double shear is

\[
P = 2 \times 0.6013 \times 21 = 25.3 \text{ kips per bolt}
\]

For design of the splice material for the Maximum Design Load, the design moment is chosen as the greater of:

- Average of the calculated moment on the section and maximum capacity of the section.
- 75% of the maximum capacity of the section.

The calculated moment is that induced by the Maximum Design Load \( 1.30(D + 5.3(L + I)) \). Splice material should have a capacity equal at least to the design moment. The section capacity is based on the gross section minus any loss in flange area due to bolt holes with area exceeding 15% of each flange area.

### Bending Moments 37 Ft from Interior Support, Kip-Ft

<table>
<thead>
<tr>
<th>For Service Loads</th>
<th>Factor</th>
<th>For Overload</th>
<th>Factor</th>
<th>Maximum Design Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>( DL_1 )</td>
<td>-100</td>
<td>1</td>
<td>-100</td>
<td>1.30</td>
</tr>
<tr>
<td>( DL_2 )</td>
<td>50</td>
<td>1</td>
<td>50</td>
<td>1.30</td>
</tr>
<tr>
<td>( + (L + I) )</td>
<td>1,390</td>
<td>5</td>
<td>2,817</td>
<td>1.30</td>
</tr>
<tr>
<td>( - (L + I) )</td>
<td>-1,050</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum</td>
<td>1,640</td>
<td></td>
<td>2,767</td>
<td></td>
</tr>
<tr>
<td>Minimum</td>
<td>-1,100</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Shears 37 Ft from Interior Support, Kips

<table>
<thead>
<tr>
<th>For Service Loads</th>
<th>Factor</th>
<th>For Overload</th>
<th>Factor</th>
<th>Maximum Design Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>( DL_1 )</td>
<td>58.0</td>
<td>1</td>
<td>58.0</td>
<td>1.30</td>
</tr>
<tr>
<td>( DL_2 )</td>
<td>13.8</td>
<td>1</td>
<td>13.8</td>
<td>1.30</td>
</tr>
<tr>
<td>( L + I )</td>
<td>47.8</td>
<td>5</td>
<td>79.7</td>
<td>1.30</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- The section at the splice is subject to the following moments:
  - Negative moment that acts only on the steel section.
  - Positive moment that acts on the composite steel-concrete section.
  - Negative moment resisted by the steel section and the concrete reinforcement. Because the effects of positive moment dominate at the splice, splice material is designed for positive moment. Also, to simplify the design procedure, the composite concrete slab is neglected.
  - Net section properties at the splice are those for the smaller section, on the positive-moment side of the splice.
BOLTED FIELD SPLICE

Net Section at Top-Flange Splice

The splice of each top flange is made with $\frac{3}{8}$-in.-dia, A325 bolts, arranged staggered in four rows. Pitch of the bolts longitudinally is $s = 3$ in. Gage $g = 2\frac{1}{8}$ in.

$$\frac{s^2}{4g} = \frac{(3)^2}{4 \times 2.375} = 0.947$$

BOLT HOLES IN TOP FLANGE

The deduction from the flange width at the section across the flange through two holes equals $2 \times 1 = 2.00$ in.

The deduction from the flange width at a section through a chain of four holes equals $4 \times 1 - 2 \times 0.947 = 2.106 > 2.00$ in. Use 2.106 in. for the deduction in computing the net flange area.

Flange Area and Deductions

Gross Area = $\frac{3}{8} \times 12 = 6.75$ in.$^2$

Area deducted for bolt holes = $\frac{3}{8} \times 2.106 = 1.18$

$-15\%$ of gross area = $-0.15 \times 6.75 = -1.01$

Net deduction for two flanges = $0.17 \times 2 = 0.34$ in.$^2$
Net Section at Bottom Flange and Stiffener Splices

Assume that the center of gravity of the stiffener coincides with the center of gravity of the bolt holes. Deduct the following areas: 16 holes in the bottom-flange plate, two holes from the flange of the stiffener and two holes from the stiffener stem.

Flange Area and Deductions

Gross area of bottom flange and stiffener = \( \frac{7}{16} \times 92 + 7.35 = 47.60 \text{ in.}^2 \)

Area deducted for bolt holes = \( \frac{7}{16} \times 16 + 2 \times 0.622 + 2 \times 0.55 = 9.34 \)

-15% of gross area = \(-0.15 \times 47.60 = -7.14 \)

Net deduction for bottom flange and stiffener = \( 2.20 \text{ in.}^2 \)

Properties of the gross cross section of the box girder are obtained from previous calculations for the section 37 ft from the interior support. The bolt holes in the flanges are deducted in the computation of properties of the net section.

Net Section at the Splice—Steel Section Only

<table>
<thead>
<tr>
<th>Material</th>
<th>( A )</th>
<th>( d )</th>
<th>( Ad )</th>
<th>( Ad^2 )</th>
<th>( I_x )</th>
<th>( I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Section</td>
<td>119.79</td>
<td>-0.34</td>
<td>28.78</td>
<td>63</td>
<td>-938</td>
<td>-885</td>
</tr>
<tr>
<td>Top Flg. Bolt Holes</td>
<td>-2.20</td>
<td>117.25</td>
<td>-10</td>
<td>-1,815</td>
<td>64,287</td>
<td>62,190</td>
</tr>
</tbody>
</table>

\( d_s = \frac{-885}{117.25} = -7.55 \text{ in.} \)

\( d_{Top of steel} = 29.06 + 7.55 = 36.61 \)

\( d_{Bot. of steel} = 28.94 - 7.55 = 21.39 \text{ in.} \)

\( S_{Top of steel} = \frac{55,508}{36.61} = 1,516 \text{ in.}^3 \)

\( S_{Bot. of steel} = \frac{55,508}{21.39} = 2,595 \text{ in.}^3 \)

Design Moments and Shears at the Field Splice

The capacity of the net section is based on the minimum section modulus of the steel section. For \( F_s = 36 \text{ ksi} \), the net section capacity is

\[ M_{net} = \frac{36 \times 1,516}{12} = 4,548 \text{ kip-ft} \]

75% \( M_{net} = 0.75 \times 4,548 = 3,411 \text{ kip-ft} \)

The average of the calculated moment for the design loads and the net capacity of the section is

\[ M_{av} = \frac{3,597 + 4,548}{2} = 4,073 > 3,411 \text{ kip-ft} \]

The design moment, therefore, is 4,073 kip-ft.

The design shear is determined by multiplying the calculated shear for the design loads, 196.9 kips, by the ratio of the design moment to the calculated moment on the section, 3,597 kip-ft.

Design Shear = \[ 196.9 \times \frac{4,073}{3,597} = 233 \text{ kips} \]

In the plane of each web,

Design Shear = \[ \frac{223}{2} \times \frac{58.69}{57} = 115 \text{ kips} \]

6/78
Web Splice

The web splice plates must carry the design shear, a moment $M_w$ due to the eccentricity of the shear, and a portion $M_o$ of the design moment on the section. The portion of the design moment to be resisted by the web is obtained by multiplying the design moment by the ratio of the moment of inertia of the web to the net moment of inertia of the entire section. The gross moment of inertia is obtained from the earlier calculation of section properties 37 ft from the interior support and adjusted for the change in position of the centroidal axis because of deductions for bolt holes in the flanges.

$$I_o = 15,891 + 58.69(7.55)^2 = 19,236 \text{ in.}^4$$

**Web Moments for Design Loads**

$$M_o = \frac{223 \times 3.25}{12} = 60$$

$$M_w = 4,073 \times \frac{19,366}{55,508} = 1,411 \text{ kip-ft, or 736 kip-ft per web}$$

Try two $\frac{3}{8} \times 55$-in. web splice plates. Assume two rows of $\frac{3}{4}$-in.-dia, A325 bolts, with 14 bolts per row, on each side of the joint. The area of one hole is 0.375 in.$^2$. The holes remove from each splice plate the following percentage of its cross-sectional area:

$$\% \text{ of plate} = \frac{14 \times 0.375}{0.375 \times 55} \times 100 = 25.5 \%$$

Consequently, the fraction of the hole area that must be deducted in determination of the net section is

$$\frac{25.5 - 15.0}{22.5} = 0.41$$

With 4-in. spacing of bolts along the slope of the web,

$$d^2 \text{ for holes} = 2^2 + 6^2 + 10^2 + 14^2 + 18^2 + 22^2 + 26^2 = 1,820$$

$$\Sigma Ad^2 = 4 \times 0.41 \times \frac{3}{8} \times 1,820 = 1,119 \text{ in.}^4$$

or, with respect to a horizontal axis

$$\Sigma Ad^2 = 1,119 \left(\frac{57}{58.69}\right)^2 = 1,055 \text{ in.}^4$$

Assume that the neutral axis of the splice coincides with the neutral axis of the net section of the box girders. The bending properties of the web splice plates with respect to a horizontal axis are then computed as follows:

The area of two bolts holes to be deducted equals $2 \times 4 \times 0.375 \times 0.41 = 4.31 \text{ in.}^2$

**Web-Splice Section**

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad^2$</th>
<th>$I_o$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Splice Pl. $\frac{3}{8} \times 55$</td>
<td>41.25</td>
<td>7.55</td>
<td>2,351</td>
<td>9,807</td>
<td>12,158</td>
</tr>
<tr>
<td>Area of Holes</td>
<td>-4.31</td>
<td>7.55</td>
<td>-246</td>
<td>-1,055</td>
<td>-1,301</td>
</tr>
</tbody>
</table>

$$d_{\text{Top of splice}} = 27.50 + 7.55 = 35.05 \text{ in.}$$

$$S_{\text{Top of splice}} = \frac{10,857}{35.05} = 310 \text{ in.}^3$$

$$d_{\text{Bot. of splice}} = 27.50 - 7.55 = 19.95 \text{ in.}$$

$$S_{\text{Bot. of splice}} = \frac{10,857}{19.95} = 544 \text{ in.}^3$$

**II/7.50**

6/78
The maximum bending stress in the plates for the Maximum Design Load therefore is

$$f_b = \frac{736 \times 12}{310} = 28.5 < 36 \text{ ksi}$$

The plates are satisfactory for bending. The allowable shear stress is

$$F_s = 0.55P_s = 0.55 \times 36 = 19.8 \text{ ksi}$$

The shear stress for the Maximum Design Shear is

$$f_s = \frac{115}{41.25} = 2.79 < 19.8 \text{ ksi}$$

The $\frac{3}{8} \times 55$-in. web splice plates are satisfactory for Maximum Design Load requirements. The plates are next checked for fatigue under service loads.

The range of moment carried by the web equals

$$M_v = (1,640 + 1,100) \frac{19,236}{55,508} = 950, \text{ or } \frac{950}{2} = 475 \text{ kip-ft per web}$$

The maximum bending-stress range in the gross section of the web splice plate then is

$$f_{bs} = \frac{475 \times 12 \times 35.05}{12,158} = 16.4 \text{ ksi}$$

**Check for Fatigue**

Fatigue in base metal adjacent to friction-type fasteners is classified by AASHTO as Category B. For 500,000 cycles of truck loading, the associated allowable stress range is 27.5 ksi. The splice plates therefore are satisfactory for fatigue.

Use two $\frac{3}{8} \times 55$-in. web splice plates.

**Web Bolts**

The 28 bolts in the web splice must carry the vertical shear, the moment due to the eccentricity of this shear about the centroid of the bolt group, and the portion of the beam moment taken by the web. These forces are induced by the Overload $D + 5/3(L + I)$. The allowable load in double shear was previously computed to be $P = 25.3$ kips per bolt.

The polar moment of inertia of the bolt group about the assumed location of the neutral axis is

$$I = 2 \times 2 \times 1.820 + 28 \left(7.55 \times \frac{58.69}{57}\right)^2 + 28(1.5)^2 = 9,035$$

**Web Moments for Overload**

$$M_v = \frac{151.5 \times 3.25}{12} = 41$$

$$M_w = 2.767 \times \frac{19,236}{55,508} = \frac{959}{1,000} \text{ or } \frac{1,000}{2} = 500 \text{ kip-ft per web}$$

Load per bolt due to shear is

$$P_s = \frac{151.5}{2(28)} \times \frac{58.69}{57} = 2.79 \text{ kips}$$

Load on the outermost bolt due to moment is

Vertical in-plane component $= \frac{500 \times 12 \times 1.5 \times 58.69}{9,035 \times 57} = 1.03 \text{ kips}$

Horizontal in-plane component $= \frac{500(58.69/57)12(26 + 7.55 \times 58.69/57)}{9,035} = 23.09 \text{ kips}$
Therefore, the total load on the outermost bolt is the resultant
\[ P = \sqrt{(2.79 + 1.03)^2 + (23.09)^2} = 23.4 < 25.3 \text{ kips} \]

Use fourteen \( \frac{3}{8} \) in.-dia, A325 bolts in two rows.

**Flange-Splice Design**

The flange splice plates are proportioned for the Maximum Design Load and checked for fatigue.

The average stress in the top flange under the Maximum Design Load is
\[ f_{s\ Top} = \frac{4,073 \times 12(28.78 + 7.55)}{55,508} = 32.0 \text{ ksi} \]

The total flange force is determined by multiplying the average stress by the net flange area.
\[ P_{\ Top} = 32.0 \left( \frac{13.50 - 0.34}{2} \right) = 211 \text{ kips} \]

The required net area of the top-flange splice plates then becomes
\[ A_{\ Top} = \frac{211}{36} = 5.86 \text{ in.}^2 \]

This value exceeds 75% of the net area of the top flange:
\[ 0.75 \left( \frac{13.50 - 0.34}{2} \right) = 4.94 < 5.86 \text{ in.}^2 \]

Try a \( \frac{5}{16} \)-in. outer splice plate and two \( \frac{3}{8} \times 5\frac{3}{4} \)-in. inner splice plates. The net area of these plates after deduction of bolt holes in excess of 15% of the plate area is
\[
\begin{align*}
\text{Top plate} &= \left( \frac{5}{16} \times 12 \right) - 2.106(1 \times \frac{5}{16}) + 0.15(\frac{5}{16} \times 12) = 3.65 \\
\text{Bot. plate} &= 2\left( \frac{3}{8} \times 5\frac{3}{4} \right) - 1.053(1 \times \frac{3}{8}) + 0.15(\frac{3}{8} \times 5\frac{3}{4}) = 3.85 \\
&= 7.50 > 5.86 \text{ in.}^2 
\end{align*}
\]

The average stress in the bottom flange under the Maximum Design Load is
\[ f_{s\ Bot.} = \frac{4,073 \times 12(28.72 - 7.55)}{55,508} = 18.6 \text{ ksi} \]

The total flange force is
\[ P_{\ Bot.} = 18.6[40.25 - 16 \times \frac{5}{16} + 0.15 \times 40.25)] = 18.6 \times 39.29 = 731 \text{ kips} \]

The required net area of the bottom flange becomes
\[ A_{\ Bot.} = \frac{731}{36} = 20.3 \text{ in.}^2 \]

This is less than 75% of the net area of the bottom flange. Therefore, the required area of the bottom plate is
\[ A_{\ Bot.} = 0.75 \times 39.39 = 29.5 \text{ in.}^2 \]

Try two \( \frac{3}{8} \times 41\frac{1}{2} \)-in. outer plates and two \( \frac{3}{8} \times 41\frac{3}{4} \)-in. inner plates. The net area after deduction of bolt holes in excess of 15% of the plate area is
\[
4\left[ \frac{3}{8} \times 41.5 - 8(1 \times \frac{3}{8}) + 0.15(\frac{3}{8} \times 41.5) \right] = 59.6 > 29.5 \text{ in.}^2 
\]

The flange splice plates are then checked for fatigue under Service Loads. The range of live-load moment at the splice equals
\[ M_L = 1,640 + 1,100 = 2,740 \text{ kip-ft} \]
And the range of average stress in the flanges is

- Top Flange: \( f_r = \frac{2.740 \times 12(28.78 + 7.55)}{55,508} = 21.5 \text{ ksi} \)
- Bot. Flange: \( f_r = \frac{2.740 \times 12(28.78 - 7.55)}{55,508} = 12.6 \text{ ksi} \)

The corresponding range of stress in the gross section of the flange splice plates is

- Top Flange: \( f_r = \frac{21.5/2(13.50 - 0.34)}{3/8 \times 12 + 2 \times 3/8 \times 5.375} = 18.2 < 27.5 \text{ ksi} \)
- Bot. Flange: \( f_r = \frac{12.6 \times 39.29}{4 \times 3/8 \times 41.5} = 7.95 < 27.5 \text{ ksi} \)

**Flange Bolts**

The number of bolts required in the flange splice is determined by the capacity needed for transmitting the flange force under the Overload \( D + 5/3(L + I) \). The total moment on the section is 2,767 kip-ft.

The average stress in the top flange is

\[
f_t = \frac{2.767 \times 12(28.78 + 7.55)}{55,508} = 21.7 \text{ ksi}
\]

And the flange force becomes

\[
P_{\text{Top}} = 21.7 \left( \frac{13.50 - 0.34}{2} \right) = 143 \text{ kips}
\]

For this flange force, the number of bolts required is

\[
\frac{143}{25.3} = 5.7 \text{ bolts}
\]

Use 8 bolts.

The average stress in the bottom flange is

\[
f_b = \frac{2.767 \times 12(28.72 - 7.55)}{55,508} = 12.7 \text{ ksi}
\]

And the bottom-flange force is

\[
P_{\text{Bot}} = 12.7 \times 39.29 = 499 \text{ kips}
\]

For this flange force, the number of bolts required is

\[
\frac{499}{25.3} = 19.7 \text{ bolts}
\]

Use 64 bolts. Details of the splice are shown on page 54.

**Stiffener Splice**

Next, a splice is designed for the ST7.5\times 25, longitudinal, bottom-flange stiffener. A splice in the stiffener is desirable to assure that the interruption of the stiffener at the field splice does not become a node for buckling. The splice is designed for the axial capacity of the ST7.5\times 25. This capacity equals the product of the critical buckling stress for the bottom flange and the area of the stiffener.
SPLICE OF ST7.5 × 25

The allowable bottom-flange stress at the field splice was determined previously for the section 37 ft from the interior support to be 9.9 ksi. The force on the stiffener therefore is

\[ P_{st} = 9.9 \times 7.35 = 72.8 \text{ kips} \]

Use \( \frac{3}{4} \)-in.-dia, A325 bolts, with an allowable stress in single shear of \( 0.442 \times 21 = 9.3 \) kips per bolt. The number of bolts required is

\[ \frac{72.8}{1.3 \times 9.3} = 6.0 \text{ bolts} \]

Use 8 bolts.

The area required for the splice plates is

\[ A_{st} = \frac{72.8}{36} = 2.02 \text{ in.}^2 \]

Try a \( \frac{3}{8} \) × 6-in. splice plate on top of the flange and a \( \frac{3}{8} \) × 5-in. plate on the stem, each with two longitudinal rows of bolts. The net area of the plates is

- Flange: \( 6 \times \frac{3}{8} - 2(\frac{3}{8} \times \frac{3}{8}) + 0.15(6 \times \frac{3}{8}) = 1.93 \)
- Stem: \( 5 \times \frac{3}{8} - 2(\frac{3}{8} \times \frac{3}{8}) + 0.15(5 \times \frac{3}{8}) = 1.50 \)

\[ 3.43 > 2.02 \text{ in.}^2 \]

STIFFENER-FLANGE SPLICING

DESIGN OF PIER

The pier that serves as the interior support of the box girder is investigated at the bottom and at the top, just below the cross girder. Two types of load, in addition to ordinary dead, live and impact loads, influence the design of the pier:

1. **Wind on the structure and on the live load.** These loads induce transverse bending moments, which are computed by treating the bridge frame as a structure loaded normal to its plane.

2. **Longitudinal loads from traction and braking.** These loads produce longitudinal bending moments, which are obtained from an analysis of the bridge as a vertical frame.

Three group loadings are considered in design of the pier:

- **Group I loading** is \( 1.30[D + 5 \times \frac{3}{3}(L + I)] \). This group includes four cases of loading:
Case 1. Three lanes of live load, symmetrically placed on the box girder, and applied on both spans for maximum axial load on the pier. (Apply a 10% reduction because of the small probability of coincident maximum loading.)

**CASE 1 LOADING**

Case 2. Three lanes of live load, symmetrically placed on the box girder, and applied on only one span to produce maximum stresses from axial load and longitudinal moment on the pier. (Apply a 10% reduction because of the small probability of coincident maximum loading.)

**CASE 2 LOADING**

Case 3. Two lanes of live load, eccentrically positioned as shown in the diagram, and applied on two spans to produce maximum stresses from axial load and transverse moment on the pier.

**CASE 3 LOADING**

Case 4. Two lanes of live load, eccentrically positioned, as shown in the diagram, and applied on only one span to produce maximum stresses from axial load, longitudinal moment and transverse moment on the pier.

**CASE 4 LOADING**

**Group II loading** is $1.30(D+W)$, where $W$ is wind on the structure. **Group III loading** is $1.30(D+L+I+0.3W+WL+LF)$, where $WL$ is wind on the live load and $LF$ is the longitudinal force due to live loads.

Axial loads and bending moments obtained from the application of the four loading cases of Group I and from Groups II and III are listed in a table.

**LOADS AT BOTTOM OF COLUMN**

<table>
<thead>
<tr>
<th>Service Loads</th>
<th>Maximum Design Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load</td>
<td>$P$, kips</td>
</tr>
<tr>
<td>$DL_1$</td>
<td>891</td>
</tr>
<tr>
<td>$DL_2$</td>
<td>211</td>
</tr>
<tr>
<td>$L+I$</td>
<td>406</td>
</tr>
<tr>
<td>$1,508$</td>
<td></td>
</tr>
</tbody>
</table>

*Includes 20 kips for weight of cross girder and column.
# LOADS AT BOTTOM OF COLUMN

<table>
<thead>
<tr>
<th>Service Loads</th>
<th>Maximum Design Loads Group I: (1.30[D+5/3(L+I)])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load</td>
<td>(P, \text{kips})</td>
</tr>
<tr>
<td>(DL_1^*)</td>
<td>891</td>
</tr>
<tr>
<td>(DL_2)</td>
<td>211</td>
</tr>
<tr>
<td>(L+I)</td>
<td>230</td>
</tr>
<tr>
<td>1,332</td>
<td>497</td>
</tr>
</tbody>
</table>

## Case 3

<table>
<thead>
<tr>
<th>Service Loads</th>
<th>Maximum Design Loads Group II: (1.30 (D+W))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load</td>
<td>(P, \text{kips})</td>
</tr>
<tr>
<td>(DL_1^*)</td>
<td>891</td>
</tr>
<tr>
<td>(DL_2)</td>
<td>211</td>
</tr>
<tr>
<td>(L+I)</td>
<td>301</td>
</tr>
<tr>
<td>1,403</td>
<td>0</td>
</tr>
</tbody>
</table>

## Case 4

<table>
<thead>
<tr>
<th>Service Loads</th>
<th>Maximum Design Loads Group III: (1.30(D+L+I+0.3W+WL+LF))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load</td>
<td>(P, \text{kips})</td>
</tr>
<tr>
<td>(DL_1^*)</td>
<td>891</td>
</tr>
<tr>
<td>(DL_2)</td>
<td>211</td>
</tr>
<tr>
<td>(W)</td>
<td>0</td>
</tr>
<tr>
<td>1,102</td>
<td>257</td>
</tr>
</tbody>
</table>

*Includes 20 kips for weight of cross girder and column.
<table>
<thead>
<tr>
<th>Load</th>
<th>P, kips</th>
<th>Mx, kip-ft</th>
<th>My, kip-ft</th>
<th>P, kips</th>
<th>Mx, kip-ft</th>
<th>My, kip-ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>DL1*</td>
<td>891</td>
<td>0</td>
<td>0</td>
<td>1,158</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>DL2</td>
<td>211</td>
<td>0</td>
<td>0</td>
<td>274</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>L+I</td>
<td>171</td>
<td>368</td>
<td>1,449</td>
<td>222</td>
<td>478</td>
<td>1,884</td>
</tr>
<tr>
<td>0.3W</td>
<td>0</td>
<td>77</td>
<td>119</td>
<td>0</td>
<td>100</td>
<td>155</td>
</tr>
<tr>
<td>WL</td>
<td>0</td>
<td>112</td>
<td>93</td>
<td>0</td>
<td>146</td>
<td>121</td>
</tr>
<tr>
<td>LF</td>
<td>0</td>
<td>110</td>
<td>0</td>
<td>0</td>
<td>143</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1,273</td>
<td>667</td>
<td>1,661</td>
<td>1,654</td>
<td>867</td>
<td>2,160</td>
</tr>
</tbody>
</table>

*Includes 20 kips for weight of cross girder and column.

The pier cross section is a rectangular steel box made up of four plates. The two plates that form the sides of the pier, or column, and are perpendicular to the longitudinal axis of the bridge are stiffened inside the box by two ST shapes, spaced equally across the width of each plate. The plates of the column section are designed for a critical buckling stress $F_{cr}$, in the same manner as the bottom compression flange of the box girder. All steel is A36.

For stiffeners, an ST7.5 x 25 is selected. This is the same shape used for stiffening the box-girder compression flange. For computation of $F_{cr}$, for the column plates, a $k$ value is computed based on the stiffener and compression-plate properties. The critical buckling stress is then determined from the graphs previously presented in the Design of Girder Sections.

For the unstiffened plates, $k$ is taken as 4, and $F_{cr}$ is obtained as for a stiffened plate.

Try a $\frac{3}{8}$-in.-thick plate for the stiffened plates and a 1-in.-thick plate for the unstiffened plates of the pier. The critical buckling stresses and section properties are calculated as follows:

**SECTION AT BOTTOM OF COLUMN**

For a $\frac{3}{8}$-in. plate,

$$k = \sqrt[3]{\frac{2,432}{0.07(2)^4(\frac{3}{8})^20}} = 3.93$$

For a 1-in. plate, $k=4$. The width-thickness ratio of the $\frac{3}{8}$-in. plate is

$$\frac{w}{t} = \frac{20}{\frac{3}{8}} = 35.6$$

From the graph of buckling stresses, $F_{cr} = 35.6$ ksi. For the 1-in. plate, $w/t = 36/1 = 36$ and $F_{cr} = 35.7$ ksi.
Section at Bottom of Column

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>$d_x$</th>
<th>$Ad_x^2$</th>
<th>$I_{xx}$</th>
<th>$I_x$</th>
<th>$d_y$</th>
<th>$Ad_y^2$</th>
<th>$I_{yy}$</th>
<th>$I_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Pl. 1/58 × 58</td>
<td>65.25</td>
<td>18.00</td>
<td>21,141</td>
<td>21,141</td>
<td>18,292</td>
<td>18,292</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 Pl. 1 × 39</td>
<td>78.00</td>
<td>9,887</td>
<td>9,887</td>
<td>29.50</td>
<td>67,880</td>
<td>67,880</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 ST7.5 × 25</td>
<td>29.40</td>
<td>12.47</td>
<td>4,572</td>
<td>162</td>
<td>4,734</td>
<td>10.00</td>
<td>2,940</td>
<td>31</td>
<td>2,971</td>
</tr>
</tbody>
</table>

172.65 in.$^2$  35,762 in.$^4$  89,143 in.$^4$

Stresses at the corners of the box are calculated taking into account axial and bending effects. The governing condition is found to be Group I, Case 3.

Group I

Case 1: $f = \frac{2,312}{172.65} = 13.4$ ksi

Case 2: $f = \frac{1,930}{172.65} + \frac{1,077 \times 12 \times 19.5}{35,762} = 18.2$ ksi

Case 3: $f = \frac{2,084}{172.65} + \frac{5,538 \times 12 \times 30}{89,143} = 34.5 < 35.6$ ksi

Case 4: $f = \frac{1,802}{172.65} + \frac{797 \times 12 \times 19.5}{35,762} + \frac{3,139 \times 12 \times 30}{89,143} = 28.3$ ksi

Group II

$f = \frac{1,432}{172.65} + \frac{334 \times 12 \times 19.5}{35,762} + \frac{515 \times 12 \times 30}{89,143} = 12.6$ ksi

Group III

Case 3: $f = \frac{1,823}{172.65} + \frac{505 \times 12 \times 19.5}{35,762} + \frac{3,599 \times 12 \times 30}{89,143} = 28.4$ ksi

Case 4: $f = \frac{1,654}{172.65} + \frac{867 \times 12 \times 19.5}{35,762} + \frac{2,160 \times 12 \times 30}{89,143} = 24.0$ ksi

Next, stresses are checked at the top of the pier. The loads at this section are shown in a table.

**LOADS AT TOP OF COLUMN**

<table>
<thead>
<tr>
<th>Service Loads</th>
<th>Maximum Design Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P$, kips</td>
</tr>
<tr>
<td><strong>Load</strong></td>
<td></td>
</tr>
<tr>
<td>$DL_1^*$</td>
<td>876</td>
</tr>
<tr>
<td>$DL_2$</td>
<td>211</td>
</tr>
<tr>
<td>$L+I$</td>
<td>406</td>
</tr>
<tr>
<td></td>
<td>1,493</td>
</tr>
<tr>
<td><strong>Case 1</strong></td>
<td></td>
</tr>
<tr>
<td>$DL_1^*$</td>
<td>876</td>
</tr>
<tr>
<td>$DL_2$</td>
<td>211</td>
</tr>
<tr>
<td>$L+I$</td>
<td>230</td>
</tr>
<tr>
<td></td>
<td>1,317</td>
</tr>
<tr>
<td><strong>Case 2</strong></td>
<td></td>
</tr>
</tbody>
</table>

*Includes 5 kips for the weight of the cross girder.
### Loads at Top of Column

<table>
<thead>
<tr>
<th>Service Loads</th>
<th>Maximum Design Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Group I: 1.30(D+D+5/3(L+I))</td>
</tr>
<tr>
<td>Load</td>
<td>P, kips</td>
</tr>
<tr>
<td>DL_1*</td>
<td>876</td>
</tr>
<tr>
<td>DL_2</td>
<td>211</td>
</tr>
<tr>
<td>L+I</td>
<td>301</td>
</tr>
<tr>
<td></td>
<td>1,388</td>
</tr>
</tbody>
</table>

**Case 3**

<table>
<thead>
<tr>
<th>Load</th>
<th>P, kips</th>
<th>M_z, kip-ft</th>
<th>M_y, kip-ft</th>
<th>P, kips</th>
<th>M_z, kip-ft</th>
<th>M_y, kip-ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>DL_1*</td>
<td>876</td>
<td>0</td>
<td>0</td>
<td>1,139</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>DL_2</td>
<td>211</td>
<td>0</td>
<td>0</td>
<td>274</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>L+I</td>
<td>171</td>
<td>368</td>
<td>1,449</td>
<td>370</td>
<td>797</td>
<td>3,139</td>
</tr>
<tr>
<td></td>
<td>1,258</td>
<td>368</td>
<td>1,449</td>
<td>1,783</td>
<td>797</td>
<td>3,139</td>
</tr>
</tbody>
</table>

**Case 4**

<table>
<thead>
<tr>
<th>Service Loads</th>
<th>Group II: 1.30(D+W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load</td>
<td>P, kips</td>
</tr>
<tr>
<td>DL_1*</td>
<td>876</td>
</tr>
<tr>
<td>DL_2</td>
<td>211</td>
</tr>
<tr>
<td>W</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1,087</td>
</tr>
</tbody>
</table>

**Service Loads**

<table>
<thead>
<tr>
<th>Load</th>
<th>P, kips</th>
<th>M_z, kip-ft</th>
<th>M_y, kip-ft</th>
<th>P, kips</th>
<th>M_z, kip-ft</th>
<th>M_y, kip-ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>DL_1*</td>
<td>876</td>
<td>0</td>
<td>0</td>
<td>1,139</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>DL_2</td>
<td>211</td>
<td>0</td>
<td>0</td>
<td>274</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>L+I</td>
<td>301</td>
<td>0</td>
<td>2,556</td>
<td>391</td>
<td>0</td>
<td>3,323</td>
</tr>
<tr>
<td>0.3W</td>
<td>0</td>
<td>77</td>
<td>182</td>
<td>0</td>
<td>100</td>
<td>237</td>
</tr>
<tr>
<td>WL</td>
<td>0</td>
<td>107</td>
<td>150</td>
<td>0</td>
<td>139</td>
<td>195</td>
</tr>
<tr>
<td>LF</td>
<td>0</td>
<td>190</td>
<td>0</td>
<td>0</td>
<td>247</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1,388</td>
<td>374</td>
<td>2,888</td>
<td>1,804</td>
<td>486</td>
<td>3,755</td>
</tr>
</tbody>
</table>

**Group III:** 1.30(D+L+I+0.3W+WL+LF)

**Case 3**

<table>
<thead>
<tr>
<th>Load</th>
<th>P, kips</th>
<th>M_z, kip-ft</th>
<th>M_y, kip-ft</th>
<th>P, kips</th>
<th>M_z, kip-ft</th>
<th>M_y, kip-ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>DL_1*</td>
<td>876</td>
<td>0</td>
<td>0</td>
<td>1,139</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>DL_2</td>
<td>211</td>
<td>0</td>
<td>0</td>
<td>274</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>L+I</td>
<td>171</td>
<td>368</td>
<td>1,449</td>
<td>222</td>
<td>478</td>
<td>1,884</td>
</tr>
<tr>
<td>0.3W</td>
<td>0</td>
<td>77</td>
<td>182</td>
<td>0</td>
<td>100</td>
<td>237</td>
</tr>
<tr>
<td>WL</td>
<td>0</td>
<td>107</td>
<td>150</td>
<td>0</td>
<td>139</td>
<td>195</td>
</tr>
<tr>
<td>LF</td>
<td>0</td>
<td>104</td>
<td>0</td>
<td>0</td>
<td>135</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1,258</td>
<td>656</td>
<td>1,781</td>
<td>1,635</td>
<td>852</td>
<td>2,316</td>
</tr>
</tbody>
</table>

*Includes 5 kips for the weight of the cross girder.*
The plates comprising the section at the top of the pier have the same thickness as those at the bottom, but the width of the stiffened plate is 9 ft, rather than 5 ft as at the bottom.

![Diagram of section at top of column]

**SECTION AT TOP OF COLUMN**

The critical buckling stress \( F_{cr} \) for the 1 × 39-in. unstiffened plates is the same at the top as at the bottom of the pier. For the \( \frac{3}{16} \)-in. plates, however, \( F_{cr} \) is considerably smaller at the top than at the bottom of the pier, because \( w/t \) is larger at the top. But the decrease in \( F_{cr} \) is compensated for by the larger area and moment of inertia at the top. The result is that stresses are smaller in the plates at the top. In the investigation of the section at the top, the governing loading condition, as at the bottom, is Group I, Case 3.

For a \( \frac{3}{16} \)-in. plate, 

\[
k = \sqrt[3]{\frac{243.2}{0.07(2)^4(\frac{3}{16})^8}} = 3.23
\]

For a 1-in. plate, \( k = 4 \). The width-thickness ratio of the \( \frac{3}{16} \)-in. plate is

\[
\frac{w}{t} = \frac{36}{\frac{3}{16}} = 64
\]

From the graphs presented earlier in this chapter, \( F_{cr} = 20.6 \) ksi. For the 1-in. plate, \( w/t = 36/1 = 36.0 \) and \( F_{cr} = 35.7 \) ksi.

**Section at Top of Column**

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>( d_x )</th>
<th>( Ad_x^2 )</th>
<th>( I_{ox} )</th>
<th>( I_x )</th>
<th>( d_y )</th>
<th>( Ad_y^2 )</th>
<th>( I_{oy} )</th>
<th>( I_y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Pl. ( \frac{3}{16} \times 106 )</td>
<td>119.25</td>
<td>18.00</td>
<td>38,637</td>
<td>38,637</td>
<td>111,658</td>
<td>111,658</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 Pl. 1 \times 39</td>
<td>78.00</td>
<td></td>
<td>9,887</td>
<td>9,887</td>
<td>53.50</td>
<td>223,256</td>
<td></td>
<td>223,256</td>
<td></td>
</tr>
<tr>
<td>4 ST ( 7.5 \times 25 )</td>
<td>29.40</td>
<td>12.47</td>
<td>4,572</td>
<td>162</td>
<td>4,734</td>
<td>18.00</td>
<td>9,526</td>
<td>31</td>
<td>9,557</td>
</tr>
</tbody>
</table>

226.65 in.\(^2\) 53,258 in.\(^4\) 344,471 in.\(^4\)

**Group I**

Case 1: \( f = \frac{2,293}{226.65} = 10.1 \) ksi

Case 2: \( f = \frac{1,911}{226.65} + \frac{1,077 \times 12 \times 19.5}{53,258} = 13.1 \) ksi

Case 3: \( f = \frac{2,065}{226.65} + \frac{5,538 \times 12 \times 54}{344,471} = 19.5 < 20.6 \) ksi

Case 4: \( f = \frac{1,783}{226.65} + \frac{797 \times 12 \times 19.5}{53,258} + \frac{3,139 \times 12 \times 54}{344,471} = 17.3 \) ksi
Group II
\[ f = \frac{1,413}{226.65} + \frac{334 \times 12 \times 19.5}{53,258} + \frac{789 \times 12 \times 54}{344,471} = 9.2 \text{ ksi} \]

Group III
Case 3: \[ f = \frac{1,804}{226.65} + \frac{486 \times 12 \times 19.5}{53,258} + \frac{3,755 \times 12 \times 54}{344,471} = 17.2 \text{ ksi} \]
Case 4: \[ f = \frac{1,635}{226.65} + \frac{852 \times 12 \times 19.5}{53,258} + \frac{2,316 \times 12 \times 54}{344,471} = 15.3 \text{ ksi} \]

**DESIGN OF CROSS GIRDER**
A cross girder is mounted on top of the pier to transfer the load from the two box girders to the column. The cross girder is integrally joined to the column and extends to the exterior webs of the box girders.

A field splice is made in the cross girder at the interior webs of the box girders. Such an arrangement allows the box girders to pass through the cross girder without interruption.

**Loads on Cross Girder**
Four loading cases are considered in design of the cross girder:

*Loading 1.* Dead, live and impact loads are applied, with live load on both box-girder spans, to produce maximum vertical load on the cross girder.

*Loading 2.* Dead, live and impact loads are applied, with the live load on one box-girder span, to produce maximum torque in the cross girder.

*Loading 3.* Longitudinal force from live load LF, taken as 5% of the live load, is combined with Loading 1.

*Loading 4.* Longitudinal force LF is combined with Loading 2.

The box girder vertical reactions \( R_A \) and \( R_B \) at the cross girder, for Loadings 1 and 2, are listed in a table. These reactions are assumed to act through the mid-depth of the exterior and interior box-girder webs, respectively. (Although the calculations are not shown here, the web live loads were determined by utilizing the influence lines for vertical reactions at the center pier.) Lane loading governs.

**Service-Load Reactions, Kips**

<table>
<thead>
<tr>
<th></th>
<th>Loading 1</th>
<th>Loading 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( R_A )</td>
<td>( R_B )</td>
</tr>
<tr>
<td>( DL_1 )</td>
<td>218</td>
<td>218</td>
</tr>
<tr>
<td>( DL_2 )</td>
<td>53</td>
<td>53</td>
</tr>
<tr>
<td>( L+I )</td>
<td>138</td>
<td>108</td>
</tr>
<tr>
<td>Total</td>
<td>409</td>
<td>379</td>
</tr>
</tbody>
</table>

Shears and moments in the cross girder at the face of the column for \( 1.30[D+5/3 \ (L+I)] \) for Loadings 1 and 2 are listed in a table. The moment arms, ft, for calculation of the moments are the distances from the center of the box-girder webs to the nearest faces of the column.
Cross-Girder Shears and Moments

**Loading 1:** \(1.30[D + 5/3(L + I)]\)

<table>
<thead>
<tr>
<th>Shear, Kips</th>
<th>Arm, Ft</th>
<th>Moment, Kip-ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_A)</td>
<td>651</td>
<td>10.25</td>
</tr>
<tr>
<td>(R_B)</td>
<td>586</td>
<td>1.58</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Loading 2:** \(1.30[D + 5/3(L + I)]\)

<table>
<thead>
<tr>
<th>Shear, Kips</th>
<th>Arm, Ft</th>
<th>Moment, Kip-ft</th>
<th>Torque, Kip-ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_A)</td>
<td>521</td>
<td>10.25</td>
<td>5,340</td>
</tr>
<tr>
<td>(R_B)</td>
<td>484</td>
<td>1.58</td>
<td>765</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1,005</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6,106</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>654</td>
</tr>
</tbody>
</table>

The torque of 654 kip-ft shown in the table is obtained by considering the behavior of the structure with the live load on one span only. Under the loading condition, the rotation of the box girders in a vertical plane, as they deflect under the unsymmetrical loading, twists the cross girder. The sum of the resulting torques in both arms of the cross girder equals the longitudinal moment in the pier. Thus, an influence line for longitudinal moment in the pier also is an influence line for the sum of the torques in the arms of the cross girder. Therefore, with the use of the influence line for longitudinal moment in the pier, the maximum torque in the cross girder may be calculated for the portion of the live load applied to one arm of the cross girder.

Similarly, moments, shears and torques in the cross girder for Loadings 3 and 4 are calculated and listed in a table. In the table, the total moment \(M_y\), about the vertical axis is reduced 50\% for the following reason: The cross-girder arms are actually not free cantilevers in the horizontal direction. The transverse rigidity of the box girders restrains rotation of the far end of the cross girder. As a result, the bending moment \(M_y\) in the cross girder at the face of the column is reduced by about one-half.

**Cross-Girder Shears, Moments and Torques**

**Loading 3:** \(1.30[D + L + I + LF]\)

<table>
<thead>
<tr>
<th>Vertical Loads</th>
<th>Horizontal Loads, (1.3LFI)</th>
<th>Moment Arm, Ft</th>
<th>(M_x), Kip-ft</th>
<th>(M_y), Kip-ft</th>
<th>Torque Arm, Ft, about C.G. of Box Girder</th>
<th>Torque, Kip-ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_A)</td>
<td>532</td>
<td>10.25</td>
<td>5,453</td>
<td>106</td>
<td>9.27</td>
<td>95</td>
</tr>
<tr>
<td>(H_A)</td>
<td>10.3</td>
<td>10.25</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_B)</td>
<td>493</td>
<td>1.58</td>
<td>779</td>
<td>13</td>
<td>9.27</td>
<td>75</td>
</tr>
<tr>
<td>(H_B)</td>
<td>8.1</td>
<td>1.58</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6,232</td>
<td>119</td>
<td></td>
<td>170</td>
</tr>
</tbody>
</table>

Use 59
**Loading 4: 1.30(D+L+I+LF)**

<table>
<thead>
<tr>
<th></th>
<th>Vertical Loads 1.3(D+L+I) Kips</th>
<th>Horizontal Loads 1.3LF Kips</th>
<th>Moment Arm, Ft</th>
<th>M_x Kip-ft</th>
<th>M_y Kip-ft</th>
<th>Torque Arm, Ft, about C.G. of Box Girder</th>
<th>Torque, Kip-ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>R_A</td>
<td>454</td>
<td>10.25</td>
<td>4,654</td>
<td>57</td>
<td>9.27</td>
<td>52</td>
<td>246</td>
</tr>
<tr>
<td>H_A</td>
<td>5.6</td>
<td>10.25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R_B</td>
<td>432</td>
<td>1.58</td>
<td>683</td>
<td>7</td>
<td>9.27</td>
<td>194</td>
<td></td>
</tr>
<tr>
<td>H_B</td>
<td>4.4</td>
<td>1.58</td>
<td></td>
<td></td>
<td></td>
<td>42</td>
<td></td>
</tr>
</tbody>
</table>

886 10.0 5,337 64 534

Use 32

**CROSS-GIRDER SECTION AT FACE OF COLUMN**

In cross section, the cross girder is a hollow box, with two \( \frac{3}{16} \)-in.-thick webs, a \( 1\frac{1}{4} \times 39 \)-in. bottom flange and two \( 2 \times 12 \)-in. top flanges. A drawing shows the cross section at the face of the column. The section satisfies requirements for a braced, noncompact section.

The \( \frac{3}{16} \)-in. webs of the section match the thickness of abutting plates in the same plane in the pier. Properties about the horizontal and vertical axes of the section are calculated for the gross and net section.
Cross-Girder Section Properties about X-X Axis

Gross Section

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d_x$</th>
<th>$Ad_x$</th>
<th>$Ad_x^2$</th>
<th>$I_x$</th>
<th>$I_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Flg. Pl. 2 × 12</td>
<td>48.00</td>
<td>29.31</td>
<td>1,407</td>
<td>41,236</td>
<td>41,236</td>
<td></td>
</tr>
<tr>
<td>2 Web Pl. $\frac{3}{4}$ × 56%</td>
<td>63.70</td>
<td>-28.94</td>
<td>-1,411</td>
<td>40,829</td>
<td>40,829</td>
<td></td>
</tr>
<tr>
<td>Flg. Pl. 1 1/4 × 39</td>
<td>48.75</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$160.45 \text{ in.}^2 \quad -4 \text{ in.}^3 \quad 99,086$

$$d_x = \frac{-4}{160.45} = -0.02$$

$$-0.02 \times 4 = 0$$

$$I_{\text{Gross}} = 99,086 \text{ in.}^4$$

Net Section

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d_x$</th>
<th>$Ad_x$</th>
<th>$Ad_x^2$</th>
<th>$I_x$</th>
<th>$I_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Section</td>
<td>160.45</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 Holes 1 1/4 × 2</td>
<td>-11.00</td>
<td>29.33</td>
<td>-9,463</td>
<td>-9,463</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15% of Top Flg. Area</td>
<td>7.20</td>
<td>29.33</td>
<td>6,194</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$$156.65 \text{ in.}^2 \quad I_{\text{Net}} = 95,817 \text{ in.}^4$$

$$d_{\text{Top of steel}} = 30.31 + 0.02 = 30.33 \text{ in.}$$

$$d_{\text{Bot of steel}} = 29.56 - 0.02 = 29.54 \text{ in.}$$

$$S_{\text{Top of steel}} = \frac{95,817}{30.33} = 3,159 \text{ in.}^3$$

$$S_{\text{Bot of steel}} = \frac{95,817}{29.54} = 3,244 \text{ in.}^3$$

The properties of the net section are obtained by deducting the area of the flange holes in excess of 15% of the flange area. The center of gravity of the gross section is used as the reference axis in locating the center of gravity of the net section.

Cross-Girder Section Properties about Y-Y Axis

Gross Section

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d_y$</th>
<th>$Ad_y^2$</th>
<th>$I_y$</th>
<th>$I_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Flg. Pl. 2 × 12</td>
<td>48.00</td>
<td>18.00</td>
<td>15,552</td>
<td>576</td>
<td>16,128</td>
</tr>
<tr>
<td>2 Web Pl. $\frac{3}{4}$ × 56%</td>
<td>63.70</td>
<td>18.00</td>
<td>20,639</td>
<td></td>
<td>20,639</td>
</tr>
<tr>
<td>Flg. Pl. 1 1/4 × 39</td>
<td>48.75</td>
<td></td>
<td></td>
<td>6,179</td>
<td>6,179</td>
</tr>
</tbody>
</table>

$$160.45 \text{ in.}^2 \quad I_{\text{Gross}} = 42,946 \text{ in.}^4$$

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d_y$</th>
<th>$Ad_y^2$</th>
<th>$I_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Section</td>
<td>160.45</td>
<td></td>
<td></td>
<td>42,946</td>
</tr>
<tr>
<td>2 Outer Holes 1 1/4 × 2</td>
<td>-5.50</td>
<td>21.00</td>
<td>-2,426</td>
<td>-2,426</td>
</tr>
<tr>
<td>2 Inner Holes 1 1/4 × 2</td>
<td>-5.50</td>
<td>15.00</td>
<td>-1,238</td>
<td>-1,238</td>
</tr>
<tr>
<td>15% of Top Flg. Area</td>
<td>7.20</td>
<td>18.00</td>
<td>2,333</td>
<td>2,333</td>
</tr>
</tbody>
</table>

$$I_{\text{Net}} = 41,615 \text{ in.}^4$$

Because the top flange is in tension, there is no restriction on the width-thickness ratio of this flange.
Web Thickness and Stiffeners at Face of Column

The web-thickness ratio of the cross girder at the face of the column is

$$\frac{D}{t_w} = \frac{56.63}{\frac{9}{16}} = 101 < 150$$

This ratio is satisfactory for an unstiffened web. The maximum design shear, however, exceeds the buckling capacity for an unstiffened web. Consequently, transverse stiffeners are necessary on the web.

The maximum shear permissible is

$$V_s = \frac{3.5Et_w}{D} = \frac{3.5 \times 29,000 \left(\frac{9}{16}\right)^3}{56.63} = 315 \text{ kips per web}$$

For Loading 1, the maximum shear is

$$V = \frac{1.237}{2} = 619 > 315 \text{ kips per web}$$

Hence, transverse stiffeners are needed. But a longitudinal stiffener is not required because

$$\frac{D}{t_w} = \frac{36,500}{\sqrt{F_y}} = \frac{36,500}{\sqrt{36,000}} = 192 > 101$$

The shear capacity is calculated with the assumption that stiffeners will be spaced about 3 ft apart. A reduction in moment capacity is required where the shear exceeds 60% of the web shear capacity.

The maximum shear capacity of one web is

$$V_p = 0.58F_yDt_w = 0.58 \times 36 \times 56.63 \times \frac{9}{16} = 665 \text{ kips}$$

For computation of the web shear capacity $$V_w$$ with the web stiffener spacing $$d_w = 36$$ in.,

$$C = 18,000 \frac{t_w}{D} \sqrt{\frac{1+(D/d_w)^2}{F_y}} - 0.3$$

$$= 18,000 \times \frac{9}{16} \frac{1+(56.63)^2}{56.63} - 0.3 = 1.45 > 1$$

Because $$C$$ is larger than unity, the shear strength of the web $$V_w = V_p = 665$$ kips. Hence, where the shear on both webs $$V$$ exceeds $$0.60 \times 2 \times 665 = 798$$ kips, the bending-moment capacity is reduced as required by

$$\frac{M}{M_u} = 1.375 - 0.625 \frac{V}{V_w} = 1.375 - 0.625 \times \frac{V}{2 \times 665} = 1.375 \frac{1.237}{2,128} = \frac{V}{2,128}$$

Check of Bending Stresses in Cross Girder—Loading 1

Next, bending stresses are checked for Loading 1. The ultimate moment capacity $$M_u$$ of the section is controlled by the allowable tensile strength $$F_y$$ of the top flange. (Note that the critical buckling stress in the bottom flange also is $$F_y$$, as obtained for a $$w/t$$ ratio of 36/1.25 = 28.8 from the curves for $$F_{cr}$$ previously presented.

For Loading 1, at the face of the column,

$$V = 1.237 \text{ kips} > (0.60V_w = 798 \text{ kips})$$

Therefore, the moment capacity is reduced by the fraction:

$$\frac{M}{M_u} = 1.375 - \frac{1.237}{2,128} = 0.794$$

For the bending moment of 7,602 kip-ft at the face of the column, the stress at the top of the steel section is

$$f_b = \frac{7,602 \times 12}{3,159} = 28.9 \approx (36 \times 0.794 = 28.6 \text{ ksi})$$
The stress at the bottom of the section is

\[ f_b = \frac{7,602 \times 12}{3,244} = 28.1 < 28.6 \text{ ksi} \]

The section is satisfactory in bending under Loading 1.

**Shear and Torque in Cross Girder**

Under Loading 2, the cross girder is subjected to torque as well as to shear and moment. The torque is resisted by shear stresses in the box section, including the concrete slab. The total shear stress equals the sum of the torsional shear and the shear stress due to flexure.

**CROSS-GIRDER TORSIONAL STRESSES**

The following calculations indicate that for Loading 2 the total shear in a web, from vertical loads and torsion, is less than the shear capacity of the web, \( V_u = 665 \) kips. The torsional shear in the web is

\[ q = \frac{T}{2bd} = \frac{654 \times 12}{2 \times 36 \times 64} = 1.70 \text{ kips per in.} \]

The shear in a web due to flexure for Loading 2 is

\[ V = \frac{1.005}{2} = 502.5 \text{ kips} \]

The total web shear then is

\[ V_r = 502.5 - 1.70 \times 56.63 = 599 < 665 \text{ kips} \]

The section is satisfactory for the combination of torsional and flexural shears.

**Cross-Girder Bending about Two Axes**

The section next is checked for bending about both the X-X and Y-Y axes under Loading 3. The shear stress in the webs is observed to be not critical and therefore need not be checked.

The shear due to flexure for Loading 3 is

\[ V = 1,025 > (0.60 V_u = 798 \text{ kips}) \]

The bending moment about the X-X and Y-Y axes are, respectively, \( M_x = 6,232 \) kip-ft and \( M_y = 59 \) kip-ft. The moment capacity of the section because of the high shear is reduced as required by

\[ \frac{M}{M_u} = 1.375 \times \frac{1.025}{2,128} = 0.893 \]
The bending stress at the bottom of the section is

\[
f_b = \frac{6,232 \times 12 + 59 \times 12 \times 19.50}{3,244 + 41,615} + 23 - 0.3 = 23.3 < (36 \times 0.893 = 32.2 \text{ ksi})
\]

Loading 4 is eliminated by inspection.

**Check of Cross Girder for Fatigue**

Fatigue is checked at the top of the cross-girder web where the flange-to-web fillet weld is terminated for the field splice at the interior webs of the box girders. The fatigue stress range in the web adjacent to the terminated fillet weld is governed by AASHTO Category E. The allowable range is 21 ksi for 100,000 cycles of lane loading. Because the section being checked is at a point of high stress produced by combined shear and bending, the range of principal stress is determined instead of the range of normal stresses due to bending.

For Loading 1, the live-load moment range is

\[
M_L = 138 \times 10.25 + 108 \times 1.58 = 1,585 \text{ kip-ft}
\]

The bending-stress range at the top of the web is

\[
f_{br} = \frac{M_L c}{I} = \frac{1,585 \times 12 \times 28.33}{95,817} = 5.62 \text{ ksi}
\]

The live-load shear range is \( V_L = 138 + 108 = 246 \) kips. The shear-stress range at the top of the web is

\[
f_{sr} = \frac{V_L Q}{I} = \frac{246(1.15 \times 48 - 11.0)29.33}{95,817 \times 0.56} = 5.94 \text{ ksi}
\]

The principal-stress range then is

\[
f_p = \frac{f_{br}}{2} + \sqrt{\left(\frac{f_{br}}{2}\right)^2 + f_{sr}^2} = \frac{5.62}{2} + \sqrt{\left(\frac{5.62}{2}\right)^2 + 5.94^2} = 9.38 < 21 \text{ ksi}
\]

Next, the flange-to-web weld is investigated for fatigue in the usual manner and found to be governed by thickness of material, rather than strength under maximum design loads. Also, fatigue in the weld metal is not critical.

**Check of Weld at Top Flange**

The section at the face of the column is investigated for Loading 1, for which the maximum shear is 1,237 kips. The horizontal shear flow in each web is

\[
S = \frac{VQ}{I} = \frac{1,237(1.15 \times 48 - 11.0)29.33}{95,817 \times 2} = 8.37 \text{ kips per in.}
\]

For two welds, the shear flow in each weld is \( 8.37/2 = 4.19 \) kips per in. The weld capacity is \( 0.45F_s \times 0.707 = 0.45 \times 58 \times 0.707 = 18.5 \text{ ksi} \).

Weld size required = \( \frac{4.19}{18.5} = 0.23 \text{ in.} \)

This, however, is less than the minimum weld size required by AASHTO Specifications for thickness of the top flange. Therefore, use a \( \frac{3}{8} \)-in. fillet weld.

The shear range at the weld is \( V_L = 138 + 108 = 246 \) kips. The range of horizontal shear flow in each web is

\[
s_r = \frac{V_L Q}{I} = \frac{246(1.15 \times 48 - 11.0)29.33}{95,817 \times 2} = 1.66 \text{ kips per in.}
\]

For 100,000 cycles of lane loading, the allowable shear-stress range on the throat of the fillet welds is 15 ksi. The actual stress range in the \( \frac{3}{8} \)-in. weld is

\[
f_{sr} = \frac{1.66}{0.375 \times 0.707 \times 2} = 3.13 < 15 \text{ ksi}
\]
Check of Weld at Bottom Flange

The horizontal shear flow in each web at the bottom flange is

$$S = \frac{1.237 \times 39 \times 1.25 \times 28.92}{95,817 \times 2} = 9.10 \text{ kips per in.}$$

For two welds, the shear flow in each weld is $9.10 / 2 = 4.55 \text{ kips per in.}$

Weld size required $= \frac{4.55}{18.5} = 0.25 \text{ in.}$

Because this is less than the minimum weld size required by AASHTO Specifications for the thickness of the bottom flange, use that minimum, $\frac{3}{16} \text{ in.}$

The shear-flow range at the weld is

$$s_r = \frac{V_4 Q}{I} = \frac{246 \times 39 \times 1.25 \times 28.92}{95,817} = 3.62 \text{ kips per in. per web}$$

For two welds, the stress range in each weld is

$$f_r = \frac{3.62}{0.313 \times 0.707 \times 2} = 8.18 < 15 \text{ ksi}$$

Web Thickness within Box Girders

In the preceding calculations, the design section for the cross girder is at the face of the pier. The shear capacity was computed for a $\frac{3}{16} \text{-in.}-\text{thick web and 36-in.}$ stiffener spacing for the region of the cross girder between the box girders. For the region of the cross girder within a box girder, however, the web thickness is reduced to $\frac{1}{16} \text{ in.}$ This can be done because the cross-girder web in this region carries only the load from the exterior web of the box girder. Thus, the maximum shear is only about one-half the shear at the face of the pier.

Connection of Cross Girder at Exterior Webs of Box Girders

No flange splice is provided at the junction of the cross-girder flanges with the exterior top flange of a box girder, because there is no stress at the ends of the cross-girder flanges. But even if the cross-girder flanges carried stress, welding into the side of the box-girder flange, which is in tension, should be avoided.

The connection of the cross-girder web to the box-girder exterior web must transmit the shear from the box-girder web to the cross-girder web. A $\frac{1}{4} \text{-in.}$ fillet weld on each side of the cross-girder webs is more than adequate. Details and calculations for this region follow.
Welds between Webs of Girders

The vertical shear in the box-girder web for maximum design loads is

\[ V = \frac{1.3}{2} \left[ 218 + 53 + \frac{5}{3} \times 138 \right] = 326 \text{ kips} \]

The shear along the slope of the web is

\[ V' = \frac{58.69}{57} \times 326 = 336 \text{ kips} \]

For two welds, the shear on each weld is \( \frac{336}{2} = 168 \) kips. The length of the welds is \( 58.69 - 1.5 \times 58.69 / 57 = 57 \) in. Capacity of a weld is 18.5 ksi.

\[ \text{Weld size required} = \frac{168}{57 \times 18.5} = 0.16 \text{ in.} \]

The minimum size of weld permitted for the \( \frac{3}{4} \)-in. box-girder web is \( \frac{3}{4} \)-in. Therefore, use a \( \frac{3}{4} \)-in. fillet weld on each side of the cross-girder web at the junction with the exterior web of the box girder.

Biaxial Stresses at Interior Support

In the discussion earlier in this chapter of design of negative-moment sections of the box girders, it was pointed out that the bottom flange of those box girders is subjected to a biaxial state of stress at the cross-girder connection. Actually, the state of stress in the flange is complicated, as can be see from a sketch of the box-girder bottom flange at the connection. In the sketch:

\[ f_{bf} = \text{longitudinal bending stress in the box girder under maximum design load} \]
\[ f_{bc} = \text{bending stress in the bottom flange of the cross girder under maximum design load} \]
\[ V_c = \text{shear stress delivered to the flange plate from the cross-girder webs under maximum design load} \]

To insure stability of the bottom flange, the following interaction equation should be satisfied:

\[ \frac{f_{bf}}{F_{bf}} + \frac{f_{bc}}{F_{bc}} \leq 1 \]

where \( F_{bf} = \text{critical buckling stress in the longitudinal direction of the box girder} \)
\( F_{bc} = \text{maximum allowable compressive stress in the cross-girder bottom flange} \)

A drawing shows a section of a box girder at the interior support. The section is hybrid, with \( F_v = 50 \text{ ksi} \) for the flanges and \( F_v = 36 \text{ ksi} \) for the webs. A \( 1\frac{3}{4} \)-in.-thick bottom flange with a single longitudinal ST7.5 \( \times \) 25 stiffener is assumed.
BOX-GIRDER SECTION AT INTERIOR SUPPORT

For use in the interaction equation, the critical buckling stress of the bottom flange is determined without the reduction factor $R$ normally employed in hybrid design. This approach is justified when the yield strength of the lower-strength webs of a hybrid section is not exceeded under maximum design loads.

Loading 1 produces the most critical state of biaxial stress in the bottom-flange plate at the cross-girder connection.

### Moments in Box Girder 1.5 Ft from Interior Support

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$L+I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$, kip-ft</td>
<td>5,950</td>
<td>1,350</td>
<td>2,211</td>
</tr>
</tbody>
</table>

### Steel Section at Interior Support

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_o$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 T. Flg. Pl. 2×15</td>
<td>60.00</td>
<td>29.50</td>
<td>1,770</td>
<td>52,215</td>
<td>20</td>
<td>52,235</td>
</tr>
<tr>
<td>2 Web Pl. ½ × 58.69</td>
<td>58.69</td>
<td>-29.13</td>
<td>-3,350</td>
<td>97,584</td>
<td>15</td>
<td>97,599</td>
</tr>
<tr>
<td>Bot. Flg. Pl. 1¼ × 92</td>
<td>115.00</td>
<td>-29.13</td>
<td>-3,350</td>
<td>97,584</td>
<td>15</td>
<td>97,599</td>
</tr>
<tr>
<td>Stiff. ST7.5×25</td>
<td>7.35</td>
<td>-23.25</td>
<td>-171</td>
<td>3,973</td>
<td>41</td>
<td>4,014</td>
</tr>
</tbody>
</table>

\[
d_s = \frac{-1.751}{241.04} = -7.26 \text{ in.}
\]

\[
I_{NA} = \frac{-7.26 \times 1.751}{241.04} = -12,712 \text{ in.}^4
\]

\[
d_{\text{Top of steel}} = 30.50 + 7.26 = 37.76 \text{ in.}
\]

\[
d_{\text{Bot. of steel}} = 29.75 - 7.26 = 22.49 \text{ in.}
\]

\[
S_{\text{Top of steel}} = \frac{157,027}{37.76} = 4,159 \text{ in.}^3
\]

\[
S_{\text{Bot. of steel}} = \frac{157,027}{22.49} = 6,982 \text{ in.}^3
\]

\[
d_{\text{Top of web}} = 28.50 + 7.26 = 35.76 \text{ in.}
\]

\[
S_{\text{Top of web}} = \frac{157,027}{35.76} = 4,391 \text{ in.}^3
\]
Steel Section with Reinforcing Steel at Interior Support

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad^2</th>
<th>I_s</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>241.04</td>
<td>35.03</td>
<td>-1,751</td>
<td>532</td>
<td>169,739</td>
<td>169,739</td>
</tr>
<tr>
<td>Reinforcement</td>
<td>15.19</td>
<td>35.03</td>
<td>532</td>
<td>18,640</td>
<td>16,840</td>
<td>16,840</td>
</tr>
</tbody>
</table>

\[
d_s = \frac{-1,219}{256.23} = -4.76 \text{ in.}
\]

\[
d_{Top \ of \ steel} = 30.50 + 4.76 = 35.26 \text{ in.}
\]

\[
d_{Bot. \ of \ steel} = 29.75 - 4.76 = 24.99 \text{ in.}
\]

\[
S_{Top \ of \ steel} = \frac{182,577}{35.26} = 5,178 \text{ in.}^3
\]

\[
S_{Bot. \ of \ steel} = \frac{182,577}{24.99} = 7,306 \text{ in.}^3
\]

\[
d_{Top \ of \ web} = 28.50 + 4.76 = 33.26 \text{ in.}
\]

\[
S_{Top \ of \ web} = \frac{182,577}{33.26} = 5,489 \text{ in.}^3
\]

Stresses 1.5 Ft from Interior Support Due to Maximum Design Loads

<table>
<thead>
<tr>
<th>Top of Web</th>
<th>Bottom of Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>(DL_1: F_s = \frac{5,950 \times 12}{4,391} \times 1.30 = 21.1)</td>
<td>(F_s = \frac{5,950 \times 12}{6,982} \times 1.30 = 13.3)</td>
</tr>
<tr>
<td>(DL_2: F_s = \frac{1,350 \times 12}{5,489} \times 1.30 = 3.8)</td>
<td>(F_s = \frac{1,350 \times 12}{7,306} \times 1.30 = 2.9)</td>
</tr>
<tr>
<td>(L + I: F_s = \frac{2,211 \times 12}{5,489} \times 1.30 \times \frac{5}{3} = 10.5)</td>
<td>(F_s = \frac{2,211 \times 12}{7,306} \times 1.30 \times \frac{5}{3} = 7.9)</td>
</tr>
</tbody>
</table>

\[35.4 < 36 \text{ ksi}\]

\[f_{ys} = 24.1 \text{ ksi}\]

Because the maximum bending stress in the web is less than \(F_s = 36 \text{ ksi}\) for the web, no reduction in allowable stress is required for the hybrid section. From the preceding calculations, for use in the interaction equation, \(f_{ys} = 24.1 \text{ ksi}\).

For determination of the critical buckling stress in the bottom flange,

\[k = \sqrt{\frac{8f_{ys}}{\sqrt{w^4}}} = \sqrt{\frac{8 \times 243.2}{\sqrt{45(1.25)}}} = 2.81\]

The width-thickness ratio of the bottom flange is \(w/t = 45/1.25 = 36\). From the curves for critical buckling stress presented previously, \(F_{cr} = 44.3 \text{ ksi} = F_{ys}\) in the interaction equation.

The bending moment in the cross girder at the edge of the box-girder bottom flange equals the product of the reaction \(R_A\) at the outer web of the box girder and the horizontal distance between the center of gravity of the web and the edge of the bottom flange. For maximum design loads and loading 1, this moment is

\[M = 651 \times 8.08 = 5,260 \text{ kip-ft}\]

Properties of the cross-girder section within the box girder are computed next. The width of the bottom flange of the section is set equal to the width of the cross-girder bottom flange between the box girder and the column.
Cross-Girder Steel Section Within Box Girder

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 T. Flg. Pl. 2 x 12</td>
<td>48.00</td>
<td>29.5</td>
<td>1,416</td>
<td>41,772</td>
<td>15</td>
<td>41,788</td>
</tr>
<tr>
<td>4 Holes</td>
<td>-11.00</td>
<td>29.5</td>
<td>-324</td>
<td>-9,573</td>
<td>-9,573</td>
<td></td>
</tr>
<tr>
<td>15% of Flange Area</td>
<td>7.20</td>
<td>29.5</td>
<td>212</td>
<td>6,266</td>
<td>6,266</td>
<td></td>
</tr>
<tr>
<td>2 Web Pl. 7/16 x 57</td>
<td>49.88</td>
<td>29.5</td>
<td>13,504</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bot. Flg. Pl. 1 1/4 x 39</td>
<td>48.75</td>
<td>-29.13</td>
<td>-1,389</td>
<td>41,367</td>
<td>5</td>
<td>41,373</td>
</tr>
</tbody>
</table>

\[ d_s = \frac{-116}{142.83} = -0.81 \text{ in.} \]
\[ I_{NA} = \frac{-0.81 \times 116}{93,264} = -94 \text{ in}^4 \]

\[ d_{Top \text{ of steel}} = 30.50 + 0.81 = 31.31 \text{ in.} \]
\[ d_{Bot. \text{ of steel}} = 29.75 - 0.81 = 28.94 \text{ in.} \]

\[ S_{Top \text{ of steel}} = \frac{93,264}{31.31} = 2,979 \text{ in}^2 \]
\[ S_{Bot. \text{ of steel}} = \frac{93,264}{28.94} = 3,223 \text{ in}^2 \]

\[ d_{Top \text{ of web}} = 28.50 + 0.81 = 29.31 \text{ in.} \]
\[ S_{Top \text{ of web}} = \frac{93,264}{29.31} = 3,182 \text{ in}^2 \]

**Stresses in Cross Girder Due to Maximum Design Loads**

**Top of Web**

\[ F_v = \frac{5,269 \times 12}{3,182} = 19.8 < 36 \text{ ksi} \]

**Bottom of Steel**

\[ F_v = \frac{5,260 \times 12}{3,223} = 19.6 \text{ ksi} \]

The maximum bending stress in the cross-girder web is less than \( F_v = 36 \) ksi. Hence, no reduction in allowable flange stress is required. From the preceding calculations, for use in the interaction equation, \( f_v = 19.6 \text{ ksi} \).

To increase the stiffness of the cross-girder bottom flange, a \( 1/2 \times 5\)-in. plate, to serve as a longitudinal stiffener, is fillet welded to the bottom flange between the box-girder longitudinal stiffener and the box-girder web.
The moment of inertia of the stiffener is
\[ I_s = \frac{0.5(5)^3}{3} = 20.8 \text{ in.}^4 \]

For calculation of the critical buckling stress in the bottom flange,
\[ k = \frac{3\sqrt{I_s}}{\sqrt{wT^2}} = \frac{3\sqrt{20.8}}{\sqrt{18(1.25)^3}} = 1.68 \]

The width-thickness ratio of the bottom flange is 18/1.25 = 14.4. From the curves for critical buckling stress, \( F_{cr} = 50 \text{ ksi} = F_s \) in the interaction equation.

All required stresses for the interaction equation have now been determined. For the bottom flange plate then,
\[ \frac{f_{br}}{F_{br}} + \frac{f_{bc}}{F_{bc}} = \frac{24.1}{44.3} + \frac{19.6}{50} = 0.936 < 1 \]

**Check of Shear in Bottom Flange**

The cross-girder webs apply shear to the cross-girder bottom flange. The shear flow in each web of the cross girder at the junction of the webs and flange is
\[ S = \frac{651 \times 39 \times 1.25 \times 28.32}{2 \times 93264} = 4.81 \text{ kips per in.} \]

The corresponding flange shear is
\[ v_c = \frac{4.81}{1.25} = 3.85 \text{ ksi} \]

The maximum allowable bottom-flange shear is assumed to be given by
\[ F_{s'} = 0.58F_s = 0.58 \times 50 = 29.0 > 3.85 \text{ ksi} \]

**Check of Fatigue in Bottom Flange**

Fatigue should be checked in the bottom flange at the end of the fillet welds for the cross-girder longitudinal stiffener. The allowable stress range for 100,000 cycles of lane loading is 21 ksi. In the direction of the stiffener, the range of live-load moment due to service loads is the product of the change in box-girder reaction \( R_a \) and the horizontal distance between the center of gravity of the web and the edge of the bottom flange nearest the column.
\[ M_L = 138 \times 8.08 = 1,115 \text{ kip-ft} \]

The corresponding stress range in the bottom flange at the end of the stiffener fillet welds is
\[ f_{st} = \frac{1,115 \times 12 \times 27.69}{93264} = 3.97 < 21 \text{ ksi} \]

The cross-girder section therefore is satisfactory.

**Cross-Girder Transverse Web Stiffeners**

The transverse web stiffeners for the cross girder are designed next. Calculations are made for four stiffeners, designated \( a, b, c \) and \( d \).
**Stiffener \(a\)**

The shear under Loading 1 is found to exceed the buckling capacity of the un-stiffened \(\frac{3}{4}\)-in. web of the cross girder. Hence, transverse stiffeners are necessary. A single stiffener \(a\) is tried at mid-width of the box girder.

The shear per web at the stiffener location is

\[
V = \frac{Ra}{2} = \frac{651}{2} = 362 \text{ kips}
\]

The shear capacity of the \(\frac{3}{4}\)-in. web is

\[
V_u = \frac{3.5 \times 29,000 (0.44)^2}{57} = 152 < 326 \text{ kips}
\]

Therefore, a stiffener is necessary. A stiffener at mid-width of the box girder satisfies the spacing requirement for the first stiffener near the end of the cross girder. A plate \(\frac{3}{8} \times 4\frac{1}{2}\) in. provides a satisfactory section for the stiffener.

The maximum permissible spacing for the stiffener is

\[
d_s = 14,500 \sqrt{\frac{D u^3}{V}} = 14,500 \sqrt{\frac{57 (0.44)^3}{326,000}} = 56.0 \text{ in.}
\]

The actual spacing measured from the end of the cross girder at mid-depth is

\[
d_s = \frac{3}{2} \times \frac{118 + 90}{2} = 52 < 56 \text{ in.}
\]

For determination of the ultimate shear capacity,

\[
C = 18,000 \times \frac{0.44}{57} \sqrt{\frac{1 + (57/52)^2}{36,000}} - 0.3 = 0.786 < 1
\]

\[
V_p = 0.58 F_p D u = 0.58 \times 36 \times 57 \times 0.44 = 524 \text{ kips}
\]

\[
V_u = V_p \left[ C + \frac{0.87 (1 - C)}{\sqrt{1 + (d_s/D)^2}} \right] = 524 \left[ 0.786 + \frac{0.87 (1 - 0.786)}{\sqrt{1 + (52/57)^2}} \right] = 484 > 326 \text{ kips}
\]

The depth-thickness ratio of the web is limited by

\[
\frac{D}{t_w} = \frac{36,500}{\sqrt{F_u}} = \frac{36,500}{\sqrt{36,000}} = 192
\]

The actual web depth-thickness ratio is \(57/0.44 = 130 < 192\).

Required area of stiffener is

\[
A = Y \left[ 0.15 B D u (1 - C) \frac{V}{V_u} - 18 t_w^2 \right]
\]

where \(B = 2.4\) for a single-plate stiffener.

\[
Y = \text{ratio of yield strength of web to that of stiffener}
\]

\[
A = \frac{36}{36} \left[ 0.15 \times 2.4 \times 57 \times 0.44 (1 - 0.786) \frac{326}{484} - 18 \times (0.44)^2 \right] = -2.18 \text{ in.}^2
\]

The negative result indicates that the web contribution is larger than the required area of stiffener.

The width-thickness ratio of the \(\frac{3}{8} \times 4\frac{1}{2}\)-in. stiffener plate is

\[
\frac{b'}{t} = \frac{4.5}{\frac{3}{8}} = 12
\]

The maximum permissible ratio is

\[
\frac{b'}{t} = \frac{2.600}{\sqrt{F_u}} = \frac{2.600}{\sqrt{36,000}} = 13.7 > 12
\]
The moment of inertia of the stiffener plate about the edge connected to the web is

\[ I = \frac{0.375(4.5)^3}{3} = 11.4 \text{ in.}^4 \]

The minimum moment of inertia required is computed as follows:

\[ J = 2.5 \left( \frac{D}{d_e} \right)^2 - 2 = 2.5 \left( \frac{57}{52} \right)^2 - 2 = 1.0 \]

\[ I = d_ch_w^2J = 52(0.44)^2 1.0 = 4.43 < 11.4 \text{ in.}^4 \]

**Stiffener b**

In design of stiffener \( b \), which is placed over the column, the presence of stiffener \( d \), which is placed diagonally over the column, will be ignored. Also, it is assumed that stiffener \( b \) traverses the full height of the panel, 56.63 in. Because it has already been shown that a spacing of 36 in. provides adequate shear capacity for the region of the cross girder over the pier, only the required properties of this stiffener need be calculated.

For the stiffener, assume that an ST7.5\( \times \)25 stiffener of the column is extended above the column. The ST7.5\( \times \)25 provides a moment of inertia equal to

\[ I = 40.6 + 7.35(5.25)^2 = 243 \text{ in.}^4 \]

The required moment of inertia is calculated as follows:

\[ J = 2.5 \left( \frac{56.63}{36} \right)^2 - 2 = 4.19 \]

\[ I = 36(0.56)^2 4.19 = 26.5 < 243 \]

Hence, the ST7.5\( \times \)25 is satisfactory.

**Stiffener c**

Stiffener \( c \) is, in effect, an extension of an unstiffened side of the column. A solid diaphragm is used for the stiffener. It serves both as a bearing stiffener, to transfer the load from the cross girder to the column, and as a transverse stiffener of the \( \frac{3}{16} \)-in. cross-girder web. Stiffener \( c \) is assumed to transmit all the load from the cross girder into the column, because of the tendency of the cross girder to rotate about the column face under negative moment. (The assumption is conservative, because some of the cross-girder load will be transferred into the column through the cross-girder web.)

The stiffener is designed as a column to carry maximum design loads. It is checked for local buckling with the width-thickness-ratio criterion for bottom compression flanges of a box girder.

For the stiffener, try a 1\( \frac{3}{4} \)-in.-thick plate. Width of the diaphragm is 36 - \( \frac{3}{16} \) = 35\( \frac{1}{16} \) in. Width-thickness ratio is 35.44/1.25 = 28.4. The maximum permissible ratio is

\[ \frac{b}{t} = \frac{6.140}{\sqrt{F_y}} = \frac{6.140}{\sqrt{36,000}} = 32.4 > 28.4 \]

---

![Diagram of horizontal section through cross girder at stiffener c](image-url)
Because the region of the cross girder in which stiffener \( c \) is located is subject to high shear and bending, the web will not be included with the stiffener as part of a column. The area of the stiffener alone is \( 1.25 \times 35.44 = 44.3 \) in. The moment of inertia of the stiffener is

\[
I = \frac{1.25(35.44)^3}{12} = 4.637 \text{ in.}^4
\]

The radius of gyration of the stiffener is

\[
r = \sqrt{\frac{I}{A}} = \sqrt{\frac{4.637}{44.3}} = 10.23 \text{ in.}
\]

The length of the diaphragm is

\[L' = 56.63 - 4 \times \frac{9}{16} = 54.38 \text{ in.}\]

Consequently, the slenderness ratio of the stiffener is

\[
\frac{L'}{r} = \frac{54.38}{10.23} = 1.23
\]

The critical strength of the stiffener as a column then is

\[F_{cr} = F_y \left[ 1 - \frac{F_v}{4\pi^2E} \left( \frac{L'}{r} \right)^2 \right] = 36 \left[ 1 - \frac{36}{4\pi^2 \times 29,000} (1.23)^2 \right] = 36.0 \text{ ksi}\]

For Loading 1, the shear at the face of the column is 1,237 kips. The capacity of the stiffener as a column is

\[P_u = 0.85AF_{cr} = 0.85 \times 44.3 \times 36.0 = 1,356 > 1,237 \text{ kips}\]

The 1 1/4-in. plate is satisfactory.

**Stiffener \( d \)**

The cross-girder web near the column face is subject to shear, bending and axial compression stresses simultaneously from two directions. The high principal stresses that would normally occur in this region can be reduced, however, by use of a compression stiffener that acts like a truss diagonal. Hence, stiffener \( d \) is incorporated as an inclined, solid diaphragm. This plate stiffens the cross-girder web and reduces principal web stresses in the combined-stress region. In design of such a member, the assumption is made that the flange forces carried by the cross girder and the unstiffened side of the column are resisted by compression in the diagonal stiffener. For computation of these forces, the web is neglected and the structure is assumed to behave essentially as a truss.

For stiffener \( d \), try a 1 1/4-in. plate. Width of the diaphragm is 35.44 in. and length is

\[L' = \sqrt{\left( \frac{108}{2} \right)^2 + 56.63^2} = 78.2 \text{ in.}\]

As for stiffener \( c \), the area of the stiffener is 44.3 in.\(^2\), \( r = 10.23 \) in. and \( L'/r = 7.64\).
The force in the bottom flange of the cross girder is

\[ F_{cy} = f_y A = 28.1 \times 1.25 \times 39 = 1,370 \text{ kips} \]

The force in the unstiffened side of the column is

\[ F_{ct} = f_y A = 19.5 \times 1 \times 39 = 761 \text{ kips} \]

Assume that stiffener \( d \) carries the smaller of the following:
One-half of the horizontal load \( F_{cy} \). (The cross-girder bottom flange above the column carries the other half.)

\[ F = \frac{78.2}{54} F_y = \frac{78.2}{54} \times \frac{1,370}{2} = 992 \text{ kips} \]

The entire vertical load on the unstiffened side of the column.

\[ F = \frac{78.2}{56.63} F_y = \frac{78.2}{56.63} \times 761 = 1,051 > 992 \text{ kips} \]

The stiffener is then designed as a column in the same manner as stiffener \( c \).
Again, the web is neglected in determining the radius of gyration. The critical stress in the stiffener as a column is

\[ F_{cr} = F_y \left[1 - \frac{F_y}{4 \pi^2 E \left( \frac{L}{r} \right)^2} \right] = 36 \left[1 - \frac{36}{4 \pi^2 \times 29,000 \times (7.64)^3} \right] = 35.9 \text{ ksi} \]

The capacity of the stiffener therefore is

\[ P_s = 0.85 AF_{cr} = 0.85 \times 44.3 \times 35.9 = 1,352 > 992 \text{ kips} \]

The 1¼-in. plate is satisfactory.

Access to Cross-Girder Interior

Provision should be made for access to the cross-girder interior for inspection and maintenance. There is no need to provide access to the column interior, because the column will be fabricated as a completely sealed unit.

For entrance into the cross girder, a 14×26-in., screen-covered manhole is centered between the box girders in the 3/8-in. web of the cross girder. Also, open 12×20-in. manholes are provided at the center of stiffeners \( c \) and \( d \) and in the box-girder interior webs.

The 1¼-in.-thick stiffeners are assumed to be one-third unloaded at the manhole. Thus, 67% of the load is carried by the net section, which is 100(35.44 − 12)/35.44 = 66% of the gross section. As a result, no increase in the stiffener thickness is necessary to make up for the loss in section because of the opening. Details of the region are shown in a drawing.
FIELD SPLICE

As shown in a drawing, the column and the portion of the cross girder directly above it are shop fabricated for erection as a unit. When the box girders are fabricated, the portion of the cross girder within them is incorporated. The remainder of the cross girder is connected between the box girders and the pier with field splices.
FIELD-SPlice SCHEME—BOX GIRDErs TO PIER

In design of the field splices, it is assumed that all the bending moment is taken by the flange splices and that all of the shear is taken by the connection of the box-girder web to the cross-girder webs.

For the top-flange splice, a splice plate connecting the pier and box-girder segments passes over the top flange of the box girder but is not attached to it structurally. The flange of the box girder passes through this region without interruption.

At the bottom flange, a positive connection is not needed. Because this flange is always in compression, the plates may be simply butted together, and the stress is transferred in bearing. This joint will be discussed later.

The field splice is designed in the same manner as the field splice for the box girder, described previously. The splice material is proportioned to carry the larger of 75% of the member capacity or the average of the member capacity and the bending moment due to maximum design loads. The fasteners are designed for overload, with a maximum shear stress of 21 ksi. Finally, the splice material is investigated for fatigue in base metal adjacent to friction-type fasteners. Details of the splice are shown in a drawing.

CROSS-GIRDER BOLTED FIELD SPlice
Design of the splice begins with a tabulation of the applied shears and bending moments. The moments are computed for a section through the middle of the inner top flange of the box girder. The box-girder reactions \( R_d \) and \( R_b \) are assumed to act at middepth of the box-girder webs. Loading 1 controls for connector and splice plate designs.

### Service Loads

<table>
<thead>
<tr>
<th>Shear, Kips</th>
<th>Moment, Kip-ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>( DL_1 ): 218+218 = 436</td>
<td>218\times9.25+218\times0.58 = 2,143</td>
</tr>
<tr>
<td>( DL_2 ): 53 + 53 = 106</td>
<td>53\times9.25 + 53\times0.58 = 521</td>
</tr>
<tr>
<td>( L+I ): 138+108 = 246</td>
<td>138\times9.25+108\times0.58 = 1,339</td>
</tr>
<tr>
<td>788</td>
<td>4,003</td>
</tr>
</tbody>
</table>

### Maximum Design Loads: \( 1.30[D+5/3(L+I)] \)

<table>
<thead>
<tr>
<th>Shear, Kips</th>
<th>Moment, Ft-kips</th>
</tr>
</thead>
<tbody>
<tr>
<td>( DL_1 ): 436 \times 1.30 = 567</td>
<td>2,143 \times 1.30 = 2,786</td>
</tr>
<tr>
<td>( DL_2 ): 106 \times 1.30 = 138</td>
<td>521 \times 1.30 = 677</td>
</tr>
<tr>
<td>( L+I ): ( 246 \times 1.30 \times \frac{5}{3} = 533 )</td>
<td>( 1,339 \times 1.30 \times \frac{5}{3} = 2,901 )</td>
</tr>
<tr>
<td>( 1,238 )</td>
<td>( 6,364 )</td>
</tr>
</tbody>
</table>

To determine the design moment for the splice, the moment capacity of the section is calculated. It is controlled by the section modulus and allowable stress for the top flange.

### Maximum Strength of Member

Previous calculations indicated that at the section at the face of the column the cross girder has a section modulus \( S = 3,159 \text{ in.}^2 \). The moment capacity of the section therefore is

\[
M_a = \frac{36 \times 3,159}{12} = 9,477 \text{ kip-ft}
\]

\[
0.75M_a = 7,108 \text{ kip-ft}
\]

The design moment is the larger of 75% of the moment capacity or the average of this capacity and the moment due to the Maximum Design Load.

\[
M_{av} = \frac{9,477 + 6,364}{2} = 7,921 > 7,108 \text{ kip-ft}
\]

The design moment for the splice therefore is 7,921 kip-ft.

### Design of Bolted Top-Flange Splice Plates

The two top-flange splice plates are to be made of A572, Grade 50, steel. The splice-plate area required for each flange is

\[
A = \frac{M_{av}}{F_d} = \frac{7,921 \times 12}{50 \times 59.75} = 15.9 \text{ in.}^2
\]

Try a 1 \times 17-in. plate on each flange with a gross area per plate of 17-in.\(^2\). The net area is

\[
A = 17 - (2 \times 1\frac{1}{4} \times 1 - 0.15 \times 17) = 16.8 > 15.9 \text{ in.}^2
\]

![1" x 17" Splice Plate (A572, Grade 50)](image)

1¼" A325 Bolt

SECTION THROUGH TOP-FLANGE SPLICE
The number of 1½-in.-dia, A325 bolts required in the flange splice for Overload is determined next. The Overload moment \( D + 5/3(L + I) = 2,143 + 521 + 5/3 \times 1,339 = 4,896 \) kip-ft, or 4,896/2 = 2,448 kip-ft per flange. The force on the flange is
\[
F = \frac{M}{d} = \frac{2,448 \times 12}{58.25} = 504 \text{ kips}
\]
Allowable stress on a bolt under Overload is 21 ksi. The bolt area is 1.23 in.². The total number of bolts required then is
\[
N = \frac{504}{21 \times 1.23} = 20 \text{ bolts}
\]
Use twenty 1½-in.-dia bolts on each side of the joint.

**Web Splice**

The connection of the cross girder to the box-girder web is assumed to carry all the shear on the splice but no bending moment. As shown in the drawing of the cross-girder field splice, ½-in. connection plates are welded to the webs and bottom flange of the cross girder. These plates are to be field bolted to the box-girder interior web at the field splice.

The Overload shear \( D + 5/3(L + I) = 436 + 106 + 5/3 \times 246 = 952 \) kips.
The shear on the sloped box-girder web then is
\[
V' = 952 \times \frac{58.69}{57} = 980 \text{ kips}
\]
For the connection, 3/8-in.-dia, A325 bolts will be used. Bolt area is 0.60 in.². Allowable stress in the bolts for Overload is 21 ksi. The number of bolts required is
\[
N = \frac{980}{21 \times 0.60} = 77 \text{ bolts}
\]
Use 78 bolts, 3/8 in. in diameter, in two rows of 18 bolts each along each cross-girder web and six along the bottom flange.

**Check of Fatigue in Bolted Top-Flange Splice**

Fatigue under Service Loads is checked in the metal adjacent to friction-type fasteners in the top-flange splice plates. Fatigue for this condition is classified by AASHTO as Category B. For 100,000 cycles of lane loading, the associated allowable stress range is 45 ksi. The range of live-load moment at the field splice is
\[
M_L = 1,339 - 0 = 1,339 \text{ kip-ft}
\]
The range of force in a flange splice plate is
\[
F_r = \frac{M_L}{d} = \frac{1,339 \times 12}{59.75} = 135 \text{ kips}
\]
The actual stress range in the gross section of the splice plate therefore is
\[
f_r = \frac{F_r}{A} = \frac{135}{17 \times 1} = 7.9 < 45 \text{ ksi}
\]
The plate is satisfactory for fatigue.

**Sealing Studs**

For sealing purposes, four 5/8-in.-dia, 2½-in.-long, threaded studs are placed on the upper side of the box-girder flange for bolting to the splice plate, as shown in a drawing.
SEALING STUDS ON BOX-GIRDER TOP FLANGE

**Bottom-Flange Joint**

As noted previously, the bottom flange is always in compression, so that a positive connection between the cross-girder and box-girder bottom flanges is not needed. The flange compressive force is assumed to be transmitted in bearing. For the purpose, abutting plate edges are ground smooth.

**DETAIL AT BOTTOM FLANGE**

To guard against buckling due to possible nonuniform bearing stress in the abutting flange plates, an additional \( \frac{3}{8} \times 5 \)-in. stiffener plate is provided at midwidth of the cross-girder bottom flange between the inner web of the box girder and the face of the column.

**Alternative Welded Field Splice**

As an alternative to the bolted splice, a welded splice may be used, as shown in a drawing.

Welding to the side of the top flange of the box girder creates an undesirable fatigue condition. To avoid it, the welded splice utilizes at the top flange a single \( 1 \times 17 \)-in. splice plate, fillet welded to the cross-girder flanges on each side of the splice and passing over but not connected to the box-girder flange. The cross-girder flanges are widened from 12 to 18 in. to accommodate the 17-in.-wide splice plates. To maintain about the same flange area, a \( 1\frac{3}{4} \times 18 \)-in. plate is used for the flanges. Eight \( \frac{3}{4} \)-in.-dia, threaded studs are used for sealing purposes over the box-girder flange.
WELDED FIELD SPLICE

Design of Flange-Splice Weld

For the design moment for the splice of 7,921 kip-ft, the force on the flange splice plate is

\[ F = \frac{1}{2} \times \frac{7,921 \times 12}{59.12} = 804 \text{ kips} \]

For a weld along each 28-in.-long side and along the 17-in.-wide end of the splice plate, the shear flow is

\[ S = \frac{804}{2 \times 28 + 17} = 11.0 \text{ kips per in.} \]

The weld capacity is \(0.45F_v \times 0.707 = 0.45 \times 58 \times 0.707 = 18.5 \text{ ksi}\).

Weld size required = \(\frac{11.0}{18.5} = 0.60 \text{ in.}\)

Use a \(\frac{3}{8}\)-in. fillet weld.
Check of Fatigue in Welded Top-Flange Splice

Fatigue under Service Loads is investigated at the splice plate fillet weld. Fatigue in base metal adjacent to a transverse flange fillet weld is classified by AASHTO as Category E. For 100,000 cycles of lane loading, the allowable stress range is 21 ksi. The range of force in the flange at the splice plate is

\[ F_r = \frac{M_l}{d} = \frac{1}{2} \times \frac{1,339 \times 12}{57.94} = 139 \text{ kips} \]

The actual stress range in the flange at the fillet weld is

\[ f_{a} = \frac{139}{0.375 \times 18} = 5.6 < 21 \text{ ksi} \]

Fatigue stress range in the longitudinal fillet weld is limited to 15 ksi for 100,000 cycles of lane loading. Actual stress range in the fillet weld is

\[ f_{a} = \frac{139}{0.625 \times 0.707(2 \times 28 + 17)} = 4.31 < 15 \]

The splice is satisfactory in fatigue.

Other Welded-Splice Details

The web splice is made with full-penetration butt welds.

As with the bolted design, the connection of the bottom flanges of the box girders to the bottom flange of the cross girder is an unwelded butt joint, stiffened to prevent buckling from possible uneven bearing pressure.

The welded splice is completed by reinforcing the region around the 12\times20-in. manhole in the box-girder web.

PIER ALTERNATIVE—REINFORCED CONCRETE

Next, an alternative pier design is prepared for reinforced concrete with the working-stress method of design. With this type of pier, the bottom flange of the cross girder is greatly increased in thickness in the region over the pier. Serving primarily as a masonry plate, the bottom flange transfers the load from the cross girder to the pier in bearing and is restrained against uplift by anchor bolts embedded in the pier concrete.

The design of the cross girder is similar to that previously covered in the steel-pier design calculations and is not treated in the following. One notable difference in the cross girder of the alternative design is that a thicker web is employed over the pier instead of a diagonal stiffener. High principal stresses in the cross-girder webs require the use of additional material.

The dimensions of the concrete pier at the top are set at 9 ft by 4.75 ft to accommodate a masonry plate of 8.5 ft by 3.25 in.

Design of Anchor Bolts

Anchor bolts are designed for maximum uplift forces under the larger of the loadings \(1.5(D+L+I)\) or \(D+2(L+I)\). Allowable stresses for elements designed for either of these loadings may be increased by 50%. Eight anchor bolts are used, as shown in a drawing.

PLAN OF MASONRY PLATE
The net area of the masonry plate is

\[ A = 102 \times 39 - 8 \times 2.76 = 3,956 \text{ in.}^2 \]

The moment of inertia and section modulus of the plate with respect to the Y-Y axis are

\[ I_y = \frac{39(102)^3}{12} - 4 \times 2.76(28)^2 - 4 \times 2.76(36)^2 = 3,426,000 \text{ in.}^4 \]

\[ S_y = \frac{3,426,000}{51} = 67,176 \text{ in.}^3 \]

Group I loading, Case 3, controls the anchor-bolt design. The anchorage should be capable of resisting the larger of the following:

150% of the calculated uplift caused by Service Loads: \( D + L + I \).
100% of the calculated uplift with double the live plus impact loads: \( D + 2(L + I) \).

**Loads at Top of Concrete Pier**

<table>
<thead>
<tr>
<th></th>
<th>( DL_1 )</th>
<th>( DL_2 )</th>
<th>( L + I )</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P ), kips</td>
<td>891</td>
<td>211</td>
<td>301</td>
<td>1,403</td>
</tr>
<tr>
<td>( M_y ), kip-ft</td>
<td>2,556</td>
<td>2,556</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the table, \( DL_1 \) includes 20 kips for the weight of the cross girder.

Stresses beneath the cross-girder masonry plate are then computed as follows (minus indicates uplift):

Under \( D + L + I \),

\[ f_t = \frac{P}{A} \pm \frac{M_y}{S_y} = \frac{1,403}{3,956} \pm \frac{2,556 \times 12}{67,176} = 0.355 \pm 0.457 = -0.102; 0.812 \text{ ksi} \]

**STRESSES UNDER MASONRY PLATE FOR \( D + L + I \)**

The distance of the neutral axis from the uplift end is

\[ d = \frac{0.102}{0.102 + 0.812} \times 102 = 11.38 \text{ in.} \]

The uplift force for \( D + L + I \) is

\[ F_{up} = \frac{1}{2} \times 0.102 \times 11.38 \times 39 = 22.6 \text{ kips} \]

\[ 1.5F_{up} = 1.5 \times 22.6 = 33.9 \text{ kips} \]

Under \( D + 2(L + I) \),

\[ f_t = \frac{1,403 + 301}{3,956} \pm \frac{2 \times 2,556 \times 12}{67,176} = 0.431 \pm 0.913 = -0.482; 1.344 \text{ ksi} \]
The distance of the neutral axis from the uplift end is
\[ d_u = \frac{0.482}{1.344 + 0.482} \times 102 = 26.92 \text{ in.} \]

The uplift force for \( D + 2(L + I) \) is
\[ F_u = \frac{1}{2} \times 0.482 \times 26.92 \times 39 = 253 \times 33.9 \text{ kips} \]

The anchorage should therefore be designed to resist an uplift of 253 kips.

The 1\( \frac{1}{2} \)-in.-dia. A325 anchor bolts have a yield strength of 105 ksi and may be pretensioned up to 70% of this. After being pretensioned, the bolts can sustain additional service-load tensile stress up to 36 ksi.

To eliminate uplift of the cross-girder masonry plate, each bolt should be pretensioned to 253/4 = 63 kips, say 65 kips. The bolt pretension stress then is
\[ f_t = \frac{65}{1.77} = 36.7 < (0.7 \times 105 = 73.5 \text{ ksi}) \]

Additional bolt tension under uplift is
\[ f_t = -\frac{253}{4 \times 1.77} = 35.7 < (1.5 \times 36 = 54 \text{ ksi}) \]

A sufficient length of the anchor bolts should be embedded in the concrete of the pier to develop the uplift force in the bolts.

\[ \text{Concrete Pier} \quad \text{Anchor Bolts} \]

Assume that the pier has no shear reinforcement. The allowable shear stress then is, for 4,000-psi concrete,
\[ \nu_s = 1.8 \sqrt{f_c} = 1.8 \sqrt{4,000} = 114 \text{ psi} \]

This may be increased 50% for the loading \( D + 2(L + I) \) to \( 1.5 \times 114 = 171 \text{ psi} \). The maximum load from four anchor bolts for this loading is
\[ \text{Prestress: } 4 \times 65 = 260 \]
\[ \text{Uplift: } 253 \]
\[ \text{Total: } 513 \text{ kips} \]

The embedment length required for the anchor bolts then is
\[ L = \frac{513,000}{171 \times 39} = 76.9 \text{ in.} \]

Use a 7-ft embedment length for the anchor bolts.
**Check of Bearing Stresses on Concrete Pier**

AASHTO Specifications state that an allowable concrete bearing stress of $0.3f'_c$ may be used for the loaded area. When the supporting surface is wider on all sides than the loaded area, however, the allowable stress may be increased by a factor equal to the square root of the ratio of supporting area to loaded area, but not more than two. When the loaded area is subject to high edge stresses, the allowable bearing stress should be also multiplied by 0.75.

The allowable bearing stress for 4,000-psi concrete not subject to high edge stresses is for the 3-ft 9-in. x 9-ft pier top and 8-ft 6-in. x 3-ft 3-in. masonry plate:

$$ F_b = 0.3 \times 4,000 \times \sqrt{\frac{3.75 \times 9.0}{3.25 \times 8.5}} = 1,326 \text{ psi} $$

For high edge stresses, however,

$$ F_b = 0.75 \times 1,326 = 995 \text{ psi} $$

Because of high edge stresses, the allowable bearing stress on the concrete pier is taken as 995 psi. Concrete stresses under the cross-girder masonry plate are as follows:

- Pretension: $\frac{8 \times 65}{3,956} = 0.131$

  $$ D + L + I: \quad 0.812 $$

  Total: $0.943 < 0.995 \text{ ksi}$

- Pretension: 0.131

  $$ D + 2(L + I): \quad 1.344 $$

  Total: $1.475 < (1.5 \times 0.995 = 1.493 \text{ ksi})$

The masonry plate is satisfactory in bearing on the pier.

**Design of Anchor-Bolt Stools**

The nuts of the anchor bolts are tightened against steel stools, 16 in. above the top of the masonry plate. In addition, a built-up plate section, or center beam, is attached to the masonry plate along its midpoint, as shown in a drawing. There are several reasons for this arrangement. One reason is that it should be easier to pretension the bolts from a higher vantage point within the cross girder. A more important reason is that the stools and center beam serve as a stiff grillage to distribute the pretension and uplift forces more uniformly, thus reducing stresses in the masonry plate.
Each stool has a top seat, or beam plate, and three legs, or webs. A stool may be considered conservatively to act in two different ways:

1. The entire load from the bolts, taken as the interior reaction of a two-span continuous beam, is transmitted directly to the masonry plate in axial compression.
2. The entire load from the bolts is transmitted in shear to the cross-girder web or to the center beam.

![Diagram of stool load distribution]

**LOADS ON STOOL**

Try ¾-in. plates for the stool webs and a 2-in. plate for the stool beam plate. Allowable compressive stress in the webs is 20 ksi and allowable shear stress is 12 ksi.

The maximum bolt force equals the sum of the pretension and uplift forces. For \( D+L+I \),

\[
F_B = 65 + \frac{22.6}{4} = 70.7 \text{ kips}
\]

For \( D+2(L+I) \), with the allowable compressive stress increased 50%,

\[
F_B = \frac{65 + 253/4}{1.5} = 85.5 > 70.7 \text{ kips}
\]

With the beam plate spanning the three webs treated as a two-span continuous beam with equal loads \( P \) at the middle of each span, the load on the center web is \( 11P/8 = 11 \times 85.5/8 = 117.6 \) kips. The axial stress in the center web then is

\[
f_{cu} = \frac{117.6}{0.75 \times 12} = 13.1 < 20 \text{ ksi}
\]

The shear stress in the center web is

\[
f_{vu} = \frac{117.6}{0.75 \times 14} = 11.2 < 12.0 \text{ ksi}
\]

Next, the beam plate is investigated as a two-span continuous beam. Stresses are checked in the net section at the bolt locations and in the gross section at the center web. Try a 2×12-in. plate. Allowable bending stress is 20 ksi.

![Diagram of beam plate stresses]
The moments of inertia of the gross and net section are

\[ I_{\text{Gross}} = \frac{12(2)^3}{12} = 8.0 \text{ in.}^4 \]
\[ I_{\text{Net}} = \frac{(12-1.88)(2)^3}{12} = 6.75 \text{ in.}^4 \]

The maximum positive bending moment is \( 5PL/32 \) and it produces a maximum stress in the net section of

\[ f_b = \frac{M_t}{I} = \frac{(\frac{5}{32})85.5 \times 8 \times 1.0}{6.75} = 15.8 < 20 \text{ ksi} \]

The maximum negative bending moment is \( 3PL/16 \) and it produces a maximum stress in the gross section of

\[ f_t = \frac{(\frac{3}{16})85.5 \times 8 \times 1.0)}{8.0} = 16.0 < 20 \text{ ksi} \]

Use a 2\times12-in. beam plate.

**Design of Center Beam**

The built-up plate section welded to the masonry plate is investigated as a center beam. This beam acts as the center support for the masonry plate, which is treated as a two-span continuous beam spanning between the webs of the cross girder and under the center beam. Group I loading, Case 1, controls the center-beam design. The load at the top of concrete below the cross girder for Case 1 is \( D+L+I = 1,508 \) kips, including 20 kips for the weight of the cross girder.

The bearing stress on the bottom of the masonry plate is

\[ f_b = \frac{P}{A} = \frac{1,508 + 8 \times 65}{3,956} = 0.513 \text{ ksi} \]

With the masonry plate treated as a two-span continuous beam with uniform load \( f_s L \) on each span, the upward load on the center beam is \( 1.25f_s L \). The upward load per unit length on the beam therefore is

\[ w = 1.25 \times 0.513 \times 18 = 11.5 \text{ kips per in.} \]

The downward loads consist of the reactions \( R_A \) and \( R_B \) at diaphragms at the ends of the center beam and the bolt loads, which are assumed to be concentrated 15\div4 = 19 in. from each end of the beam. The bolt load taken by the center beam is

\[ P = 4 \times 65 \times \frac{7.5}{18} = 108 \text{ kips} \]

![Center Beam Diagram]

The reactions of the center beam are

\[ R_A = R_B = \frac{1}{2}(11.5 \times 102 - 2 \times 108) = 478 \text{ kips} \]

The bending moment at midspan is

\[ M = 478 \times 51 + 108 \times 32 - \frac{1}{2} \times 11.5(51)^2 = 12,878 \text{ kip-in.} \]
Use a 1\times6-in. flange plate and a 1\frac{\pi}{4}\times33-in. web plate welded to the masonry plate, as shown in a drawing. Assume an effective bottom-flange width for the center beam of 1.25\times18=22.5 in. The section modulus for the top portion of the beam is computed to be 628 in.\textsuperscript{3} The tensile bending stress in the top flange is
\[ f_t = \frac{12,878}{628} = 20.5 = 20 \text{ ksi} \]
Shear stress in the web at the end of the beam is
\[ f_s = \frac{478}{1.25 \times 33} = 11.6 < 12 \text{ ksi} \]

**Design of Masonry Plate**

The masonry plate is analyzed next. Consider a 1-in.-wide transverse strip of the plate. As computed previously, under \( D+2(L+I) \), the strip has a maximum bearing stress of 1.475 ksi, which causes bending stresses along the strip. Axial stresses delivered to the ends of the masonry plate from the bottom flange of the cross girder acts at right angles to the bending stresses. By inspection, the axial stresses are not critical.

Try a 3\frac{\pi}{4}-in.-thick masonry plate. The maximum moment in the 1-in. strip is
\[ M = \frac{1.475(18)^2}{8} = 59.7 \text{ kip-in.} \]

![Diagram of masonry plate with cross girder and section](image)

The moment of inertia of the strip is
\[ I = \frac{1.0(3.5)^3}{12} = 3.57 \text{ in.}\textsuperscript{4} \]

The bending stress in the strip therefore is
\[ f_s = \frac{59.7 \times 1.75}{3.57} = 29.3 < (1.5 \times 20 = 30 \text{ ksi}) \]

Use a 3\frac{\pi}{4}-in. masonry plate.

**FINAL DESIGN**

Drawings of the box-girder bridge of this design example are shown on the following sheets.
Box Girder Design Example
Two-Span Rigid Frame Bridge
Framing Plan
Box Girder Design Example
Two-Span Rigid Frame Bridge
Steel Details
Box Girder Design Example
Two-Span Rigid Frame Bridge
Steel Details
Box Girder Design Example
Two-Span Rigid Frame Bridge
Center Pier Details—Steel Design
Box Girder Design Example
Two-Span Rigid Frame Bridge
Center Pier Details—Concrete Design
Composite: Curved Box Girder Load Factor Design

Introduction
Chapter 7 illustrates the design of a two-span, rigid-frame, box-girder bridge on straight alignment. This chapter illustrates the design of the same bridge but with horizontally curved, two-span box girders and without the rigid-frame construction at the center pier.

Horizontally curved box girders are applicable for simple and continuous spans of lengths similar to those for which straight box girders are applicable, as outlined in Chapter 7. Curved box girders are used for grade-separation and elevated bridges where the structure must coincide with the curved roadway alignment. This condition occurs frequently at urban crossings and interchanges but may also be found at rural intersections where the structure must conform with the geometric requirements of the highway.


General Design Considerations
Curved box girders are of the same general construction as straight box girders, consisting of a bottom flange, two webs, which may be either vertical or sloped, and top flanges attached to the concrete deck with shear connectors. In negative-bending regions, where the bottom flange is in compression, it is usually stiffened by longitudinal stiffeners or both longitudinal and transverse stiffeners.

Curved box girders differ from straight box girders in that the curved boxes normally have internal diaphragms or cross frames at regular intervals along the span and lateral bracing at the top flange. The cross frames maintain the shape of the cross section and are spaced at such intervals as to keep the transverse distortional stresses and lateral bending stresses in the flanges at acceptable levels. Cross frames are discussed in more detail later.
The principles of composite construction as applied to flexure in curved box girders are assumed to be the same as for straight box girders. These have been discussed in Chapter 7 and in more detail in Chapters 3, 3A, 4 and 4A in connection with rolled beams and plate girders.

LOADS, LOAD COMBINATIONS AND LOAD FACTORS
Loads and load factors are considered to be the same as those given in the AASHTO Specifications for straight bridges. Curvature, however, introduces additional effects, such as forces due to roadway superelevation, centrifugal forces and thermal forces. For box girders, the Guide Specifications account for centrifugal forces by means of special impact factors, which are given later. Centrifugal forces, therefore, need not be considered in any other way. Thermal forces may be neglected if the support system is designed to permit thermal movements.

The following load combinations should be considered:

A. Construction Loads. A partial dead load \( D_p \) and a live load due to construction vehicles \( C \) comprise the total construction load. At each construction stage, the strength of a member must be sufficient to resist the effects of the load combination \( 1.3(D_p + C) \).

B. Service Loads. These consist of the total dead load \( D \) plus the total design live load \( L_T \).

\[
L_T = L + I
\]

where \( L \) = basic live load from vehicles that may operate on a highway legally without a specific load permit

\( I \) = impact loads

The service loads are multiplied by the appropriate load factors for Maximum Design Load and Overload and then combined into group loadings in accordance with the AASHTO Specifications and as outlined in Chapters 3A, 4A, 5, and 7.

Impact is an important consideration in design of curved box girders, because of the uplift and vibrations that may occur. The Guide Specifications assign impact factors for design of components of curved box girders as given in the following table. As stated previously, these impact factors include the effects of centrifugal forces.

<table>
<thead>
<tr>
<th>Condition to Be Determined</th>
<th>Impact Factor ( I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reactions</td>
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<tr>
<td>Direct stresses in box webs and bottom plates</td>
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<td>Direct stresses in concrete slab</td>
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<tr>
<td>Shear stresses in box web</td>
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<td>Stresses in diaphragms</td>
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<tr>
<td>Deflections</td>
<td>0.30</td>
</tr>
</tbody>
</table>
The impact factors are valid within the following parameter ranges:

\[ 100 \text{ ft} \leq L \leq 300 \text{ ft} \]
\[ 300 \text{ ft} \leq R_c \leq 1,000 \text{ ft} \]
\[ v \leq 70 \text{ mph} \]
Number of box girders \( \leq 3 \)
Number of continuous spans \( \leq 2 \)
\[ \frac{\text{Weight of vehicles}}{\text{Weight of bridge}} \leq 0.3 \]

where \( L = \) girder span, ft
\( R_c = \) radius, ft, of centerline of bridge
\( v = \) vehicle speed, mph

The Guide Specifications require that a dynamic analysis be made if the above ranges are exceeded.

**LATERAL DISTRIBUTION OF DEAD AND LIVE LOAD**

Initial dead load and superimposed dead load are made up of the same items that constitute the dead loads for a straight bridge. Also, the lateral distribution is the same as that illustrated in Chapter 7. The live-load distribution factor for moment for a curved box girder, however, is different. This factor can be expressed as a modification of the distribution factor for straight box girders given in the AASHTO Specifications. Studies of curved box-girder bridges with radii ranging from 200 to 10,000 ft have shown that the moments are related to straight-girder moments by

\[ W_{Lc} = (1,440X^2 + 4.8X + 1)W_L \]

where \( W_{Lc} = \) distribution factor for live-load moments in curved box girders
\( W_L = \) distribution factor for live-load moments in straight box girders
\[ X = 1/R_c \]
\( R_c = \) radius of the centerline of the bridge, ft

This distribution factor is assumed in this chapter to be applicable also to shear, torque and deflection.

**STRUCTURAL ANALYSIS**

Any of several different approaches may be used to analyze curved box-girder bridges of the type presented in the following design example. For instance, if the various elements of the structure were idealized as line elements, it could be treated as a planar grid and analyzed by a classical stiffness method for moment, shear and torque. The grid might be taken as the two box girders, considered totally independent of each other, or as the two girders interacting through the deck slab and diaphragms.

Idealization of the girders into line elements is appropriate when the transverse dimensions of the members are small relative to the length. If the member cross sections do not deform, the unit stresses are assumed to be obtainable by ordinary flexural theory, as illustrated in Chapters 3, 4, 5 and 7.

Finite-element methods are more general and may be used for a wide variety of structural analysis problems. Such programs require moderate- to large-size computer systems.

Other solutions for curved box-girder bridges include the use of finite-difference and folded-plate techniques.
A major disadvantage of many rigorous computer programs is that they analyze only one specific loading condition at a time. This makes it difficult to obtain maximum-stress curves for live loads, because each point on a curve represents the effects of a separate loading condition, which involves its own input and for which the load position for maximum or minimum effect is generally not known. Either trial-and-error loading, or the generation of influence lines or surfaces, is required for development of the necessary curves (see W. F. Till, W. N. Poellot, Jr., and A. W. Hedgren, Jr., "Curved Girder Workshop," textbook prepared for Federal Highway Administration, Washington, D.C., 1976).

A finite-difference program developed at the University of Maryland, however, does provide automatic generation of maximum stress curves (see C. P. Heins and C. Yoo, "User's Manual for the Static Analysis of Curved Girder Bridges," Report No. 55, Sept., 1973, Civil Engineering Department, University of Maryland).

The $M/R$ method is an appropriate calculation that makes use of the conjugate-beam analogy. It is readily understood and adaptable to any design office operation. The computations may be performed longhand or may be partly or fully programmed for a computer or electronic desk calculator. (See D. H. H. Tung and R. S. Fountain, "Approximate Torsional Analysis of Curved Box Girders by the $M/R$ Method," Engineering Journal, July, 1970, American Institute of Steel Construction.)

The method loads a conjugate simple span beam with a distributed loading, which is equal to the moment in the real simple or continuous span induced by the applied load divided by the radius of curvature of the girder.

The resulting shears in the conjugate span are then numerically equal to the internal torques in the real span.

The following tables compare influence ordinates computed by the $M/R$ method with those obtained by the finite-element program NASTRAN (using beam elements) and the University of Maryland finite-difference program, for the outer girder of the design example of this chapter. The comparison indicates that, for bending moment at the maximum-positive bending section (Joint 5), the maximum differences of the $M/R$ calculations from NASTRAN and finite-difference moments are 1.4% and 3.9%, respectively. Average differences are 0.5% and 1.6%.

### Comparison of Moments at Joint 5

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<th>$M/R$</th>
<th>NASTRAN</th>
<th>Finite-Diff.</th>
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</tbody>
</table>
For girder torques at the end support, the maximum differences of the $M/R$ calculations from NASTRAN and finite-difference torques are 19.2% and 18.7%, respectively. Average differences are 11.2% in both cases.

**Comparison of Torques at Joint 1**

<table>
<thead>
<tr>
<th>Load at</th>
<th>$M/R$</th>
<th>NASTRAN</th>
<th>Finite-Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>10.2</td>
<td>9.86</td>
<td>11.4</td>
</tr>
<tr>
<td>3</td>
<td>16.6</td>
<td>15.96</td>
<td>18.98</td>
</tr>
<tr>
<td>4</td>
<td>17.4</td>
<td>18.94</td>
<td>21.39</td>
</tr>
<tr>
<td>5</td>
<td>20.4</td>
<td>19.39</td>
<td>21.79</td>
</tr>
<tr>
<td>6</td>
<td>18.9</td>
<td>17.90</td>
<td>20.35</td>
</tr>
<tr>
<td>7</td>
<td>15.9</td>
<td>15.07</td>
<td>17.03</td>
</tr>
<tr>
<td>8</td>
<td>12.1</td>
<td>11.48</td>
<td>12.68</td>
</tr>
<tr>
<td>9</td>
<td>7.98</td>
<td>7.55</td>
<td>8.17</td>
</tr>
<tr>
<td>10</td>
<td>3.79</td>
<td>3.6</td>
<td>3.84</td>
</tr>
<tr>
<td>11</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>-3.5</td>
<td>-2.95</td>
<td>-3.0</td>
</tr>
<tr>
<td>13</td>
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<td>-5.2</td>
<td>-5.3</td>
</tr>
<tr>
<td>14</td>
<td>-8.0</td>
<td>-6.78</td>
<td>-6.9</td>
</tr>
<tr>
<td>15</td>
<td>-8.98</td>
<td>-7.58</td>
<td>-7.8</td>
</tr>
<tr>
<td>16</td>
<td>-8.8</td>
<td>-7.57</td>
<td>-7.9</td>
</tr>
<tr>
<td>17</td>
<td>-8.0</td>
<td>-6.89</td>
<td>-7.1</td>
</tr>
<tr>
<td>18</td>
<td>-6.6</td>
<td>-5.66</td>
<td>-5.7</td>
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<td>-4.66</td>
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</tr>
<tr>
<td>21</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

These comparisons provide some measure of the accuracy with which curved-box girder behavior can be predicted by currently available methods. The reasonable consistency of results should enhance the designer's confidence in selection of a method.

This chapter utilizes the $M/R$ approach because of its ease of application and satisfactory results. The background, rationale and application of the method are fully described in the Tung and Fountain paper previously mentioned.

**Torsional Effects**

The applied torque in a curved box girder is resisted by a combination of two kinds of internal torsion: pure, or St. Venant, torsion and warping torsion. St. Venant torsion provides most of the resistance.

The Guide Specifications state that if the box girder does not have a full-width steel top flange, the girder must be treated under initial dead load (wet-concrete stage) as an open section. This means that the St. Venant torsion constant $K_T$ (from elementary mechanics) is given by

$$K_T = \frac{1}{3} \Sigma b t^3$$

where $b =$ width of an individual plate element

$t =$ thickness of the plate element

If the section is closed, however,

$$K_T = \frac{4A^2}{\Sigma b/t}$$

where $A =$ enclosed area of the section.
A closed box-girder section is usually several thousand times stiffer than an open section. For this reason, if a curved box girder does not have a permanent, solid, top-flange plate, the girder is braced by a lateral system at or near the top flange, to “quasi-close” the box during the wet-concrete stage of construction (see “Steel/Concrete Composite Box-Girder Bridge—A Construction Manual,” ADUSS 88-7493-01, Dec., 1978, United States Steel Corporation).

For analysis purposes, top lateral bracing may be transformed to an equivalent thickness of plate \( t_{eq} \), in., by

\[
t_{eq} = \frac{E}{G} \frac{2A_d}{b} \cos^2 \alpha \sin \alpha
\]

where \( E \) = steel modulus of elasticity, ksi  
\( G \) = steel shearing modulus of elasticity, ksi 
\( A_d \) = area of lateral-bracing diagonal, sq in.  
\( b \) = clear box width, in., between top flanges  
\( \alpha \) = angle of lateral-bracing diagonal with respect to transverse direction

To properly close the section and minimize warping stresses, the cross-sectional area of the lateral-bracing diagonal should be at least

\( A_d = 0.03b \)

The internal stresses produced by St. Venant torsion in a closed section are shearing stresses around the perimeter, as shown in the following sketch and defined by

\[
\tau = \frac{T}{2At}
\]

where \( \tau \) = St. Venant shear stress in any plate, ksi  
\( T \) = internal torque, in.-kips  
\( A \) = enclosed area within box girder, sq in.  
\( t \) = thickness of plate, in.

These shearing stresses add to the vertical shearing stresses in one of the girder webs and subtract from the vertical shearing stresses in the other girder web.

Normal stresses as shown in the following sketch result from warping torsion restraint and from distortion of the cross-section. The Guide Specifications state that “the effect of normal stresses due to nonuniform torsion (warping torsion) and cross-sectional deformation shall be included in the design of curved box-girder bridges unless a rational analysis indicates that these effects are small.”
WARPING STRESSES IN A BOX GIRDER

Researchers have determined that warping stresses may be neglected for single-box, closed cross sections but may have to be taken into account for twin-box structures. Methods are available for computing warping stresses at torsionally fixed supports (see W. T. Till, W. N. Poelot, Jr., and A. W. Hedgren, Jr., "Curved Girder Workshop," Federal Highway Administration, Washington, D.C., 1976.

Warping stress due to distortion can be reduced to negligible levels through the use of internal crossframes. This is covered in detail on page 24, and has been accounted for in the design by limiting the crossframe spacing. Warping torsional stress has been neglected in the example because of its complexity and because the design of the negative bending section at the pier has otherwise been treated conservatively.

It should also be recognized that lateral bending stress in the top flange due to curvature, discussed on pages 19 and 24, is a form of local warping torsional stress and has been fully accounted for in the example.

WEBS

The Guide Specifications require that the maximum calculated shear for design of the box-girder webs be the sum of the vertical shear $V_v$ associated with bending moment and the shear $V_T$ due to St. Venant torsion. If the web is inclined, the design shear associated with bending moment is

$$V_w = \frac{V_v}{\cos \theta}$$

where $\theta =$ angle of inclination of web plate with the vertical.

No web stiffeners are required if

$$\frac{D}{t} \leq 150$$

where $D =$ web depth, in. (depth along the slope for sloping webs)

$t =$ web thickness, in.

For an unstiffened web, the ultimate shear capacity, kips, is the smaller of the following:

$$V_{ul} = \frac{3.5Et^4}{D}$$

$$V_{uw} = 0.58F_yDt$$

where $E =$ modulus of elasticity of web steel, ksi

$F_y =$ yield strength of web steel, ksi
When the maximum design shear exceeds $V_u$, transverse stiffeners are required on the web. A transversely stiffened web must satisfy

$$\frac{D}{t} \leq \frac{1.154}{\sqrt{F_y}} [1 - 8.6 \frac{d_o}{R} + 34 \left( \frac{d_o}{R} \right)^4]^*$$

where $d_o =$ spacing, in., of transverse stiffeners
$R =$ radius of web curvature

The ultimate shear capacity of the stiffened web is given by

$$V_u = 0.58F_yDtC$$

where $C = 569.2 \frac{t}{D} \sqrt{\frac{1 + (D/d_o)^3}{F_y}} - 0.3 \leq 1.0$

The stiffener spacing, however, is not permitted to exceed the depth of the web.

For proportioning the stiffener, the Guide Specifications limit the width to $82.2/\sqrt{F_y}$ times its thickness. The moment of inertia of the transverse stiffener with respect to the midplane of the web must be at least

$$I = d_o t^3 J$$

where $J = 2.5 \left( \frac{D}{d_o} \right)^3 - 2^3 X$

$$X = 1.0 \text{ when } d_o/D \leq 0.78$$
$$= 1 + \left( \frac{d_o/D - 0.78}{1.775} \right) \text{ when } 0.78 \leq d_o/D \leq 1 \text{ and } 0 \leq Z \leq 10$$

$$Z = 0.95d_o^3/4Rt$$
$R =$ radius of curvature of the web

When the girder section is unsymmetrical, as is normally the case with composite girders, and when $D_o$, the clear distance between the neutral axis and the inside face of the compression flange, exceeds $D/2$, then $D_o/2$ for the compressed web must not exceed the preceding limits with $D$ taken as $2D_o$. In this case, the location of the neutral axis should be taken as that for the short-term composite section.

**BOTTOM FLANGES**

The maximum normal tension stress $F_o$, ksi, including warping normal stress, is limited to

$$F_o = F_y \sqrt{1 - 3(f_o/F_y)^2}$$

where $F_y =$ yield strength, ksi
$f_o =$ shear stress, ksi

*The design equations presented in this section are those given for Load Factor Design in the Guide Specifications. Where applicable, the constants in the equations have been modified to give results in kips per square inch rather than pounds per square inch.*
The allowable compression stress for flanges involves several parameters, which are defined as follows:

\[ R_1 = \frac{97.08 \sqrt{K}}{\sqrt{\frac{1}{2} \left[ \Delta + \sqrt{\Delta^2 + 4\left(\frac{f_y}{F_y}\right)^2 (K/K_s)^2} \right]} } \]

\[ R_2 = \frac{210.3 \sqrt{K}}{\sqrt{\frac{1}{1.2} \left[ \Delta - 0.4 + \sqrt{(\Delta - 0.4)^2 + 4\left(\frac{f_y}{F_y}\right)^2 (K/K_s)^2} \right]} } \]

where \( \Delta = \sqrt{1 - 3\left(\frac{F_s}{F_y}\right)^2} \)

- \( K \) = buckling coefficient
  - = 4 when \( n = 0 \)
  - = any assumed value less than 4 when \( n > 0 \)
- \( n \) = number of equally spaced longitudinal flange stiffeners
- \( K_s \) = buckling coefficient
  - = 5.34 when \( n = 0 \)
  - = \( 5.34 + 2.84\left(\frac{f_y}{b^2}\right)^{1/2n} \) when \( n > 0 \)
- \( I_s \) = moment of inertia, in.\(^4\) of a longitudinal flange stiffener about an axis parallel to the flange and at the base of stiffener

The maximum allowable compression stress \( F_s \) for flanges depends on the magnitude of the St. Venant shear stress \( f_y \) across the flange. One case is that for which \( f_y \) is less than 0.75\( F_s / \sqrt{3} \). In this case, there are several expressions for \( F_s \), depending on the ratio \( w/t \) of width to thickness of the flange between longitudinal stiffeners:

- If \( \frac{w}{t} \sqrt{F_y} \) does not exceed \( R_1 \),
  \[ F_s = F_y \Delta \]

Note that, in this case, \( F_s \) is the same as the allowable tension stress for a flange.

- If \( \frac{w}{t} \sqrt{F_y} \) lies between \( R_1 \) and \( R_2 \),
  \[ F_s = F_y \left[ \Delta - 0.4 \left( 1 - \sin \frac{\pi}{2} \frac{R_2 - \frac{w}{t} \sqrt{F_y}}{R_2 - R_1} \right) \right] \]

- If \( \frac{w}{t} \sqrt{F_y} \) equals or exceeds \( R_2 \),
  \[ F_s = 26.210K\left( \frac{t}{w} \right)^2 - \frac{f_y^2K}{26.210K_s^2(t/w)^2} \]
A second case is that for which \( f_c \) lies between \( 0.75F_c/\sqrt{3} \) and \( F_c/\sqrt{3} \). In this case, \( F_c \), \( \sqrt{F_c} \) is not permitted to exceed \( R \), nor is \( w/t \) allowed to exceed 60, except in regions of low compression stress near points of dead load contraflexure. The maximum allowable compression stress is given by

\[
F_c = F_c \Delta
\]

which again is the same as the allowable tension stress for a flange.

The longitudinal bottom-flange stiffeners should be equally spaced between the girder webs. These stiffeners must be proportioned so that the moment of inertia about the base of the stiffener is at least equal to

\[
I_s = \phi t^2 w
\]

where \( \phi = 0.07K^2n^4 \) for \( n > 1 \)

\( = 0.125K^2 \) for \( n = 1 \)

\( n = \) number of stiffeners

\( K = \) buckling coefficient \( \leq 4 \)

The Guide Specifications also state that, when longitudinal stiffeners are used, a transverse stiffener must be placed between the longitudinal stiffeners at the point of maximum compression stress and near points of dead-load contraflexure, as shown in the following sketch. The transverse stiffeners must be the same size as the longitudinal stiffeners. In addition, the Guide Specifications require that the transverse stiffeners be connected only to the bottom flange. The connection should be designed for a force equal to the calculated bending stress in the longitudinal stiffener times the stiffener area.

**TOP FLANGES**

Under total design loading, the narrow top flanges of a box girder work compositely with the concrete deck with an initial locked-in stress due to \( DL_1 \). The effective slab width for the composite section is computed in the same manner as for straight I and box girders, as shown in Chapters 3, 4, 5 and 7.

The following definitions and limits apply to all the following allowable-stress criteria for narrow top flanges:

(a) The absolute value of the ratio of the normal stress \( f_w \) due to nonuniform torsion (lateral bending) to the normal stress \( f_b \) due to flexure shall not exceed 0.5 anywhere along the length of the girder; that is \( f_w/f_b \leq 0.5 \).

(b) The unbraced length of flange \( l \) shall not exceed 25 times the width of the compression flange \( b \).

(c) The unbraced length \( l \) shall not exceed 0.1\( R \), where \( R \) is the radius of curvature of the flange.
(d) The unbraced length of flange is the distance between cross frames or diaphragms.

(e) The ratio \( f_w/f_b \) is positive when \( f_w \) is compressive on the flange tip farthest from the center of curvature. The average flexural stress \( f_b \) shall be computed using the larger of the two bending moments at either end of the braced segment of the flange, and \( f_w \) is the corresponding value of \( f_w \) at that location.

The maximum allowable total stress for top flanges for composite construction is the same as that specified in the Guide Specifications for curved, composite I girders.

**Compression in Top Flange—Total Design Loading**

The average normal stress, ksi (exclusive of lateral bending stress) is limited to

\[
F_{bu} = F_y \bar{\rho}_B \bar{\rho}_w
\]

where \( F_y = \) yield strength of top-flange steel, ksi

\[
f = 1 - 3 \frac{F_y}{E} \left( \frac{l}{b} \right)^2
\]

\[
\bar{\rho}_B = \frac{1}{1 + \frac{l}{b} \left( 1 + \frac{l}{6b} \right) \left( \frac{l}{R} - 0.01 \right)^2}
\]

\[
\bar{\rho}_w = 0.95 + 18 \left( 0.1 - \frac{l}{R} \right)^2 + \frac{(f_w/f_b) \left( 0.3 - 0.1 \frac{l}{R} \right)}{\bar{\rho}_B/f}
\]

\( l = \) unbraced length of compression flange, in.

\( b = \) flange width, in.

\( E = \) modulus of elasticity, ksi, of flange steel

\( R = \) radius of curvature of flange, in.

\( f_w = \) lateral bending stress due to all causes, ksi

\( f_b = \) ordinary bending stress due to vertical loading, ksi

If \( \bar{\rho}_B \bar{\rho}_w \) exceeds unity, \( \bar{\rho}_B \bar{\rho}_w = 1.0 \) should be used.

**Tension in Top Flange—Total Design Loading**

The average normal stress is limited to

\[
F_{tu} = F_y f
\]

**Compact Flanges Under Construction Loading**

Under construction loading at the wet-concrete stage, the top flanges should be considered to act as noncomposite, I-girder flanges. The allowable stress under this condition depends on whether the flange is compact or noncompact as defined by the ratio \( b/t \). For compactness, \( b/t \leq 101.2/\sqrt{F_y} \). When this property is checked, if the ratio \( b/t \) changes between points of bracing, the larger value of \( b/t \) should be used.

Under construction loading, if the flange is compact, the average normal stress is limited as follows:

**Compression (Compact Flange)—Construction Loading**

The allowable stress is \( F_{bu} \) as specified for composite flanges in compression under total design loading.

**Tension (Compact Flange)—Construction Loading**

\[
F_{tu} = F_y \bar{\rho}_B \bar{\rho}_w
\]

where \( \bar{\rho}_B \) and \( \bar{\rho}_w \) are as defined for total design loading.
Noncompact Flanges under Construction Loading

If $b/t$ lies between $101.2/\sqrt{F_y}$ and $139.1/\sqrt{F_y}$, the flange is noncompact. For noncompact flanges under construction loading, the average normal stress is limited to

$$F_{by} = F_y \rho_B \rho_w$$

where $F_{by} = F_y$ for tension flanges

$$F_y \left[ 1 - 3 \frac{F_y}{E t^2} \left( \frac{l}{b} \right) \right]$$

for compression flanges

$$\rho_B = \frac{1}{1 + (l/R)(l/b)}$$

$\rho_u = \rho_{uw}$ or $\rho_{ub}$, whichever is smaller, if $f_u/f_b$ is positive

$$= \rho_{uw} \text{ if } f_u/f_b \text{ is positive}$$

$$= \rho_{ub} \text{ if } f_u/f_b \text{ is negative}$$

$$\rho_{ul} = \frac{1}{1 - (f_u/f_b) \left( 1 - l/75b \right)}$$

$$\rho_{wb} = \frac{0.95 + \frac{l/b}{30 + 8,000 \left( 0.1 - l/R \right)^2}}{1 + 0.6 \left( f_u/f_b \right)}$$

Furthermore, for noncompact flanges, the tip stress $f_u + f_w$ is not permitted to exceed $F_y$.

**SHEAR CONNECTORS**

Design of shear connectors for fatigue is the same as that for straight girders as given in the AASHTO Specifications. For ultimate strength, the Guide Specifications require that the number of shear connectors between points of maximum positive moment and the end supports or dead-load inflection points be sufficient to satisfy

$$P \leq \phi S_u$$

where $\phi =$ reduction factor $= 0.85$

$S_u =$ ultimate strength, kips, of the shear connector as given in the AASHTO Specifications for straight girders

$P =$ force, kips, on the connector

$$= \sqrt{P^2 + F^2 + 2PF \sin \frac{\theta}{2}}$$

$$P = \frac{P}{N}$$

$P = 0.85 f'_c b c$ or $A_r F'_y$, whichever is smaller, at points of maximum positive moment

$$= A_r F'_y \text{ at points of maximum negative moment as defined by the AASHTO Specifications for straight girders}$$

$N =$ number of connectors between points of maximum positive moment and adjacent end supports or dead-load inflection points, or between points of maximum negative moment and adjacent dead-load inflection points

$$F = \frac{P (1 - \cos \theta)}{4K_N \sin \frac{\theta}{2}}$$

$\theta =$ angle extended between point of maximum moment (positive or negative) and adjacent point of contraflexure or support

$f'_c =$ 28-day compressive strength of concrete slab, ksi

$b =$ effective width, in., of slab

$c =$ thickness, in., of slab

$A_r =$ total area, sq. in., of steel section, including cover plates
\[ A'_t = \text{total area, sq. in., of longitudinal reinforcing steel at the interior support within the effective width of flange} \]
\[ f'_{y} = \text{yield strength, ksi, of the reinforcing steel} \]
\[ K = 0.166 \left( \frac{N}{N_t} - 1 \right) + 0.375 \]
\[ N_s = \text{number of connectors at a section} \]

**INTERNAL DIAPHRAGMS**

Curved box girders require internal diaphragms at the supports to resist transverse rotation, displacement and distortion and to transmit the girder torque to the substructure. In addition, intermediate diaphragms or cross frames should be provided unless a rational analysis indicates that they are not needed. Diaphragms or cross frames serve to limit the normal and transverse bending stresses due to distortion and the lateral bending stresses in the narrow top flanges during the wet-concrete stage of construction. Formulas for the spacing of intermediate cross frames and for the required cross-sectional area of cross-frame diagonals have been derived based on limitation of the distortion stress to 10% of the stress due to ordinary bending. For this limitation (according to Heins, page 2), cross-frame spacing should not exceed

\[ S = L \sqrt{\frac{R}{200L - 7,500}} \leq 25 \text{ ft} \]

and the cross-sectional area, sq. in., of the diagonal should be at least

\[ A_b = 750 \frac{Sb}{d^2} \frac{t^3}{d + b} \]

where
\[ S = \text{diaphragm spacing, in.} \]
\[ d = \text{depth of box, in.} \]
\[ b = \text{width of box, in.} \]
\[ t = \text{thickness, in., of thickest component of box-girder cross section} \]
\[ R = \text{radius of girder, in.} \]
\[ L = \text{span of girder, in.} \]

**LATERAL BENDING STRESSES UNDER DL₁**

Two kinds of lateral bending occur in the top flanges under the initial dead-load condition, during which the flanges are not supported by the concrete deck. The first kind is lateral bending due to the horizontal component of the shear force in the sloping webs (see Chapter 7). The second kind of lateral bending is that due to curvature. The equations used for calculating lateral-bending effects are as follows:

\[ M_{LC} = \frac{M_d d^2}{10Rh} \]

where
\[ M_{LC} = \text{lateral bending moment, kip-in., due to curvature} \]
\[ M_d = DL₁ \text{ moment, kip-in.} \]
\[ d = \text{distance, in., between diaphragms} \]
\[ R = \text{radius of curvature of the girder, in.} \]
\[ h = \text{depth of girder, in.} \]

The corresponding stress at the flange tips is

\[ f_{wc} = \frac{6M_{LC}}{2b^2t} \]
where $f_{ac} =$ lateral bending stress, ksi, due to curvature

$b =$ flange width, in.

$t =$ flange thickness, in.

Lateral bending stresses due to the effects of both curvature and the sloping webs are assumed to be proportional to the square of the unbraced length of flange. Thus, lateral flange bending stresses, as well as distortional stresses, may influence selection of cross-frame spacing.

Lateral bending stress due to curvature also occurs in the flanges of the longitudinal stiffeners attached to the bottom flange. These stiffener flanges participate with the girder flanges in resisting bending moments and carry a stress $f_s$, ksi, as shown in the following sketch and given by

$$f_s = \frac{y_b - y_s}{y_b} f_b$$

where $f_b =$ maximum bending stress, ksi, in the girder bottom flange

$y_b =$ distance, in., from neutral axis to bottom of girder

$y_s =$ distance, in., from neutral axis to top of stiffener flange

![Girder Bending Stresses](image)

**BENDING STRESSES IN LONGITUDINAL STIFFENER**

Since the stiffener is curved, its flange is subjected to a lateral bending moment

$$M_{LC} = \frac{f_b b t d^2}{10 R}$$

where $d =$ unbraced length, in., of stiffener flange

$t =$ thickness, in., of stiffener flange

$b =$ width, in., of stiffener flange

$R =$ radius of curvature, in., of stiffener

The corresponding lateral bending stress is

$$f_{ac} = \frac{6 f_b d^2}{10 R b}$$

With the direct stress and the lateral bending stress in the stiffener flange known, $f_s$ may be checked against the allowable stresses for noncomposite I-girder flanges given previously for top flanges of the girder under construction loading.
Design Example—Two-span Curved Box Girder (120-120 Ft) Composite for Positive and Negative Bending

The following data apply to this design:

Roadway Section: See typical bridge cross section.


Loading: HS20-44.

Structural Steel: ASTM A36 and A572, Grade 50.

Concrete: $f_c = 4,000$ psi, modular ratio $n = 8$.

Slab Reinforcing Steel: ASTM A615, Grade 40, with $F_y = 40$ ksi.

Loading Conditions:

Case 1—Weight of girder and slab ($DL_1$) supported by the steel girder alone.

Case 2—Superimposed dead load ($DL_2$) (parapets and railings) supported by the composite section with the modular ratio $n = 8$. (Used in design of web-to-flange fillet welds.)

Case 3—Superimposed dead load ($DL_3$) (parapets and railings) supported by the composite section with the increased modular ratio $3n = 3 \times 8 = 24$.

Case 4—Live load plus impact ($L + I$) supported by the composite section with the modular ratio $n = 8$.

Fatigue—500,000 cycles of truck load
100,000 cycles of lane loading $\{\text{Redundant load-path structure.}\$

Loading Combinations:

Combination A = Case 1 + 3 + 4
Combination B = Case 2 + 4
Combination C = Case 1 + 2 + 4
GEOMETRY OF BRIDGE
The following plan and elevation views show the geometric layout for the example structure of this chapter.

TWO-LANE CURVED OVERPASS STRUCTURE FOR 4-LANE DIVIDED HIGHWAY—90-FOOT MEDIAN

LOADS, SHEARS, MOMENTS AND TORQUES FOR BOX GIRDER
The initial dead load and superimposed dead load are the same as those of the design in Chapter 7. Initial dead load consists of the weight of the girder, concrete slab and haunches. The superimposed dead load consists of the weight of the parapet, wearing surface and railing.

Dead Load on Steel Box Girder
Slab=0.63×20.9×0.150=1.976
0.12×4.83×0.150=0.087
Haunches=0.19×1.67×0.150×2=0.095
Girder (assumed weight)=0.475
DL₁ per girder=2.633 k/ft

Dead Load Carried by Composite Section
Parapet=1.50×0.92×0.150=0.207
0.37×0.50×0.150=0.028
0.17×1.42×0.150=0.036
Wearing surface=0.020×19.50=0.390
Railing=0.020
DL₂ per girder=0.681 k/ft
Live Load on Box Girder

The live-load distribution factor for the curved box girders is the factor for straight girders, from Chapter 7, modified by the curvature factor 1.440X² + 4.8X + 1, where X is the reciprocal of the centerline radius of the bridge. For a roadway width Wc = 40 ft,

\[ N_w = \frac{W_c}{12} = 3.33 \]

This is reduced to the integer 3. Because there are two box girders, the factor R used in computation of live-load distribution is

\[ R = \frac{N_w}{2} = \frac{3}{2} \]

The distribution factor for moment in a straight box girder is

\[ W_L = 0.1 + 1.7R + \frac{0.85}{N_w} = 0.1 + 1.7 \left( \frac{3}{2} \right) + \frac{0.85}{3} = 2.933 \]

Hence, the distribution factor for moment in the curved box girders, with X = 1/400 = 0.0025, is

\[ W_L = W_L(1.440X^2 + 4.8X + 1) \]
\[ = 2.933[1.440(0.0025)^2 + 4.8 \times 0.0025 + 1] = 2.995 \text{ wheels} = 1.497 \text{ lanes} \]

The bridge is analyzed by the M/R method outlined previously in General Design Considerations. For computation of moments and shears, the girders are treated as straight but their developed lengths are used. From the bridge centerline radius of 400 ft, centerline span of 120 ft and the geometry of the bridge cross section, the span of the outer girder is calculated to be 123.115 ft, and the span of the inner girder, 116.885 ft (see drawing showing plan geometry).

![Plan Geometry at Top of Webs](image)

The following curves of maximum longitudinal moments and vertical shears were computed on this basis. Because the Guide Specifications impose a variety of impact factors, as tabulated previously in General Design Considerations, these curves do not include impact. It is taken into account at later points in the design.
Curves of maximum longitudinal moment (without impact) for outer box girder

Curves of maximum longitudinal moment (without impact) for inner box girder
CURVES OF MAXIMUM VERTICAL SHEAR (WITHOUT IMPACT) IN ONE WEB FOR OUTER BOX GIRDER

CURVES OF MAXIMUM VERTICAL SHEAR (WITHOUT IMPACT) IN ONE WEB FOR INNER BOX GIRDER
The curves of maximum torque are calculated by the $M/R$ method outlined previously in General Design Considerations. For example, to obtain the $DL_2$ torque diagram for the outer girder, the $DL_1$ ordinates of the curves of maximum longitudinal moment for the outer girder are divided by 410.3835, the centerline radius of that girder. The girder is then assumed loaded with the resulting $M/R$ diagram and the shears are computed. These shears are equivalent to the $DL_2$ torques.

To determine live-load torques in the outer girder, a unit load is placed at the first panel point in the span and the resulting longitudinal bending moments are calculated. These moments are divided by the girder radius to obtain the $M/R$ diagram. The span is then loaded with this diagram. Each of the resulting shears in the girder represent one influence ordinate for each of a series of torque influence lines for the girder. Placement of the unit load at the next panel point and repetition of the procedure yields another influence ordinate for each of the series of influence lines. A full set of torque influence lines may be obtained in this fashion. AASHTO truck or lane loading is then applied to the influence diagrams to obtain maximum values for live-load torque at each panel point for plotting the curves of maximum live-load torque (see following graphs). The computational steps are well suited to execution on programmable desk calculators or full-size computers in design offices.

![CURVES OF MAXIMUM TORQUE FOR OUTER BOX GIRDER](image-url)
DESIGN OF GIRDER SECTIONS

The effective slab width to be used for the composite section in the positive-moment region and the area of steel reinforcement to be used for the composite, negative-moment section are the same as for the straight bridge of Chapter 7.

SLAB HALF SECTION

Effective Slab Width

1. One-fourth the span: $\frac{1}{4} \times \frac{3}{4} \times 120 \times 12 \times 2 = 540$ in.
2. Center to center of girders: $12(9.83 + \frac{1}{2}(9.83 + 11)) = 243$ in.
3. $12 \times$ slab thickness: $12 \times 7.5 \times 2 = 180$ in. (governs)
Area of Slab Reinforcement for Negative-Moment Section

<table>
<thead>
<tr>
<th>Bar Location</th>
<th>No. of Bars</th>
<th>Area per Bar</th>
<th>Total Area</th>
<th>d</th>
<th>Ad</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top row</td>
<td>31</td>
<td>0.31</td>
<td>9.61</td>
<td>4.313</td>
<td>41.45</td>
</tr>
<tr>
<td>Bottom row</td>
<td>18</td>
<td>0.31</td>
<td>5.58</td>
<td>2.188</td>
<td>12.21</td>
</tr>
</tbody>
</table>

15.19 in.²       53.66 in.²

\[d_{\text{Reinf.}} = \frac{53.66}{15.19} = 3.63 \text{ in.}\]

The maximum allowable effective width of the bottom flange is one-fifth the span, or about 24 ft. The 7-ft-6-in. width of the flange is considerably less than this. Hence, the entire width of the bottom flange is considered effective.

Allowable Bottom-Flange Compression Stress

The allowable bottom-flange compression stress \( F_s \) is defined by complex expressions and parameters. For a better understanding of \( F_s \), it is convenient to show its variation graphically with the flange width-thickness ratio \( w/t \). Families of curves may be plotted for specific values of \( F_v \), the St. Venant shearing stress, for \( K=1 \) and \( K=4 \) (see following graphs).
ALLOWABLE STRESS FOR BOTTOM FLANGE OF LONGITUdINALLY STIFFENED, CURVED BOX GIRDER

A single longitudinal stiffener, as used in the bridge of Chapter 7, is efficient for flanges 7 ft 6 in. wide. The curves are therefor shown for a flange with one ST 7.5 \times 25 longitudinal stiffener, with moment of inertia $I_s = 243.2$ in.$^4$. One graph is based on $F_y = 36$ ksi and the second graph, on $F_y = 50$ ksi. The curves are terminated at points where $I_s \geq 0.125K^2t^4b$ is no longer satisfied.

To make practical use of the allowable-stress equation in design, some of the parameters may be eliminated. For example, the graphs reveal that the effect of $f_o$ is negligible at values lower than 1 ksi. (The allowable stress converges to a limit at this low range of $f_o$.) Trial also shows that, in fact, this is the range of $f_o$ to be expected in box girders of this size and type. For preliminary design, therefore, allowable stresses may be based on an arbitrary value of $f_o = 0.5$ ksi. This assumption eliminates $f_o$ as a variable in determination of $F_b$.

Another parameter is the buckling coefficient $K$. If one longitudinal stiffener is attached to the bottom flange, the minimum permissible moment of inertia of the stiffener about its base $I_s$ is related to $K$ by

$$I_s \geq 0.125K^2t^4b$$

where $t =$ flange thickness, in.

$b =$ flange width, in.

Transposition yields the maximum $K$ value for a given size of longitudinal stiffener with a moment of inertia $I_s$ as

$$K_{\text{max}} = \sqrt{\frac{I_s}{0.125t^4b}}$$

A lower value of $K$ may be used, but at the sacrifice of efficiency, because of lower allowable stress would result. Use of a larger value of $K$ would require a larger stiffener.
For convenience in design, the allowable stresses, based on $K_{max}$, may be tabulated for possible combinations of bottom-flange thickness and ST stiffeners. The following ST shapes are chosen as possible longitudinal stiffeners, and the moment of inertia $I_t$ about the base of the stem of each stiffener is calculated.

<table>
<thead>
<tr>
<th>ST Shape</th>
<th>Moment of Inertia, In.²</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.5 × 25</td>
<td>40.6 + 7.35(5.25)² = 243.2</td>
</tr>
<tr>
<td>6 × 25</td>
<td>25.2 + 7.35(4.16)² = 152.4</td>
</tr>
<tr>
<td>6 × 20.4</td>
<td>18.9 + 6.00(4.42)² = 136.1</td>
</tr>
<tr>
<td>5 × 17.5</td>
<td>12.5 + 5.15(3.44)² = 73.4</td>
</tr>
<tr>
<td>4 × 11.5</td>
<td>5.03 + 3.38(2.85)² = 32.5</td>
</tr>
<tr>
<td>3.5 × 10</td>
<td>3.36 + 2.94(2.46)² = 21.2</td>
</tr>
</tbody>
</table>

For a single longitudinal stiffener on the 7.5-ft-wide bottom flange, the stiffener spacing is $w = 7.5 \times \sqrt[3]{12} = 45$ in. For this spacing, the value of the buckling coefficient $K_{max}$ furnished by each size of stiffener is calculated for several thicknesses of bottom flange. Allowable stresses are then computed from the governing equations given previously in General Design Considerations, with a low value of $f_y$, such as 0.5 ksi, and tabulated. The resulting table can be used at later points of the design as an aid in selection of bottom-flange thicknesses.

### Allowable Compression Stress $F_b$ for Bottom Flange with One Longitudinal Stiffener ($f_y=0.5$ KSI)

<table>
<thead>
<tr>
<th>$t$</th>
<th>$w/t$</th>
<th>ST Stiffener</th>
<th>$I_t$</th>
<th>$K_{min}$</th>
<th>$K_t$</th>
<th>$F_b$, KSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{13}{16}$</td>
<td>55.4</td>
<td>6 × 25</td>
<td>152.4</td>
<td>3.70</td>
<td>2.65</td>
<td>28.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6 × 20.4</td>
<td>136.1</td>
<td>3.56</td>
<td>2.60</td>
<td>28.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5 × 17.5</td>
<td>73.4</td>
<td>2.90</td>
<td>2.36</td>
<td>24.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4 × 11.5</td>
<td>32.5</td>
<td>2.20</td>
<td>2.12</td>
<td>18.7</td>
</tr>
<tr>
<td>$\frac{7}{16}$</td>
<td>51.4</td>
<td>7.5 × 25</td>
<td>243.2</td>
<td>4.00</td>
<td>2.76</td>
<td>31.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6 × 25</td>
<td>152.4</td>
<td>3.43</td>
<td>2.55</td>
<td>29.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6 × 20.4</td>
<td>136.1</td>
<td>3.30</td>
<td>2.51</td>
<td>29.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5 × 17.5</td>
<td>73.4</td>
<td>2.69</td>
<td>2.29</td>
<td>25.9</td>
</tr>
<tr>
<td>$\frac{13}{16}$</td>
<td>48.0</td>
<td>7.5 × 25</td>
<td>243.2</td>
<td>3.74</td>
<td>2.66</td>
<td>32.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6 × 25</td>
<td>152.4</td>
<td>3.20</td>
<td>2.47</td>
<td>30.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6 × 20.4</td>
<td>136.1</td>
<td>3.08</td>
<td>2.43</td>
<td>30.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5 × 17.5</td>
<td>73.4</td>
<td>2.51</td>
<td>2.23</td>
<td>27.1</td>
</tr>
<tr>
<td>1</td>
<td>45.0</td>
<td>7.5 × 25</td>
<td>243.2</td>
<td>3.50</td>
<td>2.58</td>
<td>33.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6 × 25</td>
<td>152.4</td>
<td>3.00</td>
<td>2.40</td>
<td>31.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6 × 20.4</td>
<td>136.1</td>
<td>2.27</td>
<td>2.36</td>
<td>27.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5 × 17.5</td>
<td>73.4</td>
<td>2.35</td>
<td>2.17</td>
<td>28.1</td>
</tr>
</tbody>
</table>

### Selection of Structural Steel for Girders

The girders are designed for A36 steel in the positive-bending region and for steel with a yield strength of 50 ksi in the negative-bending region. The first step is selection of the web depth and preliminary thickness.
GIRDER DEPTH AND WEB DESIGN

The girder depth is assumed the same as that for the straight bridge of Chapter 7. A web thickness of \( \frac{1}{2} \) in. is investigated for the outer girder.

**Unstiffened Web in Positive-Moment Region—Outer Girder**

For shear stresses, the impact factor is 0.50. At the end bearing, the maximum design shear along the sloped web is

\[
V_u = \frac{58.69}{57} \times 1.3[55.5 + 14.9 + \frac{5}{3}(58.5 \times 1.50)] = 290.0 \text{ kips}
\]

Maximum capacity for buckling of the unstiffened web is

\[
V_u = \frac{3.5Es^3}{D} = \frac{3.5 \times 29,000(\frac{1}{2})^3}{58.69} = 216.2 < 290.0 \text{ kips}
\]

Therefore stiffeners are required near the end bearing. The maximum shear strength of the web is

\[
V_{ul} = 0.58F_yD_t = 0.58 \times 36 \times 58.69 \times \frac{1}{2} = 612.7 > 290.0 \text{ kips}
\]

Hence, a \( \frac{3}{4} \)-in. A36 web plate is satisfactory for ultimate strength. It also meets the requirement \( D/t \leq 150 \) for unstiffened webs.

The inner box girder, with slightly smaller shear forces and a \( \frac{1}{2} \)-in. web, also satisfies these conditions.

**Stiffened Web—Outer Girder**

The shear forces in the negative-bending region and also adjacent to the end bearing exceed the buckling capacity of the unstiffened \( \frac{1}{2} \)-in. web for both girders. For the outer girder, the webs will be stiffened for a distance of at least 39 ft from the interior support and at least 10 ft from the end bearing. Corresponding distances for the inner girder are 35 and 8 ft. (See the following shear curves.) More detailed final design calculations for the web are given later.

**Diagram:**

- **Comparison of Design Shear and Shear Capacity for Unstiffened Web of Outer Box Girder**

- **Shear, Kips**
  - 0, 0.1L, 0.2L, 0.3L, 0.4L, 0.5L, 0.6L, 0.7L, 0.8L, 0.9L
  - 290.0

- **Distance from End Bearing**
  - 39' Stiffened

- **Web Buckling Capacity**
  - 216.2

- **Total Factored Shear**
  - 0

- **Shear, Kips**
  - -371.4

---

8/81 II/7A.25
COMPARISON OF DESIGN SHEAR AND SHEAR CAPACITY FOR UNSTIFFENED WEB OF INNER BOX GIRDOR

Next, the maximum permissible depth-to-thickness ratio of the transversely stiffened web is checked, for an assumed maximum stiffener spacing of 58.69 in.

$$\frac{D}{t} = \frac{L}{t} \frac{1.154}{\sqrt{F_y}} \left[1 - 8.6 \frac{d_0}{R} + 34 \left(\frac{d_0}{R}\right)^3\right]$$

$$= \frac{1.154}{\sqrt{36}} \left[1 - 8.6 \times \frac{58.69/12}{384.67} + 34 \left(\frac{58.69/12}{384.67}\right)^3\right] = 172.4$$

The $D/t$ of the web is 58.69/(1/2) = 117.4 and therefore is satisfactory.

Because the preceding calculations were made only to determine whether the 1/2-in. flange web is a possible solution, the relatively small torsional effects have been ignored. They are considered later, however, when detailed stiffener spacing is calculated. Design computations for the stiffeners are also given later.

CROSS-FRAME SPACING

As discussed previously in General Design Considerations, internal cross frames are spaced at regular intervals within the box girders to minimize normal stresses due to distortion and to control top-flange lateral bending stresses. A tentative cross-frame spacing $S$ is calculated as follows:

$$S = L \left(\frac{R}{200L-7,500}\right)^{1/2} = 120 \left(\frac{400}{200 \times 120 - 7,500}\right)^{1/2} = 18.7 < 25 \text{ ft}$$

Additional studies of unsupported flange lengths versus lateral bending moments and required flange widths, however, establish a smaller cross-frame spacing as the best trade-off between girder-flange material and cross-frame material: Use 10 cross-frame spaces. Hence, measured along the bridge centerline,

$$S = \frac{120}{10} = 12 \text{ ft}$$
Measured along the centerline of the outer girder, the spacing is

\[ S = \frac{123.125 \times 12}{120} = 12.31 \text{ ft} \]

and measured along the centerline of the inner girder, the spacing is

\[ S = \frac{116.875 \times 12}{120} = 11.69 \text{ ft} \]

The smaller spacing is more than adequate to retain the shape of the cross section and to limit transverse distortional stresses.

It will be seen in subsequent calculations that the girder top flanges as designed are fully stressed. A larger crossframe spacing, with its corresponding larger lateral bending moments, would require larger flanges. Although crossframe material would be saved, flange material would be added. In general, reducing the unbraced flange length is a more efficient way to control lateral bending stresses than adding flange material.

LATERAL FLANGE BENDING

As discussed previously, lateral bending moments are produced by the radial component of axial force in curved flanges and by the horizontal component of shear forces in the sloping webs. The equations used to calculate lateral bending effects due to curvature were given previously in General Design Considerations.

The horizontal forces from the sloping webs act as a uniformly distributed load along both the top and bottom flanges. This load equals the change in vertical and torsional shear per foot along the girder due to DL1. It is applied as a uniform load on a continuous beam that is considered supported at the cross frames.

The change in vertical and torsional shears \( \Delta V_c \), kips per ft, due to DL along the top flange of the outer girder is

\[ \Delta V_c = \frac{V_{0.o} - V_{1.o}}{L} \]

where \( L \) = span, ft, of girder

\( V_{0.o}, V_{1.o} \) = vertical and torsional shear force, kips, at points \( k \) indicated by the subscript \( k \) is taken at the tenth points of the span.

The uniform load applied to the top flange \( \Delta V_h \), kips per ft is

\[ \Delta V_h = \frac{1}{2} \frac{h}{D} \Delta V_c \]

where \( h \) = horizontal projection of web, in.

\( D \) = depth, in., of box girder

The lateral bending moment \( M_{La} \), kip, ft, due to the sloped web is

\[ M_{La} = \frac{\Delta V_h S^2}{12} \]

where \( S \) = cross-frame spacing, ft

At the end bearing, the vertical shear due to DL1 is \( V_{0.o} = 52.7 \) kips per web. At the interior support, \( V_{1.o} = -101.2 \) kips per web.

The transverse, St. Venant shear force \( V_T \), kips, along the sloped web due to DL1 is

\[ V_T = \frac{Tl}{2A} \]

where \( T = \)

- torsional moment, kip-in., obtained from the curves of maximum torque
- \( l \) = sloped length of web, in.
- \( A \) = enclosed trapezoidal area, sq in., between girder bottom flange and top of plate diaphragm
At the end bearing, the transverse torsional shear is
\[ V_{T0} = \frac{-181.1 \times 12 \times 58.69}{2[(1/2)55(90+118)]} \times \frac{57}{58.69} = 10.8 \text{ kips} \]

At a distance of 0.7 of the span from the end bearing,
\[ V_{T0.7} = \frac{-134.4}{181.1} \times 10.8 = -8.0 \text{ kips} \]

At the interior support,
\[ V_{T1.0} = \frac{-131.5}{181.1} \times 10.8 = 7.8 \text{ kips} \]

The change in vertical and torsional shear along the girder between the end bearing (point 0.0) and the 0.7 point then is
\[ \Delta V = \frac{52.7 - (-101.2) + 10.8 - (-8)}{123.12} = 1.250 + 0.218 = 1.468 \text{ kips per ft} \]

Between the 0.7 point and the interior support, the shear change is
\[ \Delta V = 1.20 + \frac{-8.0 - 7.8}{3 \times 12.31} = 0.822 \text{ kips per ft} \]

The uniformly applied load at the top flange between the end bearing and the 0.7 point then is
\[ \Delta V_t = \frac{14}{57} \times 1.468/2 = 0.180 \text{ kips per ft} \]

The uniform load between the 0.7 point and the interior support (0.968 point) is
\[ \Delta V_t = \frac{14}{57} \times 0.822/2 = 0.101 \text{ kips per ft} \]

On the assumption that these loads are applied to the flange as a continuous beam, the lateral bending moments at the support—the cross-frame locations—are calculated. Between the end bearing and the 0.7 point,
\[ M_{Ls} = \frac{-0.180(12.31)^2}{12} = 2.27 \text{ kip-ft} \]

Between the 0.7 point and the interior support,
\[ M_{Ls} = \frac{0.101(12.31)^2}{12} = 1.28 \text{ kip-ft} \]

The nature of the allowable-stress equations is such that lateral flange bending may or may not have an effect on the design. This will be seen at a later point in the design example.

**NEGATIVE-MOMENT SECTION 2 FT FROM INTERIOR SUPPORT**

Adjacent to the center pier, the section chosen is about the same as that used for the straight bridge of Chapter 7, except that the section is composed entirely of steel with a yield strength of 50 ksi. As shown previously, in the drawing of the slab half section, the area of steel reinforcement to be used for the composite section also is the same as that for the design example of Chapter 7.

A single, longitudinal structural tee of 50-ksi steel is used to stiffen the bottom flange in the negative-moment region. Selection of this stiffener follows very closely the procedure employed for straight box girders.

The section used for maximum negative moment extends from the center pier to a transition point to be determined later. The section is investigated for negative moment occurring 2 ft from the pier. (The section directly over the pier is treated in conjunction with the pier diaphragm.) Properties are calculated for the steel section alone and for the section plus reinforcement.
SECTION 2 FT. FROM INTERIOR SUPPORT

Steel Section at Transition 2 Ft from Center of Interior Support

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>Io</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 T. Flg. Pl. 2 x 15</td>
<td>60.00</td>
<td>29.50</td>
<td>1,770</td>
<td>52,215</td>
<td>20</td>
<td>52,235</td>
</tr>
<tr>
<td>2 Web Pl. 7/8 x 58.69</td>
<td>58.69</td>
<td>-28.94</td>
<td>-2,330</td>
<td>67,421</td>
<td>15,891</td>
<td>15,891</td>
</tr>
<tr>
<td>Bot. Flg. Pl. 7/8 x 92</td>
<td>80.50</td>
<td>23.25</td>
<td>-171</td>
<td>3,973</td>
<td>41</td>
<td>67,421</td>
</tr>
<tr>
<td>Stiff. ST 7.5 x 25</td>
<td>7.35</td>
<td>-23.25</td>
<td>-171</td>
<td>3,973</td>
<td>41</td>
<td>4,014</td>
</tr>
</tbody>
</table>

\[ d_s = \frac{-731}{206.54} = -3.54 \text{ in.} \]

\[ d_{Bot. of steel} = 29.38 - 3.54 = 25.84 \text{ in.} \]

\[ \frac{I_{MA}}{I_{Top. of steel}} = \frac{136,973}{34.04} = 4,024 \text{ in.}^3 \]

\[ S_{Top. of steel} = \frac{136,973}{34.04} = 4,024 \text{ in.}^3 \]

Steel Section, with Reinforcing Steel, 2 Ft from Interior Support

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>Io</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>206.54</td>
<td>35.13</td>
<td>534</td>
<td>18,746</td>
<td>139,561</td>
<td>18,746</td>
</tr>
<tr>
<td>Reinforcement</td>
<td>15.19</td>
<td>-0.89</td>
<td>-731</td>
<td>-197</td>
<td>-197</td>
<td>-158,307</td>
</tr>
</tbody>
</table>

\[ d_e = \frac{-197}{221.73} = -0.89 \text{ in.} \]

\[ d_{Bot. of steel} = 29.38 - 0.89 = 28.49 \text{ in.} \]

\[ \frac{I_{MA}}{I_{Top. of steel}} = \frac{158,132}{31.39} = 5,038 \text{ in.}^3 \]

\[ S_{Top. of steel} = \frac{158,132}{31.39} = 5,038 \text{ in.}^3 \]

\[ d_{Top. of steel} = 30.50 + 0.89 = 31.39 \text{ in.} \]

\[ d_{Top. of steel} = 30.50 + 0.89 = 31.39 \text{ in.} \]

\[ d_{Top. of steel} = 30.50 + 0.89 = 31.39 \text{ in.} \]

\[ d_{Top. of steel} = 30.50 + 0.89 = 31.39 \text{ in.} \]
Bottom-Flange Stresses 2 Ft from Interior Support

The factored moment 2 ft from the interior support is computed from

$$M=1.3 \left( D + \frac{5}{3} L_T \right)$$

From the curves of maximum moment, the moment due to $DL_1$ is 5,820 kip-ft, the moment due to $DL_2$ is 1,370 kip-ft and the live-load moment is 2,670 kip-ft. The impact factor is 0.35.

$$L_T = L(1 + I) = 2,670 \times 1.35 = 3,604 \text{ kip-ft}$$

The longitudinal bending stress in the bottom flange then is

$$f_b = 1.3 \left( \frac{5,820 \times 12}{5,301} + \frac{1,370 \times 12}{5,550} + \frac{5}{3} \times \frac{3,604 \times 12}{5,550} \right) = 37.86 \text{ ksi}$$

For computation of the allowable bending stress for the bottom flange, the St. Venant shear stress at the section must first be calculated from

$$f_v = \frac{T}{2At}$$

with the thickness of flange $t = \frac{7}{8}$ in. The factored torque is computed from

$$T = 1.3 \left( D + \frac{5}{3} L_T \right)$$

From the curves of maximum torque, the torque due to $DL_1$ is 106.9 kip-ft, the torque due to $DL_2$ is 16.6 kip-ft and the live-load torque is 120.5 kip-ft. The impact factor is 0.50.

$$L_T = L(1 + I) = 120.5 \times 1.50 = 180.8 \text{ ft-kips}$$

The stress $f_v$ is calculated in two parts—that due to $DL_1$ torque acting on the non-composite section and that due to $DL_2$ and $L_T$ acting on the composite section. These sections are defined in the following figure.
The enclosed area \( A_1 \) to be used in computing \( f_c \) due to \( DL_1 \) is the area within the box bounded at the top by the lateral bracing, which is located 6 in. below the top flange:

\[
A_1 = \frac{1}{2} (57 - 6) (90 + 115.1) = 5,230 \text{ in.}^2
\]

The enclosed area \( A_2 \) to be used in computing \( f_c \) due to \( DL_2 \) and \( L_T \) is the area within the box bounded at the top by the mid-depth of the slab, which is 6.75 in. above the top flange:

\[
A_2 = \frac{1}{2} (57 + 6.75) (90 + 121.3) = 6,735 \text{ in.}^2
\]

The shear stress 2 ft from the interior support then is

\[
f_c = \frac{1.3 \times 12}{2 \times 7/8} \left( \frac{106.9}{5,230} + \frac{16.6}{6,735} + \frac{5}{3} \times \frac{180.8}{6,735} \right) = 0.60 \text{ ksi}
\]

Because the shear stress due to torque is much less than \( 0.75F_c/\sqrt{3} = 21.65 \text{ ksi} \), the allowable stress \( F_c \) for the bottom flange is determined by the parameters \( R_1, R_2 \) and \( \Delta \) as follows:

\[
\Delta = \sqrt{1 - 3 \left( \frac{f_c}{F_c} \right)^2} = \sqrt{1 - 3 \left( \frac{0.60}{50} \right)^2} = 0.9998
\]

For computation of \( R_1 \) and \( R_2 \), \( n = 1, b = 90/2 = 45 \text{ in.}, t = 7/8 \text{ in. and } L_s = L_o + A d^2 = 40.6 + 7.35(7.5 - 2.25)^2 = 243.2 \text{ in.}^4 \)

\[
K = \sqrt{\frac{1}{0.125b}} = \sqrt{243.2 \cdot \frac{0.125(0.875)^4}{45}} = 4.01 > 4 \text{ Use } K = 4.
\]

\[
K_i = \frac{5.34 + 2.48(f_c/bt^2)^{1/2}}{(n + 1)^2} = 2.76 < 5.34
\]

With the use of the preceding results,

\[
R_1 = \frac{97.08 \sqrt{K}}{\sqrt{\frac{1}{2} \left[ \Delta + \sqrt{\Delta^2 + 4(f_c/F_c)^2(K/K_i)^2} \right]}} = 194.2
\]

\[
R_2 = \frac{210.3 \sqrt{K}}{\sqrt{\frac{1}{1.2} \left[ \Delta - 0.4 + \sqrt{(\Delta - 0.4)^2 + 4(f_c/F_c)^2(K/K_i)^2} \right]}} = 420.4
\]

Because \( w \sqrt{F_c/2} = 45 \sqrt{50/(7/8)} = 363.7 \) falls between \( R_1 \) and \( R_2 \), the allowable compression stress in the bottom flange is given by

\[
F_b = F_c \left[ \Delta - 0.4 \left( 1 - \sin \frac{\pi}{2} \frac{R_2 - w \sqrt{F_c}}{R_2 - R_1} \right) \right] = 37.68 \text{ ksi}
\]

This represents only a \( \frac{1}{4} \% \) over stress. The \( \frac{1}{4} \)-in.-thick bottom-flange plate is considered adequate.

**Lateral Bending in the Longitudinal Tee Stiffener**

The direct stress in the flange of the longitudinal stiffener due to participation with the box-girder bottom flange in resisting bending is computed 2 ft from the interior support from

\[
f_c = \frac{y_c - y_1}{y_b} f_b
\]
with \( y_b = d_{Bot of stee} = 28.49 \text{ in.} \)
\[ y_s = \text{distance from top of stiffener to underside of girder bottom flange} = 7.50 + 0.88 = 8.38 \text{ in.} \]
\[ f_b = 37.86 \text{ ksi} \]

Hence, the direct stress in the longitudinal stiffener flange is
\[ f_s = \frac{28.49 - 8.38}{28.49} \times 37.86 = 26.72 \text{ ksi} \]

The lateral bending stress in the flange of the stiffener is computed from
\[ f_{we} = \frac{6 f_d d^2}{10 R b} \]
where \( d = l = 123.115/10 = 12.31 \text{ ft} \)
\( R = 410.38 \text{ ft} \)
\( b = 5.64 \text{ in.} \)

Substitution of these values in the equation for \( f_{we} \) yields
\[ f_{we} = \frac{6 \times 26.72(12.31)^2}{10 \times 410.38(5.64/12)} = 12.60 \text{ ksi} \]

The allowable average compression stress in the stiffener flange is given by the same equations that apply to curved I-girder flanges, outlined in General Design Considerations. For calculations of \( F_{wb} \), the following parameters are determined:
\[ f = 1 - 3 \left( \frac{E}{E_P} \right) \left( \frac{l}{b} \right)^2 = 1 - 3 \left( \frac{50}{29,000 \pi^2} \right) \left( \frac{12.31}{5.64/12} \right)^2 = 0.640 \]
\[ \bar{\rho}_B = \frac{1}{1 + \frac{l}{b} \left( 1 + \frac{l}{6b} \right) \left( \frac{l}{R} - 0.01 \right)^2} \]
\[ = \frac{1}{1 + \frac{12.31}{5.64/12} \left( 1 + \frac{12.31}{6 \times 5.64/12} \right) \left( \frac{12.31}{410.38} - 0.01 \right)^2} = 0.9468 \]
\[ \bar{\rho}_B / f = 0.9468 / 0.64 = 1.48 \]
\[ \bar{\rho}_w = 0.95 + 18 \left( 0.1 - \frac{l}{R} \right)^2 + \frac{\left( f_{we} / f_b \right) (0.3 - 0.1)}{\bar{\rho}_B / f} \]
with \( l/R = 12.31/410.38 = 0.030 \)
\[ f_{we}/f_b = 12.60 / 26.72 = 0.472 \]
\[ l/b = 12.31 / (5.64 \times 12) = 0.219 \]
\[ \bar{\rho}_w = 0.95 + 18(0.1 - 0.030)^2 \left( 0.472(0.3 - 0.1 \times 0.030 \times 26.19) \right) = 1.1088 \]
\[ \bar{\rho}_w \bar{\rho}_w = 0.9468 \times 1.1088 = 1.0499 > 1 \text{ Use 1.} \]

With the use of the preceding results, the allowable compression stress is
\[ F_{wb} = F / f \bar{\rho}_w \bar{\rho}_w = 50 \times 0.64 \times 1 = 32.02 > 26.72 \text{ ksi} \]

Braced at each cross frame, therefore, the stiffener has adequate strength for lateral bending due to curvature.

**Top-flange Stress 2 Ft from Interior Support**

The top-flange bending stress (tension) in the negative-moment section 2 ft from the interior support is
\[ F_{bs} = 1.3 \times 12 \left( \frac{5.820}{4.024} + \frac{1.370}{5.038} + \frac{5 \times 3.604}{5.038} \right) = 45.4 \text{ ksi} \]
The top-flange stress at this section may not exceed

\[ F_{ts} = F_0 \left[ 1 - 3 \left( \frac{F_0}{E_0 t_s^2} \right) \left( \frac{l}{b} \right)^2 \right] \]

With a cross-frame spacing of \( l = 12.31 \text{ ft} \) and flange width of 15 in.,

\[ F_{ts} = 50 \left[ 1 - 3 \left( \frac{50}{29,000 \pi^2} \right) \left( \frac{12.31 \times 12}{15} \right)^2 \right] = 47.5 > 45.4 \text{ ksi} \]

Hence, the top flange is adequate.

**Reinforcing Steel Stress 2 Ft from Interior Support**

The allowable tension stress in the slab reinforcing steel is 40 ksi. At the negative-moment section 2 ft from the interior support, the stress in the reinforcing steel is

\[ f_0 = 1.3 \times 12 \times \frac{1370 + (5/3)3.604}{4390} = 26.2 < 40 \text{ ksi} \]

Therefore, the reinforcing steel is not overstressed.

**INVESTIGATION OF WEBS AT INTERIOR SUPPORT**

The ratio \( D/t \) for the webs in the negative-moment region is \( 56.89/(1/2) = 113.78 < 150 \). Therefore, no web stiffeners are required in this region, if the buckling capacity of the webs is not exceeded.

The design shear per web at the center pier is a combination of direct and torsional shears. The direct shear is

\[ V_d = 1.3(106.6 + 27.1 + \frac{5}{3} \times 1.5 \times 57.5) \frac{58.69}{57} = 371 \text{ kips} \]

The torsional shear is

\[ V_T = 1.3 \times 12 \left( \frac{131.5}{5,230} + \frac{22.4}{6,735} + \frac{5}{3} \times 1.5 \times \frac{132.6}{6,735} \right)^{1/2} \times 58.69 = 36 \text{ kips} \]

The total shear then is \( V = 371 + 36 = 407 \text{ kips} \).

The ultimate shear capacity is the smaller of the following:

\[ V_{u1} = \frac{3.5E_t w^3}{D} = \frac{3.5 \times 29,000 (1/2)^4}{58.69} = 216.2 < 407 \text{ kips} \]

\[ V_{u2} = 0.58F_0 D t_w = 0.58 \times 50 \times 58.69 \times \frac{1}{2} = 851 > 407 \text{ kips} \]

Because the design shear exceeds \( V_{u1} \), the buckling capacity of the web, transverse stiffeners are required on the web. With stiffeners, the allowable shear is given by

\[ V_e = 0.58F_0 D t_w C = 0.58 \times 50 \times 58.69 \times \frac{1}{2} C = 851C \]

\[ C = 569.2 \frac{t_w}{2} \sqrt{\frac{1 + (D/d_s)^2}{F_0}} - 0.3 \leq 1.0 \]

\[ = 569.2 \times \frac{\sqrt{1 + (58.69/d_s)^2}}{58.69} - 0.3 = 0.6858 \sqrt{1 + \left( \frac{58.69}{d_s} \right)^2} - 0.3 \]

When the preceding expression for the allowable shear is equated to the required shear capacity of 407 kips, the required stiffener spacing is found to be 109 in. With this result, \( d_s/D = 109/58.69 = 1.9 \). The ratio \( d_s/D \), however, may not exceed 1. Therefore, the web stiffeners are placed 49 in. on centers, one-third the cross-frame spacing.
The design shear decreases to below the shear capacity of the unstiffened web approximately 39 ft from the interior support. Hence, all panels should be stiffened for at least this distance from the center pier. For practical reasons, transverse stiffeners are placed on the webs up to the field splice, a distance from the center pier that exceeds the 39 ft required. (The distance is measured along the outer web.)

Stiffeners also are required over a 10-ft length adjacent to the end bearing. For the purpose, two stiffeners are equally spaced in the first panel.

WEB STIFFENERS

Transverse web stiffeners consist of \( \frac{3}{4} \times 5 \)-in. plates. These are welded to the webs. The required moment of inertia of the web stiffeners, about the midplane of the web, is determined from

\[
I = d_w t^3 J
\]

where

\[
J = \left[ 2.5 \left( \frac{D}{d_w} \right)^3 - 2 \right] X
\]

Because \( d_w/D = 49/58.69 = 0.835 \) lies between 0.78 and 1, \( X \) should be determined from

\[
X = 1 + \frac{d_w/D - 0.78}{1.775} Z
\]

where \( Z = 0.95d_w^2/Rt = 0.95(49)^2/(415.3 \times 12 \times 1/2) = 0.914 \).

\[
X = 1 + \frac{0.835 - 0.78}{1.775} (0.914)^4 = 1.00
\]

\[
J = \left[ 2.5 \left( \frac{58.69}{49} \right)^3 - 2 \right] 1.00 = 1.59
\]

Hence, the required stiffener moment of inertia is

\[
I = d_w t^3 J = 49 \left( \frac{1}{2} \right)^3 \times 1.59 = 9.74 \text{ in}^4
\]

A 3/8 \times 5\)-in. plate provides a moment of inertia about the web midplane of

\[
I = \frac{0.375(5)^4}{12} + 0.375 \times 5(2.5 + 0.25)^2 = 18.08 > 9.74 \text{ in}^4
\]

Hence, this plate is adequate. Also, the width-thickness ratio of \( 5/(3/8) = 13.3 \) is below the maximum allowed of \( 82.2/\sqrt{R_t} = 82.2/\sqrt{36} = 13.7 \).

The ratio \( D/t \) for the webs is \( 58.69/(1/2) = 117.4 \). The ratio below which longitudinal web stiffeners are not required is

\[
\frac{D}{t} = \frac{1.154}{\sqrt{R_t}} \left[ 1 - 8.6 \left( \frac{d_w}{R} \right) + 34 \left( \frac{d_w}{R} \right)^2 \right] = 150 > 117.4
\]

Therefore, longitudinal web stiffeners are not required.

SHEAR-BENDING INTERACTION

The negative-moment section has been checked for both shear and bending as independent actions. AASHTO Specifications Art. 1.7.59E(4), however, limits the permissible bending moment when the design shear exceeds 60% of the critical shear so that

\[
\frac{M}{M_u} \leq 1.375 - 0.625 \frac{V}{V_u}
\]

where \( M \) = bending moment in girder

\( M_u \) = critical bending moment

\( V \) = shear in girder

\( V_u \) = critical shear

II/7A.34
From page 48, the total shear is $V = 371 + 36 = 407$ kips.

For calculation of the shear capacity with stiffeners spaced at 49 in.,

$$C = 569.2 \times \frac{1}{2} \sqrt{\frac{1 + (58.69/49)^2}{50}} - 0.3 = 0.770$$

The shear capacity then is

$$V_c = 0.58 \times 50 \times 58.69 \times \frac{1}{2} \times 0.770 = 655 \text{ kips per web}$$

and 60% of the shear capacity equals 393 kips, which is less than the 407-kip design shear. A reduction in permissible bending moment, therefore, is required. Because the section is highly stressed, an additional stiffener is needed to prevent a reduction in permissible bending stress. Hence, an extra stiffener is placed 24.5 in. from the pier. With this stiffener, 60% of the shear capacity will exceed the design shear at the pier.

Next, the design shear 49 in. from the pier is calculated and found to be less than 60% of the shear capacity of the section:

The direct shear is

$$V_d = 1.3(100.1 + 25.4 + \frac{5}{3} \times 1.5 \times 54.8) \frac{58.69}{57} = 351 \text{ kips}$$

The torsional shear is

$$V_t = 1.3 \times 12 \left( \frac{70.5}{5,250} + \frac{8.2}{6,735} + \frac{5}{3} \times 1.5 \times \frac{66.5}{6,735} \right) \frac{1}{2} \times 58.69 = 18 \text{ kips}$$

The total shear then is $351 + 18 = 369 < 393$ kips. Therefore, no bending reduction is required for the negative-moment section.

**MAXIMUM—POSITIVE MOMENT SECTION**

The section chosen for maximum positive moment is shown in the following drawing.

![Section for Maximum Positive Moment](image)

**SECTION FOR MAXIMUM POSITIVE MOMENT**

The section is composed entirely of A36 steel and is considered to act compositely with the concrete slab. A bottom-flange longitudinal stiffener is not required, because the bottom flange is in tension. The stiffener used in the negative-moment region will be terminated near the field splice.

In determination of the effective width of the concrete slab for the composite section, each half of the box girder is considered equivalent to a plate girder and the usual AASHTO criteria for effective slab width are applied. Hence, the effective slab
width for the box girder equals the sum of the effective flange widths for each flange. As shown previously in the Slab Half Section in Design of Girder Sections, the effective width of the slab for each box girder is 180 in.

Steel Section for Maximum Positive Moment

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 T. Flg. Pl. 1/4x12</td>
<td>16.50</td>
<td>28.84</td>
<td>476</td>
<td>13,278</td>
<td>15,891</td>
<td>13,728</td>
</tr>
<tr>
<td>2 Web Pl. ½x58.69</td>
<td>58.69</td>
<td>-28.75</td>
<td>-1,323</td>
<td>38,022</td>
<td>38,022</td>
<td></td>
</tr>
</tbody>
</table>

\[ d_s = \frac{-847}{121.19} = -6.99 \text{ in.} \]
\[ I_{NA} = 61,720 \text{ in.}^4 \]
\[ d_{Top of steel} = 29.19 + 6.99 = 36.18 \text{ in.} \]
\[ d_{Bot of steel} = 29.00 - 6.99 = 22.01 \text{ in.} \]
\[ S_{Top of steel} = \frac{61,720}{36.18} = 1,706 \text{ in.}^2 \]
\[ S_{Bot of steel} = \frac{61,720}{22.01} = 2,804 \text{ in.}^2 \]

Composite Section, 3n = 24, for Maximum Positive Moment

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>121.19</td>
<td>56.25</td>
<td>35.25</td>
<td>1,983</td>
<td>69,894</td>
<td>267</td>
</tr>
<tr>
<td>Conc. 180x7.5/24</td>
<td>56.25</td>
<td>-847</td>
<td>1,136</td>
<td>177,44</td>
<td>177,44</td>
<td>137,802</td>
</tr>
</tbody>
</table>

\[ d_{24} = \frac{1,136}{177.44} = 6.40 \text{ in.} \]
\[ -6.40 + 1,136 = 6.736 \]
\[ I_{NA} = 130,529 \text{ in.}^4 \]
\[ d_{Top of steel} = 29.19 \times 6.40 = 22.79 \text{ in.} \]
\[ d_{Bot of steel} = 29.00 + 6.40 = 35.40 \text{ in.} \]
\[ S_{Top of steel} = \frac{130,529}{22.79} = 5,727 \text{ in.}^3 \]
\[ S_{Bot of steel} = \frac{130,529}{35.40} = 3,687 \text{ in.}^3 \]

Composite Section, n = 8, for Maximum Positive Moment

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>121.19</td>
<td>168.75</td>
<td>35.25</td>
<td>5,948</td>
<td>209,682</td>
<td>791</td>
</tr>
<tr>
<td>Conc. 180x7.5/8</td>
<td>168.75</td>
<td>-847</td>
<td>289.94</td>
<td>5,101</td>
<td>278,114</td>
<td></td>
</tr>
</tbody>
</table>

\[ d_s = \frac{5,101}{289.94} = 17.59 \text{ in.} \]
\[ -17.59 + 5,101 = 39,727 \]
\[ I_{NA} = 188,387 \text{ in.}^4 \]
\[ d_{Top of steel} = 29.19 - 17.59 = 11.60 \text{ in.} \]
\[ d_{Bot of steel} = 29.00 + 17.59 = 46.59 \text{ in.} \]
\[ S_{Top of steel} = \frac{188,387}{11.60} = 16,240 \text{ in.}^3 \]
\[ S_{Bot of steel} = \frac{188,387}{46.59} = 4,044 \text{ in.}^3 \]

Stresses in Bottom Flange at 0.4 Point—Maximum Design Loads

Maximum positive moment occurs at a distance from the end bearing of about 0.4 the span. The bottom-flange stresses resulting from the maximum design moment for a section located at the 0.4 point are computed as follows:

\[ F_s = 1.3 \times 12 \left( \frac{2.271}{2.804} + \frac{638}{3.687} + \frac{5}{3} \times 1.35 + \frac{2.272}{4.044} \right) = 35.05 \text{ ksi} \]
The St. Venant shear stress is

\[
f_v = \frac{1.3 \times 12}{2 \times 0.5} \left(\frac{-159}{5230} - \frac{1.1}{6735} + \frac{5}{3} \times 1.5 \times \frac{50}{6735}\right) = -0.19 \text{ ksi}
\]

The allowable bending stress for a bottom flange in tension is

\[
F_b = F_y \sqrt{1 - 3(f_v/F_y)^2} = 36 \sqrt{1 - 3(0.19/36)^2} = 36.00 > 35.05 \text{ ksi}
\]

Hence, the bottom flange is adequate.

**Stresses in Top Flanges at 0.4 Point—Maximum Design Loads**

The top flanges in the positive-moment region are in compression and therefore the allowable stress is limited to

\[
F_{bu} = F_y f B \bar{B} w
\]

where \( f = 1 - 3 \left( \frac{F_y}{E \pi^2} \right) \left( \frac{l}{b} \right)^2 \).

The unbraced length \( l \) will be taken as zero, corresponding to a composite top flange that is continuously restrained by the slab. For \( l = 0, f = 1, \bar{B} = 1, \)

\[
\bar{w} = 0.95 + 18(0.1)^2 + \frac{f_B}{f_b} \times 0.3 = 1.13 + \frac{f_B}{f_b} \times 0.3 > 1
\]

Because \( F_i = F_i(1)(1)w \) and \( w > 1, F_i \) must be equal to the yield strength. This is always the case for top flanges of composite box girders. (A check should be made, however, that, before the concrete of the deck hardens, the steel section is not overstressed under \( DL_1 \). For this check, \( l \) should be taken equal to the distance between cross frames or diaphragms or other points of top-flange lateral support. This calculation is performed at a later point in the example.)

The bending stress under Maximum Design Load is

\[
f_b = 1.3 \times 12 \left( \frac{2,271}{1,706} + \frac{638}{5,727} + \frac{5}{3} \times 1.35 \times \frac{2,272}{16,240}\right) = 27.4 < 36 \text{ ksi}
\]

The stress under full design load is substantially less than the allowable stress. A reduction in the thickness of the \( \frac{1}{16} \)-in. flange plates, however, is not desirable, because a thickness if \( \frac{1}{14} \)-in. is considered the minimum desirable thickness to the top flange.

As discussed in General Design Considerations previously in this chapter, the webs will not require stiffeners if the ratio \( D/t < 150 \) and the shear is small. The \( \frac{1}{16} \)-in. web plates therefore need not be stiffened in the region of maximum positive bending.

The section designed for maximum positive moment extends from the field splice to a point to be determined later for transition to a lighter section in the positive bending region near the abutment.

**Stresses in Top Flanges at 0.4 Point—Construction Loads**

As mentioned previously, the top flanges should be checked at the 0.4 point for adequacy under \( DL_1 \) and construction loads. With the deck not in place, \( b/t \) for the \( \frac{1}{16} \times 12 \)-in. flanges equals 17.45. The flanges are noncompact because 17.45 exceeds 101.2/\( \sqrt{F_y} = 16.87 \) and is less than 139.1/\( \sqrt{F_y} = 23.19 \). Hence, the average normal stress is limited to

\[
F_{ny} = F_{ny} \rho B \rho w
\]

With \( L = \) unsupported length of flange = 12.31 ft = 148 in., \( R = \) radius of curvature of the flange = 410 ft, \( b = \) flange width = 12 in., \( l/b = 12.33 \) and \( l/R = 12.31/410 = 0.030, \)

\[
\rho_B = \frac{1}{1 + (l/R)(l/b)} = \frac{1}{1 + 0.030 \times 12.33} = 0.73
\]
For determination of $\rho_w$, the bending stress $f_b$ due to vertical loading and the lateral bending stress $f_w$ must first be computed.

The vertical-bending stresses arise from a moment of 2,271 kip-ft due to $DL_1$ plus a moment from a concentrated load. This load is taken as 4 kips, simulating a concrete screeding or finishing machine, and is placed at the 0.4 point. It produces a moment of 96 kip-ft. The total moment for construction loads then is 2,271 + 96 = 2,367 kip-ft. The resulting maximum bending stress in the top flanges is

$$f_b = 1.3 \times 12 \times \frac{2,367}{1,706} = 21.6 \text{ ksi}$$

Lateral bending is caused by curvature and web inclination. The bending moment due to curvature is

$$M_{le} = \frac{M_{le}^2}{10Rh} = \frac{2,367 \times 12(148)^2}{10 \times 410 \times 12 \times 57} = 221.9 \text{ in.-kips}$$

The lateral bending stress due to curvature then is

$$f_w = \frac{6M_{le}}{2b^2t} = \frac{6 \times 221.9}{2(12)^2(11/16)} = 6.7 \text{ ksi}$$

Web inclination causes a lateral bending moment of

$$M_{la} = 2.27 \times 12 = 27.2 \text{ in.-kips}$$

as calculated previously in LATERAL FLANGE BENDING. The lateral bending stress due to web inclination then is

$$f_w = \frac{27.2}{(12)^2(11/16)/6} = 1.7 \text{ ksi}$$

Hence, the total lateral bending stress is

$$f_w = 6.7 + 1.7 = 8.4 \text{ ksi}$$

and $f_w/f_b = 8.4/21.6 = 0.389$.

Since the ratio $f_w/f_b$ is, by definition, positive for the top flanges at the cross frames in positive-bending regions of the girder, $\rho_w$ is taken as the smaller of $\rho_{w1}$ and $\rho_{w2}$. With $l/b = 148/12 = 12.3$,

$$\rho_{w1} = \frac{1}{1 - (f_w/f_b)(1 - l/75b)} = \frac{1}{1 - 0.389(1 - 12.3/75)} = 1.48$$

$$\rho_{w2} = \frac{l/b}{0.95 + \frac{30 + 8,000(0.1 - l/r)^3}{1 + 0.6f_w/f_b}} = \frac{12.3}{0.95 + \frac{30 + 8,000(0.1 - 12.31/410)^3}{1 + 0.6 \times 0.389}} = 0.91 \text{ Governs}$$

and

$$F_{by} = F_y \left[ 1 - 3 \left( \frac{F_y}{E \pi^2} \right) \left( \frac{l}{b} \right)^2 \right] = 36 \left[ 1 - 3 \left( \frac{36}{29,000 \pi^2} \right)(12.3)^2 \right] = 33.9 \text{ ksi}$$

The maximum allowable stress is

$$F_{by} = 3.39 \times 0.73 \times 0.91 = 22.5 > 21.6 \text{ ksi O.K.}$$

The combined bending stress due to vertical and lateral bending at the flange tip is

$$f_b + f_w = 21.6 + 8.2 = 30.0 < 36 \text{ ksi}$$

The top flange is, therefore, adequate for $DL_1$ and construction loads.
FATIGUE INVESTIGATIONS

In the check of fatigue resistance, because the twin box girders have four webs interacting with the concrete deck and are continuous over two spans, the bridge is considered to have sufficient redundancy to be treated as a redundant load-path structure.

Fatigue of Stiffener Welds to Webs

The welds of the transverse stiffeners to the web near the bottom flange in the positive-moment region constitute a Category C detail, for which the allowable stress range is 19 ksi. For computation of the bending stress 1 1/2 in. above the bottom flange (near the toe of the stiffeners) section moduli are computed for the steel section alone and for the short- and long-term composite section that are used for positive bending moment.

<table>
<thead>
<tr>
<th>Section Moduli, In.³</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
</tr>
<tr>
<td>61,720</td>
</tr>
<tr>
<td>27.00−6.99</td>
</tr>
<tr>
<td>=3,084</td>
</tr>
</tbody>
</table>

At the point of maximum positive moment, the bending stress for positive live-load moment, with an impact factor of 0.35, is

\[ f_b = 1.35 \times 12 \times \frac{2.272}{4.225} = 8.7 \text{ ksi} \]

The bending stress for negative live-load moment is

\[ f_b = 1.35 \times 12 \times \frac{-579}{3,907} = -2.4 \text{ ksi} \]

Thus, the maximum stress range is 8.7−(−2.4)=11.1<19 ksi. Hence, the stiffener welds are satisfactory.

Fatigue of Shear-Connector Welds

Studies of the box girder indicate that, beginning at a distance from the end bearing of about 0.6 of the span, the section experiences sufficient negative live-load bending to put the top flange into tension. Hence, the shear connectors on the top flange at the 0.6 point should be checked for fatigue as a Category C detail, for which the stress range permitted is 19 ksi. Properties needed for computation of stress range at the 0.6 point are those computed previously for the maximum-positive-moment section. In addition, properties of the section to be used for negative bending at the 0.6 point must be calculated. This section consists of the steel section plus the reinforcing steel in the concrete slab. The reinforcement area is less than that used at the interior support, because half of the top layers of bars is terminated near the point of contraflexure. The area is obtained from the size and number of bars shown in the SLAB HALF SECTION in DESIGN OF GIRDER SECTIONS and the location of the center of gravity of the reinforcement relative to the bottom of the concrete slab is obtained as follows:

<table>
<thead>
<tr>
<th>Reinforcement at 0.6 Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bar Location</td>
</tr>
<tr>
<td>Top row</td>
</tr>
<tr>
<td>Bottom row</td>
</tr>
</tbody>
</table>

\[ d_{Reinf.} = \frac{29.59}{9.61} = 3.08 \text{ in.} \]

\[ 9.61 \text{ in.}² \quad 29.59 \text{ in.}² \]

8/81 11/7A.39
Steel Section, with Reinforcing Steel, at 0.6 Point

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>121.19</td>
<td>9.61</td>
<td>34.58</td>
<td>332</td>
<td>11,491</td>
<td>...</td>
</tr>
<tr>
<td>Reinforcement</td>
<td>9.61</td>
<td>34.58</td>
<td>332</td>
<td>11,491</td>
<td>...</td>
<td>11,491</td>
</tr>
</tbody>
</table>

\[ d_s = \frac{-515}{130.80} = -3.94 \text{ in.} \]
\[ d_{Bot. of steel} = 29.00 - 3.94 = 25.06 \text{ in.} \]
\[ S_{Bot. of steel} = \frac{77,104}{25.06} = 3,077 \text{ in}^3 \]

The positive live-load moment, with an impact factor of 0.35, at the point is
\[ M = 1.35 \times 1,948.3 = 2,630 \text{ kip-ft} \]

The negative live-load moment is
\[ M = 1.35(-867.9) = -1,172 \text{ kip-ft} \]

The bending stress due to positive live-load moment is
\[ f_b = \frac{2,630 \times 12}{16,240} = 1.9 \text{ ksi} \]

The bending stress due to negative live-load moment is
\[ f_b = \frac{-1,172 \times 12}{2,327} = -6.0 \text{ ksi} \]

Thus, the stress range is \( 1.9 - (-6.0) = 7.9 < 19 \text{ ksi} \). Therefore, the shear-connector welds are satisfactory.

Fatigue Weld at End of Longitudinal Stiffener

Another fatigue consideration in the positive-bending region is the weld at the termination of the bottom-flange longitudinal stiffener. A square termination of the stiffener is a Category E detail, for which the allowable stress range is 12.5 ksi. The stiffener can be terminated at a point at which the bottom-flange compression is within the allowable stress for an unstiffened flange. The 0.6 point of the span is tried.

First, the normal stress in the bottom flange is checked, requiring computation of the shear stress due to torsion. Then, the fatigue stress range at the stiffener termination is checked.

### Moments and Torques at 0.6 Point

<table>
<thead>
<tr>
<th></th>
<th>DL₁</th>
<th>DL₂</th>
<th>+(L+I)</th>
<th>-(L+I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M, kip-ft</td>
<td>1,013</td>
<td>338</td>
<td>2,630</td>
<td>-1,172</td>
</tr>
<tr>
<td>T, kip-ft</td>
<td>-121.6</td>
<td>-31.9</td>
<td>1.5 \times 5.2 = 7.8</td>
<td>1.5(-84.6) = -126.9</td>
</tr>
</tbody>
</table>

At the 0.6 point, the shear flow at the bottom flange due to torsion under \( DL₁ \) is
\[ \tau = -\frac{121.6 \times 12}{2 \times 51(90+118)/2} = -0.14 \text{ kips per in.} \]

The shear flow due to torsion under \( DL₂ \) is
\[ \tau = -\frac{-31.9 \times 12}{2 \times 63.75(90+121.3)/2} = -0.03 \text{ kips per in.} \]
For live-load, the negative shear flow is
\[ r = \frac{-126.9 \times 12}{2 \times 6.735} = -0.11 \text{ kips per in.} \]
The total shear flow then is 0.14 + 0.03 + 0.11 = 0.28 kips per in.

The shear stress in the bottom flange at the 0.6 point is therefore
\[ f_c = \frac{0.28}{1/2} = 0.56 \text{ ksi} \]

The vertical-bending stress in the bottom flange at this point is
\[ f_b = 1.3 \times 12 \left( \frac{1.013}{2.804} + \frac{338}{3.687} + \frac{5}{3} \times \frac{-1.172}{3.077} \right) = -2.84 \text{ ksi} \]

For computation of the critical compression stress for the bottom flange, assume for \( R_2 \) its maximum value:
\[ R_2 = \frac{210.3}{1/4} = 420.6 \]

\[ \frac{w}{t} \sqrt{R_2} = \frac{90}{1/2} \sqrt{36} = 1,080 > (R_2 = 420.6) \]

Hence, the maximum allowable normal stress in the bottom flange is
\[ f_c = 26,210K \left( \frac{t}{w} \right)^2 - \frac{f_c^2K}{26,210K^4(t/w)^4} \]
\[ = 26,210 \times 4 \left( \frac{1/2}{90} \right)^2 - \frac{(0.56)^4}{26,210(5.34)^4(0.5/90)^4} = 3.18 > 2.84 \text{ ksi} \]

Since the critical compression stress is larger than the design bending stress, the bottom flange is adequate without a stiffener, and the stiffener may be terminated at the 0.6 point.

The bending stress at the bottom of steel for positive live-load moment is
\[ f_b = \frac{2.630 \times 12}{4,044} = 7.8 \text{ ksi} \]
and for negative live-moment is
\[ f_b = \frac{-1.172 \times 12}{3,077} = -4.6 \text{ ksi} \]

Thus, the stress range is 7.8 - (-4.6) = 12.4 < 12.5 ksi. By a narrow margin, a square termination of the longitudinal stiffener can be made at the 0.6 point.

Fatigue characteristics of the termination, however, may be improved considerably by introducing a radius at the end of the stiffener, as shown in the following drawing. The 7-in. radius coping as shown is considered to upgrade the detail to Category D, with an allowable stress range of 16 ksi. Inasmuch as there are only four of these details in the structure, the coped termination is judged to be worth the small extra cost for providing a more fatigue-resistant design.
The longitudinal stiffener is terminated 11 ft from the field splice, so that it ends near the 0.6 point.

**POSITIVE-MOMENT TRANSITION 20 FT FROM END SUPPORT**

At a distance of 20 ft from the end bearing, the section used for maximum positive moment may be reduced. The top-flange thickness is decreased from $\frac{1}{4}\text{in.}$ to $\frac{1}{8}\text{in.}$ and the bottom flange is reduced from $\frac{1}{4}\text{in.}$ to $\frac{1}{8}\text{in.}$ The section is made of A36 steel.

### Steel Section Adjacent to end Support

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_o$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 T. Flg. Pl. $\frac{1}{4}\text{in.} \times 12$</td>
<td>13.50</td>
<td>28.78</td>
<td>389</td>
<td>11,182</td>
<td>11,182</td>
<td></td>
</tr>
<tr>
<td>2 Web Pl. $\frac{1}{8}\text{in.} \times 58.69$</td>
<td>58.69</td>
<td>-28.66</td>
<td>-824</td>
<td>23,615</td>
<td>15,891</td>
<td>15,891</td>
</tr>
<tr>
<td>Bot. Flg. Pl. $\frac{1}{8}\text{in.} \times 92$</td>
<td>28.75</td>
<td>-28.66</td>
<td>-824</td>
<td>23,615</td>
<td></td>
<td>23,615</td>
</tr>
</tbody>
</table>

$d_s = -\frac{435}{100.94} = -4.31\text{ in.}$

$I_{NA} = 50,688$ in.$^4$

$d_{\text{Top of steel}} = 29.06 + 4.31 = 33.37$ in.

$d_{\text{Bot. of steel}} = 28.81 - 4.31 = 24.50$ in.

$S_{\text{Top of steel}} = \frac{48,813}{33.37} = 1,463$ in.$^3$

$S_{\text{Bot. of steel}} = \frac{48,813}{24.50} = 1,992$ in.$^3$

### Composite Section, $3n=24$, Adjacent to End Support

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_o$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>100.94</td>
<td>35.25</td>
<td>-435</td>
<td>69,894</td>
<td>267</td>
<td>70,161</td>
</tr>
<tr>
<td>Conc. 180×7.5/24</td>
<td>56.25</td>
<td>35.25</td>
<td>1,983</td>
<td>69,894</td>
<td>267</td>
<td>70,161</td>
</tr>
</tbody>
</table>

$d_{34} = \frac{1,548}{157.19} = 9.85$ in.

$I_{NA} = 120,849$ in.$^4$

$d_{\text{Top of steel}} = 29.06 + 9.85 = 19.21$ in.

$d_{\text{Bot. of steel}} = 28.81 + 9.85 = 38.66$ in.

$S_{\text{Top of steel}} = \frac{105,601}{19.21} = 5,497$ in.$^3$

$S_{\text{Bot. of steel}} = \frac{105,601}{38.66} = 2,732$ in.$^3$

### Composite Section, $n=8$, Adjacent to End Support

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_o$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>100.94</td>
<td>35.25</td>
<td>-435</td>
<td>209,682</td>
<td>791</td>
<td>210,473</td>
</tr>
<tr>
<td>Conc. 180×7.5/8</td>
<td>168.75</td>
<td>35.25</td>
<td>5,948</td>
<td>209,682</td>
<td>791</td>
<td>210,473</td>
</tr>
</tbody>
</table>

$d_s = \frac{5,513}{269.69} = 20.44$ in.

$I_{NA} = 261,161$ in.$^4$

$d_{\text{Top of steel}} = 29.06 + 20.44 = 8.62$ in.

$d_{\text{Bot. of steel}} = 28.81 + 20.44 = 49.25$ in.

$S_{\text{Top of steel}} = \frac{148,475}{8.62} = 17,224$ in.$^3$

$S_{\text{Bot. of steel}} = \frac{148,475}{49.25} = 3,015$ in.$^3$

$d_{\text{Top of conc.}} = 39.00 - 20.44 = 18.56$ in.

$S_{\text{Top of conc.}} = \frac{148,475}{18.56} = 8,000$ in.$^3$
From the curves of maximum moment, for the section 20 ft from the end bearing, the moment due to $DL_1$ is 1,670 kip-ft, due to $DL_2$ 460 kip-ft and due to live load 1,480 kip-ft. The impact factor to be applied to live load is 0.35. The bottom-flange bending stress due to design moment then is

$$F_b = 1.3 \times 12 \left( \frac{1,670}{1,992} + \frac{460}{2,732} + \frac{5}{3} \times 1.35 \times \frac{1,480}{3,015} \right) = 32.9 \text{ ksi}$$

From the curves of maximum torque, the torque due to $DL_1$ is 137 kip-ft, due to $DL_2$ 41 kip-ft and due to live load 150 kip-ft. The impact factor is 0.50. The St. Venant shear stress in the bottom flange due to torque then is

$$f_s = \frac{1.3 \times 12}{2 \times 0.31} \left( \frac{137}{5,230} + \frac{41}{6,735} + \frac{5}{3} \times 1.50 \times \frac{150}{8,735} \right) = 2.20 \text{ ksi}$$

Accordingly, the bottom-flange allowable bending stress is

$$F_b = F_y \sqrt{1 - 3\left(\frac{f_s}{F_y}\right)^2} = 36 \sqrt{1 - 3(2.20/36)^2} = 35.8 > 32.9 \text{ ksi}$$

Hence, the bottom flange is adequate.

As indicated in the previous discussion of top-flange stresses at the 0.4 point, the allowable stress for the composite top flange under service conditions is the yield stress, 36 ksi. The design bending stress in the flange is

$$F_b = 1.3 \times 12 \left( \frac{1,670}{1,463} + \frac{460}{5,497} + \frac{5}{3} \times 1.35 \times \frac{1,480}{17,224} \right) = 22.1 < \text{ksi}$$

Hence, the $\frac{1}{2}$-in. x 12-in. flange is more than adequate.

A check of this flange under construction loads by the procedure used for bottom-flange stresses at the 0.4 point also indicates that the section has adequate strength and stability.

Inasmuch as the torsional stresses are low, the manhole detail for the bottom flange near the end bearing, as used in the example design of Chapter 7, may also be used for this example.

**NEGATIVE-MOMENT TRANSITION 13 FT FROM INTERIOR SUPPORT**

Inspection of the curves of maximum moment indicates that a smaller section than that used for maximum negative moment may be introduced at some distance from the pier. A reduced section made of A572, Grade 50, steel is chosen. Since the portion of the girder near the pier is also made of this material, the same steel is used throughout the negative-bending region, from field splice to field splice.

The reduced section is investigated at a distance from the interior support of 13 ft, where a transition is made to the heavier negative-moment section.

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_o$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 T. Flg. Pl. 1$\frac{1}{4}$ x 15</td>
<td>33.75</td>
<td>29.06</td>
<td>981</td>
<td>28,508</td>
<td>28,508</td>
<td></td>
</tr>
<tr>
<td>2 Web Pl. 1$\frac{1}{4}$ x 58.69</td>
<td>58.69</td>
<td>-28.88</td>
<td>-1,993</td>
<td>57,558</td>
<td>57,558</td>
<td></td>
</tr>
<tr>
<td>Bot. Flg. Pl. 1$\frac{3}{4}$ x 92</td>
<td>69.00</td>
<td>-23.25</td>
<td>-171</td>
<td>3,973</td>
<td>41</td>
<td>4,014</td>
</tr>
<tr>
<td>Stiff. ST 7.5 x 25</td>
<td>7.35</td>
<td>168.79 in.$^3$</td>
<td>-1,183 in.$^3$</td>
<td>105,971</td>
<td>$I_{NA} = 97,678$ in.$^4$</td>
<td></td>
</tr>
</tbody>
</table>

$$d_s = \frac{-1.183}{168.79} = -0.007 \text{ in.}$$

$$d_{Top of steel} = 29.62 + 7.01 = 36.63 \text{ in.}$$

$$d_{Bot of steel} = 29.25 - 7.01 = 22.24 \text{ in.}$$

$$S_{Top of steel} = \frac{97,678}{36.63} = 2,667 \text{ in.}^3$$

$$S_{Bot of steel} = \frac{96,678}{22.24} = 4,392 \text{ in.}^3$$
Steel Section, with Reinforcing Steel, 13 Ft from Interior Support

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>Ad²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>168.79</td>
<td>-1,183</td>
<td>534</td>
<td>18,746</td>
<td>105,971</td>
<td>18,746</td>
</tr>
<tr>
<td>Reinforcement</td>
<td>15.19</td>
<td>35.13</td>
<td>534</td>
<td>18,746</td>
<td>105,971</td>
<td>18,746</td>
</tr>
</tbody>
</table>

\[
d_e = \frac{-649}{183.98} = 3.53 \text{ in.} \quad \text{183.98 in.}^2 = -649 \text{ in.}^2 \quad -3.53 \times 649 = -2,289 \quad I_{NA} = 122,428 \text{ in.}^4
\]

\[
d_{\text{Top of steel}} = 29.63 + 3.53 = 33.16 \text{ in.} \quad d_{\text{Bot of steel}} = 29.25 - 3.53 = 25.72 \text{ in.}
\]

\[
S_{\text{Top of steel}} = \frac{122,428}{33.16} = 3,693 \text{ in.}^3 \quad S_{\text{Bot of steel}} = \frac{122,428}{25.72} = 4,760 \text{ in.}^3
\]

\[
d_{\text{Reinf}} = 35.13 + 3.53 = 38.66 \text{ in.} \quad S_{\text{Reinf}} = \frac{122,428}{38.66} = 3,167 \text{ in.}^3
\]

**Stresses in Bottom Flange**

At 13 ft from the interior support, the moment due to $DL_1$ is 3,705 kip-ft, due to $DL_2$ 845 kip-ft and due to live load 1,768 kip-ft. The impact factor is 0.35. The bending stress in the bottom flange then is

\[
F_s = 1.3 \times 12 \left( \frac{3,705}{4,392} + \frac{845}{4,760} + \frac{5}{3} \times 1.35 \times \frac{1,768}{4,760} \right) = 29.0 \text{ ksi}
\]

The torque due to $DL_1$ is $-30$ kip-ft, due to $DL_2$ $-15$ kip-ft and due to live load $-127$ kip-ft. The impact factor is 0.50. The shear stress in the bottom flange due to torque then is

\[
f_v = \frac{1.3 \times 12}{2 \times 3/4} \left( \frac{30}{5,230} + \frac{15}{6,735} + \frac{5}{3} \times 1.5 \times \frac{127}{6,735} \right) = 0.57 \text{ ksi}
\]

Because the shear stress is much less than $0.75F_s/\sqrt{3} = 15.6 \text{ ksi}$, the allowable stress $F_s$ for the bottom flange is determined by the parameters $R_1$, $R_2$ and $\Delta$ as follows:

\[
\Delta = \sqrt{1 - 3 \left( \frac{f_v}{F_s} \right)^2} = \sqrt{1 - 3 \left( \frac{0.57}{50} \right)^2} = 1.000
\]

For computation of $R_1$ and $R_2$, $n=1$, $b=90/2=45 \text{ in.}$, $t=3/4 \text{ in.}$ and $I_z=243.2 \text{ in.}^4$

\[
K = \sqrt{\frac{I_z}{0.125 t^4 b}} = \sqrt{\frac{243.2}{0.125 (0.75)^4 (45)}} = 4.68 > 4 \text{ Use } K=4.
\]

\[
K_t = \frac{5.34 + 2.84 (I_z/bt)^{1/3}}{(n+1)^2} = 3.00 < 5.34
\]

With the use of the preceding results,

\[
R_1 = \frac{97.08 \sqrt{4}}{\sqrt{\frac{1}{2} \left[ 1 + \sqrt{(1)^2 + 4 \left( \frac{0.57}{50} \right)^2 \left( \frac{4}{3} \right)^2} \right]^2}} = 194.2
\]

\[
R_2 = \frac{210.3 \sqrt{4}}{\sqrt{\frac{1}{1.2} \left[ 1 - 0.4 + \sqrt{(1-0.4)^2 + 4 \left( \frac{0.57}{50} \right)^2 \left( \frac{4}{3} \right)^2} \right]^2}} = 420.4
\]
Because \( w \sqrt{R}/t = 45 \sqrt{50}/0.75 = 424.2 > (R_2 = 420.4) \), the allowable compression stress in the bottom flange is given by

\[
f_b = 26,210K \left( \frac{t}{w} \right)^2 - \frac{f_y^2K}{26,210K^2(t/w)^2}
= 26,210 \times 4 \left( \frac{0.75}{45} \right)^2 - \frac{(0.57)^4}{26,210(3)^4(0.75/45)^2} = 29.1 > 29.0 \text{ ksi}
\]

The bottom flange is satisfactory.

**Stress in Top Flange**

The bending stress in the top flange 13 ft from the interior support is

\[
F_{bs} = 1.3 \times 12 \left( \frac{3.705}{2.667} + \frac{845}{3,693} \times \frac{5}{3} \times 1.35 \times \frac{1.768}{3,693} \right) = 42.0 \text{ ksi}
\]

The allowable stress in the top flange is computed as follows:

\[
F_b = F_y \left[ 1 - 3 \left( \frac{F_r}{E}\pi^2 \right) \left( \frac{t}{b} \right) \right]
= 50 \left[ 1 - 3 \left( \frac{50}{29,000\pi^2} \right) \left( \frac{148}{15} \right) \right] = 47.5 > 42.0 \text{ ksi}
\]

Hence, the 1% \times 15-in. top flange is adequate.

**Fatigue Check—13 Ft from Interior Support**

Fatigue is checked at the butt-welded top-flange transition. The weld is in category B, with an allowable stress range of 27.5 ksi. The positive live-load moment is 372 kip-ft at 13 ft from the interior support, and the negative live-load moment is \(-1,768\) kip-ft. Hence, the moment range is 372 - \((-1,768)\) = 2,140 kip-ft, and the stress range is

\[
f_b = 1.35 \times 12 \times \frac{2.140}{3,693} = 9.39 < 27.5 \text{ ksi}
\]

The flange weld is satisfactory for fatigue.

**FLANGE-TO-WEB WELDS**

Size of the flange-to-web welds for the straight box girder of Chapter 7 are governed by material-thickness requirements, rather than by horizontal shear flow, by a substantial margin. Torsional effects for the curved box girders do not add to the stresses in the flanges-to-web welds sufficiently to change this condition. Consequently, welds in this example are sized by material thickness.

The requirement that the web be fully developed by the flange-to-web weld, to insure adequate fatigue resistance with respect to transverse distortional stresses, should be checked, however. By AASHTO 1.749(E),

\[
\text{Weld size required} = \frac{\text{web thickness}}{0.707 \times 2 \times 1.414} = \frac{h}{2} \text{ in.} \quad \text{Governs}
\]

**SHEAR CONNECTORS**

Two \( \frac{h}{2} \)-in.-dia, 5-in.-high, welded stud shear connectors are welded per row to each flange. The 5-in. height satisfies the 2-in. minimum concrete cover over the connectors as well as the requirement for 2-in. minimum penetration into the concrete slab. The spacing of the shear connectors to meet fatigue criteria is determined at tenth points along the span. Subsequently, the connectors at the spacing that results are checked for ultimate strength.
## Computation of Shear-Connector Spacing

<table>
<thead>
<tr>
<th>Distance from End Bearing</th>
<th>Positive Live Load Shear, Kips</th>
<th>Negative Live Load Shear, Kips</th>
<th>Shear Range V, Including 50% Impact, Kips</th>
<th>Q ( \text{In.}^3 )</th>
<th>( I \text{ In.}^4 )</th>
<th>( S_c = \frac{VQ}{I} \text{ kips per in.} )</th>
<th>Spacing, ( S_r = \frac{4Z_r}{S_c} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>117.0</td>
<td>- 12.8</td>
<td>194.7</td>
<td>2,500</td>
<td>148,475</td>
<td>3.28</td>
<td>9.9</td>
</tr>
<tr>
<td>0.1L</td>
<td>83.3</td>
<td>- 14.3</td>
<td>146.4</td>
<td>2,500</td>
<td>148,475</td>
<td>2.47</td>
<td>13.1</td>
</tr>
<tr>
<td>0.2L</td>
<td>69.9</td>
<td>- 21.6</td>
<td>137.3</td>
<td>2,980</td>
<td>188,387</td>
<td>2.17</td>
<td>15.0</td>
</tr>
<tr>
<td>0.3L</td>
<td>57.1</td>
<td>- 30.9</td>
<td>132.0</td>
<td>2,980</td>
<td>188,387</td>
<td>2.09</td>
<td>15.5</td>
</tr>
<tr>
<td>0.4L</td>
<td>45.0</td>
<td>- 44.1</td>
<td>133.7</td>
<td>2,980</td>
<td>188,387</td>
<td>2.11</td>
<td>15.4</td>
</tr>
<tr>
<td>0.5L</td>
<td>34.0</td>
<td>- 56.6</td>
<td>135.9</td>
<td>2,980</td>
<td>188,387</td>
<td>2.15</td>
<td>15.1</td>
</tr>
<tr>
<td>0.6L</td>
<td>24.1</td>
<td>- 68.2</td>
<td>138.5</td>
<td>2,980</td>
<td>188,387</td>
<td>2.19</td>
<td>14.8</td>
</tr>
<tr>
<td>0.7L</td>
<td>15.4</td>
<td>- 78.7</td>
<td>141.5</td>
<td>587</td>
<td>122,428</td>
<td>0.68</td>
<td>47.7</td>
</tr>
<tr>
<td>0.8L</td>
<td>7.9</td>
<td>- 88.0</td>
<td>143.9</td>
<td>587</td>
<td>122,428</td>
<td>0.69</td>
<td>47.1</td>
</tr>
<tr>
<td>0.9L</td>
<td>2.5</td>
<td>-101.3</td>
<td>155.7</td>
<td>587</td>
<td>122,428</td>
<td>0.75</td>
<td>43.3</td>
</tr>
<tr>
<td>L</td>
<td>0</td>
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<td>172.4</td>
<td>547</td>
<td>158,132</td>
<td>0.60</td>
<td>54.1</td>
</tr>
</tbody>
</table>

Try the connector spacing shown in the following graph.

### SHEAR-CONNECTOR SPACING

**Shear Connectors—Strength Requirements**

For ultimate strength, the number of shear connectors between critical points must be such that the design load \( P_r \), kips per shear connector, does not exceed the ultimate strength, kips, of a shear connector:

\[
P_r = \phi S_u
\]

where \( \phi = 0.85 \)

\[
S_u = 0.4d^2 \sqrt{f_y E_y} = \frac{0.4(7/8)^2}{1,000} \sqrt{\frac{4,000(150)^3}{33} \times 4,000} = 37.93 \text{ kips}
\]

In a straight girder, the design load \( P \), kips per shear connector, is given by

\[
\bar{P} = \frac{P}{N}
\]
where \( N \) = number of shear connectors between point of maximum positive moment and the end bearing or dead-load inflection points, or between points of maximum negative moment and adjacent dead-load inflection points.

For positive bending moment, \( P \) is the smaller of the following:

\[
P_1 = 0.85f'bc
\]
\[
P_2 = A_yF_y
\]

where \( b \) = effective width of concrete slab = 180 in.
\( c \) = slab thickness = 7.5 in.
\( A_y \) = area of steel section = 121.19 in.\(^2\)
\( F_y \) = yield strength of the steel = 36 ksi

\[
P_1 = 0.85 \times 4 \times 180 \times 7.5 = 4,590 \text{ kips}
\]
\[
P_2 = 121.19 \times 36 = 4,363 < 4,590 \text{ kips Governors}
\]

For negative moment, with the area of longitudinal reinforcing steel in the slab \( A_y' = 15.19 \text{ in.}^2 \) and yield strength of this steel \( F_y' = 40 \text{ ksi} \),

\[
P = A_y'F_y' = 15.19 \times 40 = 608 \text{ kips}
\]

For curved box girders, the design load per shear connector is

\[
P_c = \sqrt{P^2 + F^2 + 2PF \sin \theta/2}
\]

where \( \theta \) = angle subtended between the point of maximum positive moment and the end bearing (6.87°) or the point of contraflexure (4.87°) or between the point of maximum negative moment and the point of contraflexure (5.45°)

\[
F = \frac{P(1 - \cos \theta)}{4KN_c \sin \theta/2}
\]
\( N_c \) = number of shear connectors on the two flanges at a section = 4
\( K = 0.166(N/N_c - 1) + 0.375 \)

Between the point of maximum positive moment and the end bearing, \( N = 212 \). Hence, with \( P = 4,363 \text{ kips} \),

\[
K = 0.166 \left( \frac{212}{4} - 1 \right) + 0.375 = 9.007
\]
\[
P = \frac{4,363}{212} = 20.58 \text{ kips}
\]
\[
F = \frac{4,363(1 - \cos 6.87^\circ)}{4 \times 9.007 \times 4 \sin 6.87/2} = 3.63 \text{ kips}
\]

The design load per shear connector for the curved girder then is

\[
P_c = \sqrt{(20.58)^2 + (3.63)^2 + 2 \times 20.58 \times 3.63 \sin 6.87/2} = 21.11 \text{ kips}
\]

The ultimate strength of a shear connector is

\[
P = 0.85 \times 37.93 = 32.2 > 21.11 \text{ kips}
\]

Hence, the number of shear connectors between the point of maximum moment and the end bearing is satisfactory.
Between the point of maximum positive moment and the dead-load inflection point, \( N=140 \).

\[
K = 0.166 \left( \frac{140}{4} - 1 \right) + 0.375 = 6.019
\]

\[
\bar{P} = \frac{4.363}{140} = 31.16 \text{ kips}
\]

\[
F = \frac{4.363(1 - \cos 4.77^\circ)}{4 \times 6.019 \times 4 \times \sin 4.77^\circ/2} = 3.77 \text{ kips}
\]

The design load per shear connector for the curved girder then is

\[
P = \sqrt{(31.16)^2 + (3.77)^2 + 2 \times 31.16 \times 3.77 \sin 4.77^\circ/2} = 31.54 < 32.2 \text{ kips}
\]

Hence, the number of shear connectors between the point of maximum positive moment and the dead-load inflection point is satisfactory.

Between the point of maximum negative moment and the dead-load inflection point, \( N=80 \).

\[
K = 0.166 \left( \frac{80}{4} - 1 \right) + 0.375 = 3.529
\]

\[
\bar{P} = \frac{608}{80} = 7.6 \text{ kips}
\]

\[
F = \frac{608(1 - \cos 5.45^\circ)}{4 \times 3.529 \times 4 \times \sin 5.45^\circ/2} = 1.02 \text{ kips}
\]

The design load per shear connector for the curved girder then is

\[
P = \sqrt{(7.6)^2 + (1.02)^2 + 2 \times 7.6 \times 1.02 \sin 5.45^\circ/2} = 7.72 < 32.2 \text{ kips}
\]

Hence, the number of shear connectors between the point of maximum negative moment and the dead-load inflection point is satisfactory. Thus, the spacing selected to meet fatigue requirements also satisfies ultimate-strength requirements.

CROSS FRAMES

Three different designs for intermediate cross frames of A36 steel are employed for the girders in this example. Two of these are used for regions of the box girders where a longitudinal stiffener is attached to the bottom flange. The third design is used for regions of the girder without this stiffener. The cross frame shown in the following drawing is the third type.

CROSS FRAME IN SECTIONS WITHOUT LONGITUDINAL STIFFENER
Design of Top Strut of Cross Frames

For simplicity, cross-frame members are designed with working-stress criteria. For the top strut, an angle $5\times5\times\frac{3}{16}$ in., with an area of 3.61 in.$^2$, is investigated for overall buckling ($L/r<120$), local buckling ($b/t<12$) and for capacity as a compression member. The computations show that the lateral reaction of the strut on the curved flange is 10.6 kips, considerably less than the strut capacity of 34.2 kips.

The unbraced length of the strut is 118 in. For overall buckling of the $5\times5\times\frac{3}{16}$-in. angle,

$$\frac{L}{r} = \frac{118}{0.99} = 119 < 120$$

For local buckling,

$$\frac{b}{t} = \frac{5 - 0.38}{0.38} = 12.2$$

This is close enough to the limiting value of 12 for main compression members to be acceptable.

The force acting on the strut is given by

$$R = l.w.l$$

where $w =$ load imposed during the wet-concrete condition, kips per ft

$l =$ cross-frame spacing = 12.31 ft

For the change in vertical and torsional shear between the end bearing and the 0.7 point, as computed for lateral flange bending,

$$w = \frac{14}{57} \times 1.468 \times \frac{1}{2} = 0.18 \text{ kips per ft}$$

From maximum positive moment due to $DL_1$,

$$w = \frac{2.271 \times 12}{2 \times 55 \times 4.925} = 0.05 \text{ kips per in.} = 0.60 \text{ kips per ft}$$

Hence, the force on the strut is

$$R = 1.1(0.18 + 0.60)12.31 = 10.6 \text{ kips}$$

The allowable force on the strut is

$$R = F_c.A = [16,980 - 0.53(119)^3]13.61 = 34.2 > 10.6 \text{ kips}$$

The strut is satisfactory.

Design of Cross-Frame Diagonals

An angle $3\times3\times\frac{3}{16}$ in. checked for the diagonal for overall and local buckling. The diagonal has an area of 1.44 in.$^2$ and makes an angle with the horizontal of $\alpha = \text{tan}^{-1}(51/104) = 26.12^\circ$. The minimum area permissible for the member is

$$A_b = 75 \times \frac{S_b}{\alpha^2} \times \frac{t_w^3}{d+b} = 75 \times \frac{12.31 \times 12 \times 90}{(26.12)^2} \times \frac{(\frac{3}{16})^3}{57 + 90} = 0.017 < 1.44 \text{ in.}^2$$

The diagonal is checked for local buckling as a secondary member.

$$\frac{b}{t} = \frac{3 - 0.25}{0.25} = 11 < 16$$

The $3\times3\times\frac{3}{16}$-in. angle, therefore is satisfactory.

Design of Bottom Strut Placed Above the Longitudinal Stiffener

The cross frame used in conjunction with the longitudinal stiffener on the bottom flange of the girder has a bottom strut (see the following sketch). This strut serves as a transverse bottom-flange stiffener for the girder and also as a transverse lateral
support for the curved longitudinal stiffener. Sizes of the other members of this cross frame are the same as those of the previously discussed cross frame.

CROSS FRAME IN SECTIONS WITH LONGITUDINAL STIFFENER

Since the load-factor specifications have no provisions for girders with both longitudinal and transverse bottom flange stiffeners, working-stress provisions are used to proportion the bottom strut of the cross frame. An ST 7.5×25, which has an area of 7.35 in.² and moment of inertia of 40.6 in.⁴, is investigated.

For unfactored loads, the bending stress in the bottom flange at the 0.9 point of the girder is

\[ f_b = 12 \left( \frac{3.870}{5.301} + \frac{887}{5550} + \frac{5}{3} \times \frac{1.35}{5550} \times \frac{1.819}{5550} \right) = 19.5 \text{ ksi} \]

The area of the girder bottom flange plus the longitudinal stiffener is

\[ A_f = 92 \times \frac{7}{8} + 7.35 = 87.85 \text{ in.}^2 \]

The moment of inertia required for the bottom strut of the cross frame is

\[ I_s = 0.1(n + 1)^4 \omega \frac{f_b}{E} \frac{A_f}{l} \]

\[ = 0.1(1+1)^4(45)^4 \times 19.5 \times \frac{87.85}{29000} \times \frac{12.31 \times 12}{29000} = 29.2 < 40.6 \text{ in.}^4 \]

The width-thickness ratio of the stem of the ST is

\[ \frac{b}{t} = \frac{7.5 - 0.62}{0.55} = 12.5 \]

The maximum permissible value of this ratio is

\[ \frac{b}{t} = \frac{82.2}{\sqrt{36}} = \frac{82.2}{6} = 13.7 > 12.5 \]

The ST 7.5×25 is satisfactory.

Design of Bottom Strut at Same Level as Longitudinal Stiffener

Another type of cross frame used in conjunction with the longitudinal stiffener on the bottom flange is required by the Guide Specifications at the points of maximum flange stress and dead-load contraflexure. In this type of cross frame, the bottom strut, which also serves as a transverse stiffener, is attached to the bottom flange of
the girder, at the same level as the longitudinal stiffener, as shown in the following drawing.

![Cross Frame Diagram]

**CROSS FRAME AT POINTS OF DEAD-LOAD CONTRAFLUXURE AND MAXIMUM FLANGE STRESS**

Sizes of all members of this cross frame are the same as those of the other cross frames. Because a solid plate diaphragm is placed over the interior support (point of maximum flange stress), only one cross frame of this type is needed. It is placed near the field splice.

**Cross-Frame Connections**

All cross-frame connections are made with ¼-in. fillet welds, which provide more than adequate strength.

**LATERAL BRACING**

The lateral bracing, which will be placed about 6 in. below the top flanges of the box girders, is designed to carry the St. Venant shear that exists across the top of the box due to torsion under initial dead load. For computation of this shear, a solid plate is assumed as a substitute for the open bracing actually used.

The curves of maximum torque indicate that maximum $DL_t$ torque occurs at the end bearing. The shear is obtained by multiplying the shear flow produced by the torque by the width of the box at the level of the lateral bracing (115.1 in.). This shear force is the lateral component $F_t$ of the force $F$ in the 15.6-ft-long bracing diagonal (see the following drawing).

![Plan View of Girder Lateral Bracing Diagram]
The $DL_1$ torque at the end bearing is 181.1 kip-ft. For an enclosed area of the box of $A_1=5,230$ in.$^2$, the shear flow is

$$S = \frac{T}{2A_1} = \frac{1.3 \times 181.1 \times 12}{2 \times 5,230} = 0.270 \text{ kips per in.}$$

The resulting transverse force is

$$F_t = 0.270 \times 115.1 = 31.1 \text{ kips}$$

The force is the diagonal bracing therefore is

$$F = 31.1 \times \frac{15.6}{9.6} = 50.5 \text{ kips}$$

and the longitudinal component of the force is

$$F_l = 31.1 \times \frac{12.3}{9.6} = 39.8 \text{ kips}$$

Because the lateral-bracing diagonals are considered main members, for which the slenderness ratio $L/r$ must be equal to or less than 120, the radius of gyration of the diagonal should be at least

$$r = \frac{15.6 \times 12}{120} = 1.56 \text{ in.}$$

For the diagonals, try a WT7x26.5. It has a radius of gyration about the Y-Y axis $r_y = 1.92$, area $A = 7.81$ in.$^2$ and section modulus $S_y = 4.94$ in.$^3$. The slenderness ratio for the diagonal for the Y-Y axis is

$$\frac{KL_y}{r_y} = \frac{0.75 \times 15.6 \times 12}{1.92} = 73.1$$

For computation of the critical buckling stress in the diagonal,

$$\sqrt{\frac{2\pi^2E}{F_y}} = 107 > 73.1$$

Hence, the critical buckling stress for the Y-Y axis is

$$F_{cr} = F_y \left[ 1 - \frac{F_y}{4\pi^2E} \left( \frac{KL_y}{r_y} \right)^2 \right] = 36 \left[ 1 - \frac{36}{4\pi^2 \times 9 \times 29,000} (73.1)^2 \right] = 29.95 \text{ ksi}$$

The bending strength of the diagonal as an unbraced beam is

$$M_u = F_y S \left[ 1 - \frac{3F_y}{4\pi^2E} \left( \frac{L_e}{0.95} \right)^2 \right]$$

$$= 36 \times 4.94 \left[ 1 - \frac{3 \times 36}{4\pi^2 \times 29,000} \left( \frac{15.6 \times 12}{0.9 \times 3.845} \right)^2 \right] = 128.7 \text{ kip-in.}$$

The slenderness ratio for the X-X axis is

$$\frac{KL_x}{r_x} = \frac{0.75 \times 15.6 \times 12}{1.88} = 74.7$$

The Euler buckling stress for the X-X axis is

$$\frac{F_e}{(KL_x/r_x)^2} = \frac{\pi^2 E}{(74.7)^2} = 51.29 \text{ ksi}$$
The neutral axis of the WT for bending under Maximum Design Load is located at a distance \( y_p \) below the outer surface of the flange. The area of the section above the axis equals the area below:

\[
8.06y_p = 6.30 \times 0.37 + 8.06 (0.66 - y_p)
\]

Solution of the equation yields \( y_p = 0.475 \)-in. and \( 0.66 - y_p = 0.185 \)-in.

The plastic section modulus then is

\[
Z = \frac{8.06 (0.475)^2}{2} = \frac{8.06 (0.185)^2}{2} + 6.30 \times 0.37 \times 3.335 = 8.82 \text{ in.}^3
\]

The capacity of the WT as a compact beam therefore is

\[
M_p = F_p Z = 36 \times 8.82 = 317.5 \text{ kip-in.}
\]

The maximum bending moment in the diagonal is

\[
M_{ec} = 50.5 \times 1.38 = 69.7
\]

\[
M_{DL} = 1.30 \times 0.0265 (15.6)^3 \times 12 = 12.6
\]

\[
M = 82.3 \text{ kip-in.}
\]

Substitution of the preceding results in the interaction equation for combined compression and bending yields

\[
\frac{50.5}{0.85 \times 7.81 \times 29.95} + \frac{82.3}{128.7 \left(1 - \frac{50.5}{7.81 \times 51.29}\right)} = 0.986 < 1.0
\]

\[
\frac{50.5}{0.85 \times 7.81 \times 36} + \frac{82.3}{317.5} = 0.471 < 1.0
\]

The WT7×26.5 is satisfactory.

**Bolted Gusset-Plate Connection**

The connections of the lateral bracing to the girder webs are made with bolts, because of their excellent fatigue characteristics. This type of connection corresponds to an AASHTO Category B detail, with an allowable girder stress range of 45.0 ksi (lane loading) and 27.5 ksi (truck loading).

The bolted connections of the lateral bracing at a girder are made to a 3/4-in. gusset plate, which is, in turn, welded to another 3/4-in. plate that is bolted to the girder web (see following drawing). Bolts are \( \frac{7}{8} \)-in. in diameter and have a capacity of 12.63 kips each. Maximum permissible length of the unsupported edge of the gusset is

\[
L = \frac{347.8 t}{\sqrt{F_p}} = \frac{347.8 \times 3/4}{\sqrt{36}} = 43.5 \text{ in.}
\]

The unsupported edge of the gusset is about 12 in. < 43.5 in.
TYPICAL BOLTED CONNECTION FOR LATERAL BRACING

The two plates are connected by fillet welds. The capacity of a fillet weld is

\[ 0.45f_w = 0.45 \times 70 = 31.5 \text{ ksi} \]

A \( \frac{1}{4} \)-in. fillet weld is tried. For simplicity, the longitudinal force delivered by the lateral bracing to the girder web is assumed to be twice the longitudinal component of the force on the bracing diagonal, or \( 2 \times 39.8 = 79.6 \) kips. The connected edge of the gusset plate is about 30 in. long. For two \( \frac{1}{4} \)-in. fillet welds carrying \( P_L = 79.6 \) kips, the stress on a weld is

\[ f_w = \frac{1.3 \times 79.6}{30 \times 2 \times 0.707 \times \frac{1}{4}} = 9.75 < 31.5 \text{ ksi} \]

Since the lateral bracing is carrying only dead load, the weld need not be investigated for fatigue.

The diagonals are connected to the gusset plate with \( \frac{7}{8} \)-in.-dia, A325 bolts. Each bolt has a capacity of 12.63 kips. Hence, the number of bolts required for a diagonal is

\[ \frac{1.3 \times 50.5}{12.63} = 5.2 \text{ bolts} \]

Use 6 bolts.

The plate-to-girder web attachment requires

\[ \frac{1.3 \times 79.6}{12.63} = 8.2 \text{ bolts} \]

Use 12 bolts.

The bolts in the connection to the girder web are subject to combined tension and shear. The tensile forces are caused by a direct pull and prying action. The maximum direct tensile force is 31.1 kips. Divided among the six bolts, the tension is

\[ T = \frac{31.1}{6} = 5.2 \text{ kips per bolt} \]
Prying action results from both the tension on the bolts and distortion of the connected parts. The lever arms involved are the distance \( a = 1\frac{1}{8} \) in. from the center of the bolts to the edge of the \( \frac{3}{4} \times 9 \)-in. connection plate and the distance \( b = 1\frac{3}{8} \) in. from the toe of the fillet weld between the two connection plates and the center of the bolts. The thickness of the girder web may be assumed to be 0.5 in. < 0.75 in. The prying force on the bolts then is

\[
Q = \left( \frac{3b}{8a} - \frac{t^2}{20} \right) T = \left[ \frac{3 \times 2.375}{8 \times 1.5} - \frac{(\frac{3}{4})^2}{20} \right] 5.2 = 3.0 \text{ kips per bolt}
\]

The total tension on the bolts = 5.2 + 3.0 = 8.2 kips per bolt. The tensile stress in each bolt therefore is

\[
f_t = \frac{8.2}{0.601} = 13.6 \text{ ksi}
\]

The shear stress in each bolt is

\[
f_s = \frac{39.8}{6 \times 0.601} = 11.0 \text{ ksi}
\]

The allowable shear stress is

\[
f_s = 1.33F_s \left( 1 - \frac{f_t}{159} \right) = 1.33 \times 16.0 \left( 1 - \frac{13.6}{159} \right) = 19.5 > 11.0 \text{ ksi}
\]

The connection, therefore, is satisfactory.

A similar design is made for the lateral bracing connection at the ends of the girders, where the longitudinal shear is 39.8 kips (see the following drawing). The gusset plate is about 12 in. long along the connection to the girder web. For two \( \frac{1}{4} \)-in. fillet welds, the stress in a weld is

\[
f_w = \frac{1.3 \times 39.8}{12 \times 2 \times 0.707 \times \frac{1}{4}} = 12.20 < 31.5 \text{ ksi}
\]

---

**Diaphragm**

![Diaphragm Diagram]

**BRACING CONNECTION WITH BOLTED GUSSET AT END OF GIRDER**

By inspection, the connection shown is adequate.

**END DIAPHRAGM**

The box girders are supported at the end bearings on two shoes, 5.5 ft. apart. These shoes need not be designed for uplift inasmuch as this condition cannot occur. A \( \frac{1}{4} \)-in.-thick plate diaphragm with a \( \frac{1}{2} \times 10 \)-in. top flange is used to transfer the girder-web shear and the girder torque into the shoes. Investigation of the diagram begins with a tabulation of these shears and torques and a computation of St. Venant shear flow. As noted previously, an impact factor of 1.0 is used for live-load reactions.
SECTION AT DIAPHRAGM AT END BEARING

Vertical Shear and Torque at End Bearing

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$L+I$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$, kips</td>
<td>55.5</td>
<td>14.9</td>
<td>117.0</td>
<td>187.4</td>
</tr>
<tr>
<td>$T$, kip-ft</td>
<td>181.1</td>
<td>52.6</td>
<td>355.0</td>
<td>588.7</td>
</tr>
</tbody>
</table>

The enclosed area to be used for the box girder in computing the shear due to $DL_1$ torque is

$$A_1 = \frac{1}{2} (90 + 118) 57 = 5928 \text{ in}^2$$

The enclosed area of the box girder to be used in computing the shear due to $DL_2$ and $L+I$ torque is

$$A_2 = \frac{1}{2} (90 + 118) 63.75 = 6630 \text{ in}^2$$

**Shear Flow Due to Torque**

For $DL_1$:

$$S = \frac{181.1 \times 12}{2 \times 5928} = 0.183$$

For $DL_2$:

$$S = \frac{52.6 \times 12}{2 \times 6630} = 0.048$$

For $L+I$:

$$S = \frac{355 \times 12}{2 \times 6630} = 0.321$$

The shear force among the web, therefore, is

$$V = 0.552 \times 58.69 = 32.4 \text{ kips}$$

The shoe reactions are calculated by superimposing the torque reactions on the reactions due to vertical loads.

$$\text{Torque Reaction} = \frac{588.7}{5.5} = 107.0 \text{ kips}$$
One shoe reaction then is

\[ R_L = 187.4 + 107.0 = 294.4 \text{ kips} \text{ Governs.} \]

and the other reaction is

\[ R_R = 187.4 - 107.0 = 80.4 \text{ kips} \]

SHOE REACTIONS AT END BEARING

Design of Bearing Stiffeners at End Support

Bearing stiffeners are designed on the basis of the maximum reaction of 294.4 kips, with working-stress principles. The stiffeners are then checked as columns under ultimate-strength loads.

Try two 5-in.-wide stiffeners, welded on each side of the diaphragm web, over each shoe. The required stiffener thickness is computed with 29 ksi as the allowable stress under service load. Since there are four bearing stiffeners,

\[ t = \frac{294.4}{29(5 - 0.375)4} = 0.549 \]

Try 5×\(\frac{1}{8}\)-in. stiffeners.

SECTION A-A

BEARING STIFFENERS

<table>
<thead>
<tr>
<th>End Reactions, kips</th>
</tr>
</thead>
<tbody>
<tr>
<td>( DL_1 )</td>
</tr>
<tr>
<td>Direct</td>
</tr>
<tr>
<td>Torque</td>
</tr>
<tr>
<td>Total</td>
</tr>
</tbody>
</table>

The equivalent column area of the diaphragm web and stiffeners is

\[ A = \frac{1}{2} (2 \times 4.5 + 8) + 4 \times \frac{5}{8} \times 5 = 21.0 \text{ in}^2 \]
The moment of inertia of the section is

\[ I = \frac{bd^3}{12} = \frac{0.625(10.5)^2}{12} = 120.6 \text{ in.}^4 \]

The radius of gyration of the section is

\[ r = \sqrt{\frac{I}{A}} = \sqrt{\frac{120.6}{21.0}} = 2.40 \text{ in.} \]

and the slenderness ratio is

\[ \frac{KL}{r} = \frac{D}{r} = \frac{57}{2.40} = 23.75 \]

The maximum permissible slenderness ratio is

\[ \frac{KL}{r} = \sqrt{\frac{2\pi^2E}{F_y}} = \sqrt{\frac{2\pi^2 \times 29,000}{36}} = 126.1 > 23.75 \text{ O.K.} \]

Hence, the column critical stress is

\[ F_c = F_y \left[ 1 - \frac{KL}{D} \left( \frac{D}{r} \right)^{-4} \right] = 36 \left[ 1 - \frac{36}{4 \pi^2 \times 29,000} (23.75)^4 \right] = 35.4 \text{ ksi} \]

The column load capacity then is

\[ P_c = 0.85 \times 21.0 \times 35.4 = 631.9 \text{ kips} \]

The maximum design load is

\[ V_c = 1.3 (88.4 + 24.5 + \frac{5}{3} \times 181.5) = 540 < 631.9 \text{ kips O.K.} \]

Use PL \( \frac{4}{5} \times 5 \) in. as bearing stiffeners.

Check of End Diaphragm in Bending

Next, the end diaphragm is checked in bending, beginning with computation of section properties. A 10-in.-wide strip of the bottom flange of the girder is taken as the bottom flange of the diaphragm.

Section Properties of End Diaphragm

<table>
<thead>
<tr>
<th>Material</th>
<th>( A )</th>
<th>( d )</th>
<th>( Ad )</th>
<th>( Ad^2 )</th>
<th>( I_c )</th>
<th>( I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Flg. ( \frac{1}{2} \times 10 )</td>
<td>5.00</td>
<td>28.75</td>
<td>143.8</td>
<td>4,133</td>
<td></td>
<td>4,133</td>
</tr>
<tr>
<td>Web ( \frac{1}{2} \times 57 )</td>
<td>28.50</td>
<td></td>
<td></td>
<td></td>
<td>7,716</td>
<td>7,716</td>
</tr>
<tr>
<td>Bot. Flg. ( \frac{1}{16} \times 10 )</td>
<td>3.12</td>
<td>28.66</td>
<td>-89.4</td>
<td>2,563</td>
<td></td>
<td>2,563</td>
</tr>
</tbody>
</table>

\[ y = \frac{36.62}{36.62} = 1.49 \text{ in.} \]

\[ y = \frac{54.4}{36.62} = 1.49 \text{ in.} \]

\[ I_{NA} = 14,331 \text{ in.}^4 \]

\[ d_{Top} = \frac{57/2 + \frac{1}{2} - 1.49}{2} = 27.51 \text{ in.} \]

\[ d_{Bot} = \frac{57/2 + \frac{1}{16} - 1.49}{2} = 30.30 \text{ in.} \]

\[ S_{Top} = \frac{14,331}{27.51} = 521 \text{ in.}^3 \]

\[ S_{Bot} = \frac{14,331}{30.30} = 473 \text{ in.}^3 \]

Bending moments in the end diaphragm are calculated next, beginning with those due to torque and continuing with those due to vertical loads. As tabulated previously, the shear flow due to torque under \( DL_1 \) is 0.183 kips per in. The shear along the top flange then is

\[ V = 0.183 \times 118 = 21.6 \text{ kips} \]
The vertical component of the shear along one web is

\[ V = 0.183 \times 58.69 \times \frac{57}{58.69} = 10.4 \text{ kips} \]

and the horizontal component is

\[ V = 0.183 \times 58.69 \times \frac{14}{58.69} = 2.6 \text{ kips} \]

The reactions at each shoe are computed by equating to zero the sum of the moments of the shears about each shoe. From moments about \( R_B \) (see following drawing):

\[ R_A = \frac{21.6 \times 57 - 10.4 \times 104 - 2 \times 2.6 \times 28.50}{66} = 32.8 \text{ kips} \]

**SHEAR FLOW IN DIAPHRAGM AT END BEARING**

The \( DL_1 \) bending moment taken about a point on the neutral axis above the reactions, due to the shear on the projecting portions of the diaphragm is

\[ M_A = -M_B = 0.183 \times 26 \times 27.01 + 10.4 \times 19 + 2.6 \times 1.49 + 0.183 \times 29.99 = 396 \text{ kip-in.} \]

Bending moments due to torque under \( DL_2 \) and \( L+I \) are computed on the assumption, for simplicity, that shear flows due to \( DL_2 \) and \( L+I \) act on the same perimeter as does the shear flow due to \( DL_1 \). The bending moments due to torque thus are proportional to the shear flows.

For \( DL_2 \): \( M_A = -M_B = 396 \times \frac{0.048}{0.183} = 104 \text{ kip-in.} \)

For \( L+I \): \( M_A = -M_B = 396 \times \frac{0.321}{0.183} = 695 \text{ kip-in.} \)

**Bending in Diaphragm Due to Vertical Loads (Unfactored)**

For \( DL_1 \): \( M_A = M_B = 55.5 \times (19 + \frac{14}{57} \times 1.49) = 55.5 \times 19.37 = 1,075 \text{ kip-in.} \)

For \( DL_2 \): \( M_A = M_B = 14.9 \times 19.37 = 288 \text{ kip-in.} \)

For \( L+I \): \( M_A = M_B = 117.0 \times 19.37 = 2,266 \text{ kip-in.} \)
Factored Bending Moments

For $DL_1: M = 1.3(1,075+396) = 1,912$
For $DL_2: M = 1.3(288+104) = 510$
For $L+I: M = 1.3 \times \frac{5}{3}(2,266+695) = 6,416$
8,838 kip-in.

Factored Shears

For $DL_1: V = 1.3 \left( 55.5 + 0.183 \times 58.69 \times \frac{57}{58.69} \right) = 86$
For $DL_2: V = 1.3 \left( 14.9 + 0.048 \times 58.69 \times \frac{57}{58.69} \right) = 23$
For $L+I: V = 1.3 \left( 117.0 + 0.321 \times 58.69 \times \frac{57}{58.69} \right) = 293$
402 kips

The maximum allowable shear $V_s$, kips, without stiffeners on the diaphragm, is

$$V_s = 101,500 \times \frac{t_w}{D} = 101,500 \times \frac{(1/4)^3}{57} = 222.6 < 402 \text{ kips}$$

Because the design shear exceeds the allowable shear, stiffeners are required on the diaphragm. For the purpose, the effect of the bearing stiffeners at the shoes may be taken into account. Hence, the shear capacity of the diaphragm is calculated, with the distance from mid-depth of the girder web to the exterior bearing stiffener taken as the stiffener spacing $d_w = 15$ in.

$$V_s = V_p \left[ C + \frac{0.87(1-C)}{\sqrt{1+(D/d_w)^3}} \right]$$

where $V_p = 0.58F_yD t_w = 0.58 \times 36 \times 57 \times \frac{1}{2} = 595 \text{ kips}$

$$C = 569.2 \frac{t_w}{D} \sqrt{1+(D/d_w)^3} - 0.3 \leq 1.0$$

$$= 569.2 \times \frac{1/2}{57} \sqrt{1+(57/15)^3} - 0.3 = 2.97 > 1.0$$

Use $C = 1$ for calculating the ultimate shear strength:

$$V_u = 595(1.0 + 0) = 595 > 402 \text{ kips}$$

Hence, the section is adequate for shear capacity. A reduction in calculated bending strength is required, however, because the design shear exceeds $0.6V_s$.

$$0.6V_s = 0.6 \times 595 = 357 < 402 \text{ kips}$$

The thickness of the diaphragm web meets the requirement $D/t_w \leq 150$.

$$\frac{D}{t_w} = \frac{57}{1/2} = 114 < 150$$

Since the requirement for web thickness is satisfied and the compression flange of the diaphragm can be considered supported over its full length, the diaphragm meets requirements for a braced, noncompact section. Its bending strength independent of shear, therefore, is

$$M_u = F_yS_{Bot} = 36 \times 473 = 17,030 > 8,838 \text{ kip-in.}$$
The moment reduction required is computed from

\[
\frac{M}{M_a} = 1.375 - 0.625 \frac{V}{V_a}
\]

Hence, the allowable moment is

\[
M = 17,030 \left( 1.375 - 0.625 \times \frac{402}{595} \right) = 16,220 > 8,838 \text{ kip-in.}
\]

Therefore, the \( \frac{3}{16} \)-in. diaphragm meets bending requirements.

**Expansion Dam at End Support**

The support brackets and beam developed for the straight girder in the example of Chapter 7 are adequate for the expansion dams at the end bearings.

**DIAPHRAGM AT INTERIOR SUPPORT**

The diaphragm over the pier is similar to the end diaphragm and is shown in the following drawing. Computations for the bearing stiffeners and treatment of the manhole for the pier diaphragm are not given inasmuch as they are similar to those for the diaphragm at the end bearing.

The following computations indicate that a thicker bottom flange than that used for the adjoining portion of the box girder is desirable. This change is necessary because of the combination of transverse and longitudinal bending in the diaphragm.

![Diaphragm Diagram]

**SECTION AT DIAPHRAGM AT PIER**

The following table lists design loads at the interior support with an impact factor of 1. Torque reactions are calculated by dividing the torque by 5.5 ft, the distance between shoes.

**Design Loads at Interior Support**

<table>
<thead>
<tr>
<th></th>
<th>Shear per Web, Kips</th>
<th>Torque on Section, Kip-Ft</th>
<th>Torque Reaction, Kips</th>
</tr>
</thead>
<tbody>
<tr>
<td>DL₁</td>
<td>213.2</td>
<td>265.2</td>
<td>48.2</td>
</tr>
<tr>
<td>DL₂</td>
<td>54.2</td>
<td>44.8</td>
<td>8.1</td>
</tr>
<tr>
<td>L+I</td>
<td>230.0</td>
<td>526.0</td>
<td>95.6</td>
</tr>
<tr>
<td>Total</td>
<td>497.4</td>
<td>836.0</td>
<td>151.9</td>
</tr>
</tbody>
</table>
As for the end diaphragm, the enclosed area to be used in computing the shear due to $DL_1$ torque is 5,928 in.$^2$ and the enclosed area for $DL_2$ and $L+I$ is 6,630 in.$^2$

Shear Flow Due to Torque

For $DL_1$: $S = \frac{265.2 \times 12}{2 \times 5,928} = 0.268$

For $DL_2$: $S = \frac{44.8 \times 12}{2 \times 6,630} = 0.041$

For $L+I$: $S = \frac{526 \times 12}{2 \times 6,630} = 0.476$

0.785 kips per in.

The shoe reactions equal the sum of the torque reaction and the reaction due to vertical loads. One shoe reaction then is

$$R_L = 497.4 + 151.9 = 649 \text{ kips}$$

and the other reaction is (see following drawing)

$$R_R = 497.4 - 151.9 = 346 \text{ kips}$$

![Diagram of shoe reactions at pier]

**SHOE REACTIONS AT PIER**

Check of Pier Diaphragm

Next, the pier diaphragm is checked for bending in its plane, beginning with computation of section properties. The diaphragm section has flanges of A572, Grade 50, steel, and a web of A36 steel. Allowable stresses must be reduced because this section is hybrid.

### Section Properties of Diaphragm at Pier

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad$</th>
<th>$Ad^2$</th>
<th>$I_z$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Flg. 1×18</td>
<td>18.00</td>
<td>29.00</td>
<td>522</td>
<td>15,138</td>
<td>15,138</td>
<td>15,138</td>
</tr>
<tr>
<td>Web 1×57</td>
<td>57.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bot. Flg. 1¼×26</td>
<td>29.25</td>
<td>-29.06</td>
<td>-850</td>
<td>24,703</td>
<td>15,433</td>
<td>15,433</td>
</tr>
</tbody>
</table>

$$y = \frac{-328}{104.25} = -3.146 \text{ in.}$$

$$-328 \text{ in.}^3, \quad 55,274 \text{ in.}^4$$

$$I_{NA} = \frac{54,242}{64.796} = 848 \text{ in.}^4$$

$$d_{\text{Top of steel}} = 29.5 + 3.146 = 32.65 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 29.625 - 3.146 = 26.479 \text{ in.}$$

$$S_{\text{Top of steel}} = \frac{54,242}{32.65} = 1,662 \text{ in.}^3$$

$$S_{\text{Bot. of steel}} = \frac{54,242}{26.479} = 2,048 \text{ in.}^3$$

Then, bending moments in the pier diaphragm are calculated, beginning with those due to torque and continuing with those due to vertical loads. As tabulated previously, the shear flow due to torque under $DL_1$ is 0.268 kips per in. The shear along the top flange then is

$$V = 0.268 \times 118 = 31.6 \text{ kips}$$
The vertical component of the shear along one web is

\[ V = 0.268 \times 58.69 \times \frac{57}{58.69} = 15.3 \text{ kips} \]

and the horizontal component is

\[ V = 0.268 \times 58.69 \times \frac{14}{58.69} = 3.75 \text{ kips} \]

The reactions at each shoe are computed by equating to zero the sum of the moments of the shear about each shoe. From moments about \( R_A \) (see following drawing):

\[ R_B = \frac{31.6 \times 57 + 15.3 \times 104 - 2 \times 3.75 \times 28.5}{66} = 48.2 \text{ kips} \]

**SHEAR FLOW IN DIAPHRAGM AT PIER**

The \( DL_1 \) bending moment in the plane of the diaphragm, taken about a point on the neutral axis above the reactions, due to the shears on the projecting portions of the diaphragm is \( M_B = -M_A = 0.268 \times 26 \times 31.646 + 15.3 \times 19 - 3.75 \times 3.146 + 0.268 \times 12 \times 25.354 = 581 \text{ kip-in.} \) As for the end diaphragm, moments due to torque for \( DL_2 \) and \( L+I \) are taken proportional to shear flows.

For \( DL_2 \): \( M_B = M_A = \frac{581 \times 0.041}{0.268} = 89 \text{ kip-in.} \)

For \( L+I \): \( M_B = M_A = \frac{581 \times 0.476}{0.268} = 1,032 \text{ kip-in.} \)

**Bending in Pier Diaphragm Due to Vertical Loads (Unfactored)**

For \( DL_1 \): \( M_B = M_A = 213.2 \times 19 = 4,051 \text{ kip-in.} \)

For \( DL_2 \): \( M_B = M_A = 54.2 \times 19 = 1,030 \text{ kip-in.} \)

For \( L+I \): \( M_B = M_A = 230.0 \times 19 = 4,370 \text{ kip-in.} \)

**Factored Bending Moments**

For \( DL_1 \): \( M = 1.3 \times (4,501 + 581) = 6,606 \)

For \( DL_2 \): \( M = 1.3 \times (1,030 + 89) = 1,455 \)

For \( L+I \): \( M = 1.3 \times \frac{5}{3} (4,370 + 1,032) = 11,704 \)

\[ \frac{19,765}{19,765} \text{ kip-in.} \]
Factored Shears

For $DL_v$: $V = 1.3 \left( 213.2 + 0.268 \times 58.69 \times \frac{57}{58.69} \right) = 297.0$

For $DL_e$: $V = 1.3 \left( 54.2 + 0.041 \times 58.69 \times \frac{57}{58.59} \right) = 73.5$

For $L+I$: $V = 1.3 \times \frac{2}{3} \left( 230.0 + 0.476 \times 58.69 \times \frac{57}{58.69} \right) = 557.1 \text{ kips}$

The maximum shear capacity $V_\text{u}$, kips, without stiffeners on the diaphragm web, is the smaller of the following:

$$V_\text{u} = 101,500 \frac{\frac{3}{D}}{101,500} = \frac{(1)^3}{57} = 1,781 > 927.6 \text{ kips}$$

$$V_\text{u} = 0.58F_D = 0.58 \times 36 \times 57 \times 1 = 1,190 > 927.6 \text{ kips. Governs.}$$

Inasmuch as the shear capacity exceeds the design shear, stiffeners are not required. Bearing stiffeners, however, are placed over the shoes, as in the case of the end diaphragm.

A reduction in the computed bending strength is required because the design shear exceeds $0.6V_\text{u}$.

$$0.6V_\text{u} = 0.6 \times 1,190 = 714 < 927.6 \text{ kips}$$

The moment reduction required is computed from

$$\frac{M}{M_\text{u}} = 1.375 - 0.625 \frac{V}{V_\text{u}}$$

where $M_\text{u} = \text{computed bending strength, kip-independent of shear. Hence the allowable moment is}$

$$M = \left( 1.375 - 0.625 \times \frac{927.6}{1,190} \right) M_\text{u} = 0.888M_\text{u}$$

and the reduced allowable bending stress is given by

$$F_\text{r} = 0.888F_\text{r}R$$

where $R$ is the reduction factor for a hybrid section. For computation of $R$, the following parameters are computed:

$$\rho = \frac{F_\text{w}}{F_\text{r}} = \frac{36}{50} = 0.72$$

$$\beta = \frac{A_\text{w}}{A_\text{r}} = \frac{57}{57} = 3.167$$

$$\psi = \frac{d\text{top}}{D} = \frac{32.65}{57} = 0.573$$

Substitution of these parameters yields

$$R = 1 - \frac{\beta \psi (1 - \rho) (3 - \psi + \rho)}{6 + \beta \psi (3 - \psi)}$$

$$= 1 - \frac{3.167 \times 0.573 (1 - 0.72) (3 - 0.573 + 0.72 \times 0.573)}{6 + 3.167 \times 0.573 (3 - 0.573)} = 0.961$$

For tension and compression, the allowable stress for bending in the plane of the diaphragm then is

$$F_\text{r} = 0.888 \times 50 \times 0.961 = 42.7 \text{ ksi}$$
Bending Stresses in Plane of Diaphragm

For the factored bending moment of 19,765 kip-in., the stress in the tension flange is

\[ f_t = \frac{19,765}{1,662} = 11.9 < 42.7 \quad \text{O.K.} \]

and in the compression flange,

\[ f_c = \frac{19,765}{2,048} = 9.7 < 42.7 \text{ ksi O.K.} \]

Bending Normal to Plane of Diaphragm

The preceding stresses must be combined with those, at the interior support, from longitudinal bending of the box girder. Section properties of the girder are computed for a 1-in.-thick, A572, Grade 50, bottom flange, which is thicker than the \( \frac{5}{8} \)-in. plate used for the adjoining section of box girder. The thicker flange extends for a distance of 2 ft on both sides of the interior support.

Box-Girder Steel Section at Pier

<table>
<thead>
<tr>
<th>Material</th>
<th>( A )</th>
<th>( d )</th>
<th>( Ad )</th>
<th>( Ad^2 )</th>
<th>( I_o )</th>
<th>( I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 T. Flg. Pl. 2x15</td>
<td>60.00</td>
<td>29.50</td>
<td>1,770</td>
<td>52,215</td>
<td>52,215</td>
<td></td>
</tr>
<tr>
<td>2 Web Pl. ( \frac{3}{4} \times 58.69 )</td>
<td>58.69</td>
<td></td>
<td></td>
<td></td>
<td>15,891</td>
<td>15,891</td>
</tr>
<tr>
<td>Bot. Flg. Pl. 1( \frac{1}{4} \times 92 )</td>
<td>103.50</td>
<td>-29.06</td>
<td>-3,008</td>
<td>87,404</td>
<td>41</td>
<td>87,415</td>
</tr>
<tr>
<td>Stiff. ST 7.5x25</td>
<td>7.35</td>
<td>-23.25</td>
<td>-171</td>
<td>3,973</td>
<td></td>
<td>4,014</td>
</tr>
</tbody>
</table>

\[ d_1 = \frac{-1.409}{29.54} = -6.14 \text{ in.} \]

\[ d_{top \text{ of } steel} = 30.50 + 6.14 = 36.64 \text{ in.} \]

\[ d_{bot \text{ of } steel} = 29.62 - 6.14 = 23.48 \text{ in.} \]

\[ d_{top \text{ of } steel} = \frac{150,884}{36.64} = 4,118 \text{ in}^2 \]

\[ d_{bot \text{ of } steel} = \frac{150,884}{23.48} = 6,426 \text{ in}^2 \]

Steel Section, with Reinforcing Steel, at Pier

<table>
<thead>
<tr>
<th>Material</th>
<th>( A )</th>
<th>( d )</th>
<th>( Ad )</th>
<th>( Ad^2 )</th>
<th>( I_o )</th>
<th>( I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Section</td>
<td>229.54</td>
<td></td>
<td>-1,409</td>
<td></td>
<td>159,535</td>
<td></td>
</tr>
<tr>
<td>Reinforcement</td>
<td>15.19</td>
<td>35.13</td>
<td>534</td>
<td>18,746</td>
<td></td>
<td>18,746</td>
</tr>
</tbody>
</table>

\[ d_c = \frac{-875}{244.73} = 3.58 \text{ in.} \]

\[ d_{top \text{ of } steel} = 30.50 + 3.58 = 34.08 \text{ in.} \]

\[ d_{bot \text{ of } steel} = 29.62 - 3.58 = 26.04 \text{ in.} \]

\[ S_{top \text{ of } steel} = \frac{175,153}{34.08} = 5,139 \text{ in}^2 \]

\[ S_{bot \text{ of } steel} = \frac{175,153}{26.04} = 6,724 \text{ in}^2 \]

\[ d_{Reinf} = 35.13 + 3.58 = 38.71 \text{ in.} \]

\[ S_{Reinf} = \frac{175,153}{38.71} = 4,525 \text{ in}^2 \]

Bending stresses are computed for full design load, with an impact factor of 0.35, with moments obtained from the curves of maximum moment.
## Factored Moments at Pier

<table>
<thead>
<tr>
<th></th>
<th>$DL_1$</th>
<th>$DL_2$</th>
<th>$-(L+I)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$, kip-ft</td>
<td>-6,296</td>
<td>-1,502</td>
<td>-3,838</td>
</tr>
</tbody>
</table>

### Stresses Due to Bending Normal to Plane of Diaphragm

**Top of Steel (Tension)**

For $DL_1$: $F_b = \frac{6,296 \times 12}{4,118} \times 1.30 = 23.8$

For $DL_2$: $F_b = \frac{1,502 \times 12}{5,139} \times 1.30 = 4.6$

For $L+I$: $F_b = \frac{3,838 \times 12}{5,139} \times 1.30 \times \frac{5}{3} = 19.4$

**Bottom of Steel (Compression)**

$F_b = \frac{6,296 \times 12}{6,426} \times 1.30 = 15.3$

$F_b = \frac{1,502 \times 12}{6,724} \times 1.30 = 3.5$

$F_b = \frac{3,838 \times 12}{6,724} \times 1.30 \times \frac{5}{3} = 14.8$

47.8 ksi

### Reinforcing Steel Stress (Tension) at Pier

$$f_t = \frac{1.3 \times 12 \left(1.502 + \frac{5}{3} \times 3.838\right)}{4,525} = 27.3 < 40 \text{ ksi O.K.}$$

The tension stress in the top flange of the box girder at the pier may not exceed

$$F_{st} = F_t \left[1 - 3 \left(\frac{E}{F_y}\right) \left(\frac{1}{b}\right)^3\right]$$

$$= 50 \left[1 - 3 \times \frac{50}{29,000 \pi^2} \left(\frac{12.31 \times 12}{15}\right)^2\right] = 47.5 \text{ ksi}$$

The design stress of 47.8 ksi in the top flange is close enough to the allowable stress that the flange is considered adequate. Stresses in the top flange for bending in the plane of the diaphragm and bending normal to that plane, in the longitudinal direction of the box girder, need not be combined, because these stresses occur in different plates.

For computation of the allowable bending stress, normal to the plane of the diaphragm, in the bottom flange, the St. Venant shear stress at the pier section must first be calculated. From the shear flows tabulated previously, the shear stress in the 1/8-in. flange due to torque is

$$f_s = \frac{1.3}{1.125} \left(0.268 + 0.041 + \frac{5}{3} \times 0.476\right) = 1.27 \text{ ksi}$$

The shear stress is so small that, for calculation of the allowable compression stress in the flange, the parameter $\Delta$ may be taken as unity. The other parameters are computed as follows: With $I$, for the ST7.5 × 25 stiffener equal to 243.2 in.4,

$$K = \sqrt{\frac{l_s}{0.125 t^6 b}} = \sqrt{\frac{243.2}{0.125 (1.125)^4 45}} = 3.12$$

$$K_s = \frac{5.34 + 2.84 (I_s/b)^{1/2}}{(n+1)^2} = 2.443$$

With the use of the preceding results,

$$R_1 = \frac{97.08 \sqrt{K}}{\sqrt{\frac{1}{2} \left[\Delta + \sqrt{\Delta^2 + 4f_s/F_y^3 (K/K_s)^2}\right]}} = 171.4$$

II/7A.66 8/81
\[ R_2 = \frac{210.3 \sqrt{K}}{\sqrt{\frac{1}{1.2} \left[ \Delta - 0.4 + \sqrt{(\Delta - 0.4)^2 + 4(f_0/F_0)^2(K/K_s)^2} \right]}} = 370.9 \]

Because \( w \sqrt{E_I} / t = 45 \sqrt{50}/1.125 = 282.8 \) falls between \( R_1 \) and \( R_2 \), the allowable compression stress in the bottom flange, in the direction normal to the plane of the diaphragm, is

\[ F_b = F_0 \left[ \Delta - 0.4 \left( 1 - \frac{\pi}{2} \frac{R_2 - w \sqrt{E_I} t}{R_2 - R_1} \right) \right] \]

\[ = 50 \left[ 1 - 0.4 \left( 1 - \sin \frac{\pi}{2} \frac{370.9 - 282.8}{2370.9 - 171.4} \right) \right] = 42.8 > 33.6 \text{ ksi} \]

Hence, the bottom flange is satisfactory for bending in the direction normal to the plane of the web as well as for bending in the plane of the web. The flange, however, must also be investigated for the combined bending stresses.

**Combined Bending Stresses**

An interaction equation \( f_{bx}/F_{bx} + f_{by}/F_{by} \leq 1 \) is used to determine the adequacy of the bottom flange for the combined bending stresses parallel and normal to the plane of the diaphragm.

\[ \frac{33.6}{42.8} + \frac{9.7}{42.7} = 0.785 + 0.227 = 1.01 \]

This is close enough to unity that the 1\( \frac{1}{8} \)-in. flange plate is considered satisfactory.

**BOLTED FIELD SPLICE**

For Load-Factor design of a bolted field splice, AASHTO specifications require that the splice material be proportioned for the Maximum Design Load and resistance to fatigue under Service Loads. Because friction connections must resist slip Overload, fastener size must be selected for an allowable stress of 1.33\( F_v \) under the overload of \( D = \frac{5}{3}(L+I) \), where \( F_v \) is the allowable shear stress as given in AASHTO Table 1.7.4.1C1.

Anticipating that curvature will require a heavier splice than that used for the straight bridge of Chapter 7 and attempting to maintain reasonably compact bolt patterns and plate sizes, we select 7/8-in.-dia, A490 fasteners. The allowable load in double shear is

\[ P = 2 \times 0.6013 \times 1.33 \times 20 = 32.0 \text{ kips per bolt} \]

For design of the splice material for the Maximum Design Load, the design moment is chosen as the greater of:

- Average of the calculated moment on the section and maximum capacity of the section.
- 75% of the maximum capacity of the section.

The calculated moment is that induced by the Maximum Design Load 1.3\( [D + \frac{5}{3}(L+I)] \). Splice material should have a capacity equal at least to the design moment. The section capacity is based on the gross section minus any loss in flange area due to bolt holes with area exceeding 15% of each flange area.
### Bending Moments 38 Ft from Interior Support, Kip-Ft

<table>
<thead>
<tr>
<th></th>
<th>For Service Loads</th>
<th>Factor</th>
<th>For Overload</th>
<th>Factor</th>
<th>Maximum Design Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DL_1$</td>
<td>10</td>
<td>1</td>
<td>10</td>
<td>1.30</td>
<td>13</td>
</tr>
<tr>
<td>$DL_2$</td>
<td>100</td>
<td>1</td>
<td>100</td>
<td>1.30</td>
<td>130</td>
</tr>
<tr>
<td>$+LL$</td>
<td>1,595</td>
<td>$\frac{5}{3}$</td>
<td>2,658</td>
<td>1.30</td>
<td>3,455</td>
</tr>
<tr>
<td>$-LL$</td>
<td>$-1,000$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Shears 38 Ft from Interior Support

<table>
<thead>
<tr>
<th></th>
<th>For Service Loads</th>
<th>Factor</th>
<th>For Overload</th>
<th>Factor</th>
<th>Maximum Design Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DL_1$</td>
<td>$-54.9$</td>
<td>1</td>
<td>$-54.9$</td>
<td>1.30</td>
<td>$-71.4$</td>
</tr>
<tr>
<td>$DL_2$</td>
<td>$-14.0$</td>
<td>1</td>
<td>$-14.0$</td>
<td>1.30</td>
<td>$-18.2$</td>
</tr>
<tr>
<td>$LL$</td>
<td>$-38.6$</td>
<td>$\frac{5}{3}$</td>
<td>$-64.3$</td>
<td>1.30</td>
<td>$-83.6$</td>
</tr>
</tbody>
</table>

### Torques 38 Ft from Interior Support, Kip-Ft

<table>
<thead>
<tr>
<th></th>
<th>For Service Loads</th>
<th>Factor</th>
<th>For Overload</th>
<th>Factor</th>
<th>Maximum Design Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DL_1$</td>
<td>$-134.9$</td>
<td>1</td>
<td>$-134.9$</td>
<td>1.30</td>
<td>$-175.4$</td>
</tr>
<tr>
<td>$DL_2$</td>
<td>$-37.0$</td>
<td>1</td>
<td>$-37.0$</td>
<td>1.30</td>
<td>$-48.1$</td>
</tr>
<tr>
<td>$LL$</td>
<td>$-113.4$</td>
<td>$\frac{5}{3}$</td>
<td>$-189.0$</td>
<td>1.30</td>
<td>$-245.7$</td>
</tr>
</tbody>
</table>

The section at the splice is subject to the following moments:

Negative moment that acts only on the steel section.
Positive moment that acts on the composite steel-concrete section.
Negative moment resisted by the steel section and the concrete reinforcement.

Because the effects of positive moment dominate at the splice, splice material is designed for positive moment. Also, to simplify the design procedure, the composite concrete slab is neglected.

Net section properties at the splice are those for the smaller section, on the positive-moment side of the splice.
BOLTED FIELD SPLICE

Net Section at Top-Flange Splice

The splice of each top flange is made with 7/8-in.-dia., A490 bolts, arranged staggered in four rows. Pitch of the bolts longitudinally is \( s = 3 \) in. Gage \( g = 2\frac{7}{8} \) in.

\[
\frac{s^2}{4g} = \frac{(3)^2}{4 \times 2.375} = 0.947
\]

BOLT HOLES IN TOP FLANGE

The deduction from the flange width at the section across the flange through two holes equals \( 2 \times 1 = 2.00 \) in.

The deduction from the flange width at a section through a chain of four holes equals \( 4 \times 1 - 2 \times 0.947 = 2.106 > 2.00 \) in. Use 2.106 in. for the deduction in computing the net flange area.
Flange Area and Deductions
Gross Area = \(\frac{1}{16} \times 12 = 8.25\) in.²
Area deducted for bolt holes = \(\frac{1}{16} \times 2.106 = 1.45\)
\(-15\%\) of gross area = \(-0.15 \times 8.25 = -1.24\)
Net deduction for two flanges = \(0.21 \times 2 = 0.42\) in.²

Net Section at Bottom Flange and Stiffener Splices
Assume that the center of gravity of the stiffener coincides with the center of gravity of the bolt holes. Deduct the following areas: 16 holes in the bottom-flange plate, two holes from the flange of the stiffener and two holes from the stiffener stem.

Flange Area and Deductions
Gross area of bottom flange and stiffener = \(\frac{1}{2} \times 92 + 7.35 = 53.35\) in.²
Area deducted for bolt holes = \(\frac{1}{2} \times 16 + 2 \times 0.622 + 2 \times 0.55 = 10.34\)
\(-15\%\) of gross area = \(-0.15 \times 53.35 = -8.00\) in.²
Net deduction for bottom flange and stiffener = \(2.34\) in.²

Properties of the gross cross section of the box girder are obtained from previous calculations for the maximum-positive-moment section. The bolt holes in the flanges are deducted in the computation of properties of the net section, and the ST 7.5×25 properties are added.

Net Section at the Splice—Steel Section Only

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>d</th>
<th>Ad</th>
<th>AD²</th>
<th>I₀</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pos. Mom. Gross Section</td>
<td>121.19</td>
<td>-938</td>
<td>67,641</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top Flg. Bolt Holes</td>
<td>-0.42</td>
<td>28.84</td>
<td>-349</td>
<td>-349</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bot. Flg. Bolt Holes</td>
<td>-2.34</td>
<td>28.75</td>
<td>-1,934</td>
<td>-1,934</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ST 7.5×25</td>
<td>7.35</td>
<td>-23.25</td>
<td>3,973</td>
<td>41</td>
<td>4,014</td>
<td></td>
</tr>
</tbody>
</table>

\(125.78\) in.² \hspace{1cm} \(-963\) in.² \hspace{1cm} \(69,372\) in.²
\(d_s = \frac{-963}{125.78} = -7.66\) in.
\(I_{NA} = 61,995\) in.⁴
\(d_{Top \ of \ steel} = 29.19 + 7.66 = 36.85\) in.
\(d_{Bot. \ of \ steel} = 29.00 - 7.66 = 21.34\) in.
\(S_{Top \ of \ steel} = \frac{61,995}{36.85} = 1,682\) in.²
\(S_{Bot. \ of \ steel} = \frac{61,995}{21.34} = 2,905\) in.²

Design Moments and Shears at the Field Splice
The capacity of the net section is based on the minimum section modulus of the steel section and the allowable stress for the corresponding flange. From above, the lower section modulus is 1,682 in.²—for the top flange. Because the effects of positive bending dominate, the allowable stress will be the allowable compressive stress for the top flange. This has previously been calculated for the maximum positive moment section as 36.0 ksi on page 37 in this text.
For $F_c = 36.0$ ksi, the net section capacity is

\[ M_{net} = \frac{36.0 \times 1,682}{12} = 5,046 \text{ kip-ft} \]

75% $M_{net} = 0.75 \times 5,046 = 3,785 \text{ kip-ft}$

With an impact factor of 0.35, the calculated moment is

\[ M_{calc} = 13 + 130 + 1.35 \times 3,455 = 4,807 \text{ kip-ft} \]

The average of the calculated moment and the net capacity of the section is

\[ M_{av} = \frac{4,807 + 5,046}{2} = 4,920 \text{ kip-ft} \]

The design moment, therefore, is 4,920 kip-ft.

The design vertical shear is determined by multiplying the calculated vertical shear for the design loads by the ratio of the design moment to the calculated moment on the section. With an impact factor of 0.50,

\[ V_{calc} = 71.4 + 182 + 1.50 \times 83.6 = 215.0 \text{ kips} \]

Hence, the design vertical shear is

\[ V_c = 215.0 \times \frac{4,920}{4,807} = 220 \text{ kips} \]

In the plane of each web,

\[ V_r = \frac{220}{2} \times \frac{58.69}{57} = 113 \text{ kips} \]

The design torque similarly is obtained by multiplying the calculated torque by the ratio of the design moment to the calculated moment. The design torque is resolved into a torque acting on the noncomposite section and a torque acting on the composite section.

**Design Torque, Kip-Ft**

For $DL_1$: $T = 175.4 \times \frac{4,920}{4,807} = 179.5$

For $DL_2$: $T = 48.1 \times \frac{4,920}{4,807} = 49.2$

For $L + I$: $T = 245.7 \times 1.50 \times \frac{4,920}{4,807} = 377.2 \text{ ft-kips}$

The design torsional shear on each web then is

\[ V_r = \frac{12 \times 57}{2} \left( \frac{179.5}{5,230} \times \frac{49.2}{6,735} \times \frac{377.2}{6,735} \right) = 33 \text{ kips} \]

and in the plane of each web,

\[ V_r = 33 \times \frac{58.69}{57} = 34 \text{ kips} \]

The Maximum Design Vertical and Torsional Shear then is

\[ V = V_c + V_r = 113 + 34 = 147 \text{ kips} \]
Web Splice

The web splice plates must carry the design vertical shear, design torsional shear, moments due to the eccentricities of the shears and a portion \( M_w \) of the design moment on the section. The portion of the design moment to be resisted by the web is obtained by multiplying the design moment by the ratio of the moment of inertia of the web to the net moment of inertia of the entire section. The gross moment of inertia is obtained from the earlier calculation of section properties and adjusted for the change in position of the centroidal axis because of deductions for bolt holes in the flanges.

\[
I_w = 15,891 \geq 58.69(7.66)^2 = 19,335 \text{ in}^4
\]

Web Moments for Design Loads

The bending moment due to eccentricity of the vertical shear is

\[
M_{vv} = \frac{220 \times 3.25}{12} = 60 \text{ kip-ft.}
\]

The moment due to eccentricity of the torsional shear is

\[
M_{vt} = \frac{33 \times 2 \times 3.25}{12} = 18 \text{ kip-ft}
\]

The portion of the design moment resisted by the webs is

\[
M_{uw} = 4,920 \times \frac{19,335}{61,996} = 1,534 \text{ kip-ft}
\]

The total web moment then is

\[
M_w = 60 + 18 + 1,534 = 1,612 \text{ kip-ft, or 806 kip-ft per web}
\]

Try two \( \frac{3}{4} \times 55\)-in. web splice plates. Assume two columns of \( \frac{3}{4} \)-in.-dia, A490 bolts, with 14 bolts per column, on each side of the joint. The area of one hole is 0.375 in.\(^2\) The holes remove from each splice plate the following percentage of its cross-sectional area:

\[
\% \text{ of plate} = \frac{14 \times 0.375}{0.375 \times 55} \times 100 = 25.5\%
\]

Consequently, the fraction of the hole area that must be deducted in determination of the net section is

\[
\frac{25.5 - 15.0}{25.5} = 0.41
\]

With 4-in. spacing of bolts along the slope of the web,

\[
d^2 \text{ for holes} = 2^2 + 6^2 + 10^2 + 14^2 + 18^2 + 22^2 + 26^2 = 1,820
\]

\[
\Sigma Ad^2 = 4 \times 0.41 \times \frac{3}{4} \times 1,820 = 1,119 \text{ in}^4
\]

or, with respect to a horizontal axis

\[
\Sigma Ad^2 = 1,119 \left(\frac{57}{58.69}\right)^2 = 1,055 \text{ in}^4
\]

Assume that the neutral axis of the splice coincides with the neutral axis of the net section of the box girders. The bending properties of the web splice plates with respect to a horizontal axis are then computed as follows:

The area of two bolts holes to be deducted equals \( 2 \times 4 \times 0.375 \times 0.41 = 4.31 \text{ in}^2 \)
Web-Splice Section

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$d$</th>
<th>$Ad^2$</th>
<th>$I_o$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Splice Pl. $\frac{7}{8} \times 55$</td>
<td>41.25</td>
<td>7.66</td>
<td>2,420</td>
<td>9,807</td>
<td>12,227</td>
</tr>
<tr>
<td>Area of Holes</td>
<td>-4.31</td>
<td>7.66</td>
<td>-253</td>
<td>-1,055</td>
<td>-1,303</td>
</tr>
</tbody>
</table>

\[ d_{\text{Top of splice}} = 27.50 + 7.66 = 35.16 \text{ in.} \]
\[ d_{\text{Bot of splice}} = 27.50 - 7.66 = 19.84 \text{ in.} \]
\[ S_{\text{Top of splice}} = \frac{10,919}{3.516} = 311 \text{ in}^3 \]
\[ S_{\text{Bot of splice}} = \frac{10,919}{19.84} = 550 \text{ in}^3 \]

The maximum bending stress in the plates for the Maximum Design Load therefore is
\[ f_b = \frac{806 \times 12}{311} = 31.1 < 36 \text{ ksi} \]

The plates are satisfactory for bending. The allowable shear stress is
\[ F_y = 0.58F_y = 0.58 \times 36 = 20.9 \text{ ksi} \]

The shear stress for the Maximum Design Vertical and Torsional Shear is
\[ f_c = \frac{147}{41.25} = 3.56 < 20.9 \text{ ksi} \]

The $\frac{7}{8} \times 55$-in. web splice plates are satisfactory for Maximum Design Load requirement. The plates are next checked for fatigue under service loads.

The range of moment on the section is
\[ M_r = 1.35 \times 1,595 + 135 \times 1,000 = 2,153 + 1,350 \]

The range of moment carried by the web equals
\[ M_w = (2,153 + 1,350) \frac{19,335}{61,995} = 1,093, \text{ or } \frac{1,093}{2} = 547 \text{ kip-ft per web} \]

The maximum bending-stress range in the gross section of the web splice plate then is
\[ f_w = \frac{547 \times 12 \times 35.16}{12,227} = 18.9 \text{ ksi} \]

**Check for Fatigue**

Fatigue in base metal adjacent to friction-type fasteners is classified by AASHTO as Category B. For 500,000 cycles of truck loading, the associated allowable stress range is 27.5 ksi. The plates therefore are satisfactory for fatigue.

Use two $\frac{7}{8} \times 55$-in. web splice plates.

**Web Bolts**

The 28 bolts in the web splice must carry the vertical and torsional shears, the moment due to the eccentricities of these shears about the centroid of the bolt group, and the portion of the beam moment taken by the web. These forces are induced by the Overload $D + 5/3(L + I)$. The allowable load in double shear was previously computed to be $P = 32.0$ kips per bolt.

The polar moment of inertia of the bolt group about the assumed location of the neutral axis is
\[ I = 2 \times 2 \times 1,820 + 28 \left( 7.66 \times \frac{58.69}{57} \right)^2 + 28(1.5)^2 = 9.085 \text{ in}^4 \]
Web Shears for Overload

From the previous tabulation of shears for the section 38 ft from the interior support, the vertical shear per web for Overload is, with an impact factor of 0.50,

\[
V_v = \frac{1}{2}(54.9 + 14.0 + 1.5 \times 64.3) = 82.7 \text{ kips}
\]

Also, at the section 38 ft from the interior support, the torsional shear for Overload is

\[
V_t = \frac{12 \times 57}{2} \left( \frac{134.9}{5,230} + \frac{37.0}{6,735} + 1.50 \times \frac{189.0}{6,735} \right) = 25.1 \text{ kips}
\]

The Overload Vertical and Torsional Shear then is

\[
V_o = V_v + V_t = 82.7 + 25.1 = 107.8 \text{ kips}
\]

Web Moments for Overload

The bending moment due to eccentricity of the vertical shear is

\[
M_{vw} = \frac{82.7 \times 3.25}{12} = 22 \text{ kip-ft}
\]

The moment due to eccentricity of the torsional shear is

\[
M_{wr} = \frac{25.1 \times 3.25}{12} = 7 \text{ kip-ft}
\]

The direct bending moment at the section 38 ft from the interior support is, with an impact factor of 0.35,

\[
M = 10 + 100 + 1.35 \times 2,658 = 3,698 \text{ kip-ft}
\]

The portion of this moment to be resisted by each web is

\[
M_w = \frac{1}{2} \times 3,698 \times \frac{19,335}{61,995} = 577 \text{ kip-ft}
\]

The total moment due to Overload then is

\[
M_w = 577 + 22 + 7 = 606 \text{ kip-ft per web}
\]

Load per bolt due to shear is

\[
P_s = \frac{107.8}{28} \times \frac{58.69}{57} = 3.96 \text{ kips}
\]

Load on the outermost bolt due to moment is

Vertical in-plane component = \[
\frac{606 \times 12 \times 1.5}{9,085} = \frac{58.69}{57} = 1.24 \text{ kips}
\]

Horizontal in-plane component = \[
\frac{606(58.69/57)12(26 + 7.66 \times 58.69/57)}{9,085} = 30.75 \text{ kips}
\]

Therefore, the total load on the outermost bolt is the resultant

\[
P = \sqrt{(3.96 + 1.24)^2 + (30.75)^2} = 31.2 < 32.0 \text{ kips}
\]

Use fourteen \( \frac{3}{4} \)-in.-dia, A490 bolts in two rows.

Flange-Splice Design

The flange splice plates are proportioned for the Maximum Design Load and checked for fatigue.

The average stress in the top flange under the Maximum Design Load is

\[
\bar{\sigma}_{top} = \frac{4,920 \times 12(28.84 + 7.66)}{61,995} = 34.8 \text{ ksi}
\]
The total flange force is determined by multiplying the average stress by the net flange area.

\[ P_{top} = 34.8 \left( \frac{16.50 - 0.42}{2} \right) = 280 \text{ kips} \]

The required net area of the top-flange splice plates then becomes

\[ A_{top} = \frac{280}{36} = 7.78 \text{ in}^2 \]

This value exceeds 75% of the net area of the top flange:

\[ 0.75 \left( \frac{16.50 - 0.42}{2} \right) = 6.03 < 7.78 \text{ in}^2 \]

Try a \( \frac{3}{8} \)-in. outer splice plate and two \( \frac{3}{8} \times 5\frac{1}{4} \)-in. inner splice plates. The net area of these plates after deduction of bolt holes in excess of 15% of the plate area is

- Top plate: \( \frac{3}{8} \times 12 - 2.106(1 \times \frac{3}{8}) + 0.15(\frac{3}{8} \times 12) = 4.38 \)
- Bot. plate: \( 2[(\frac{3}{8} \times 5\frac{1}{4}) - 1.0531(1 \times \frac{3}{8}) + 0.15(\frac{3}{8} \times 5\frac{1}{4})] = 3.85 \)

Total area: \( 8.23 > 7.78 \text{ in}^2 \)

The average stress in the bottom flange under the Maximum Design Load is

\[ f_{bot} = \frac{4.920 \times 12(28.75 - 7.66)}{61.995} = 20.1 \text{ ksi} \]

The total flange force is

\[ P_{bot} = 20.1[46.00 - 16 \times \frac{3}{8} + 0.15 \times 46.00] = 20.1 \times 44.90 = 902 \text{ kips} \]

The design torsional shear across the bottom flange is

\[ V_t = \frac{12 \times 90}{2} \left( \frac{179.5}{5.230} + \frac{49.2}{6.735} + \frac{373.2}{6.735} \right) = 53 \text{ kips} \]

For the bottom-flange splice, a trial is made with two \( \frac{3}{8} \times 41\frac{1}{4} \)-in. outer plates and two \( \frac{3}{8} \times 41\frac{1}{4} \)-in. inner plates. For one pair of plates (half box), assume two rows of bolts with 8 bolts per row, on each side of the joint. The area of one bolt hole is 0.375 in.\(^2\) The holes remove from each splice plate the following percentage of its cross-sectional area:

\[ \% \text{ of plate} = \frac{8 \times 0.375}{0.375 \times 41.5} \times 100 = 19.3\% \]

Consequently, the fraction of the hole area that must be deducted in determination of the net section is

\[ \frac{19.3 - 15.0}{19.3} = 0.22 \]

With 5\( \frac{1}{2} \)-in. spacing of bolts,

\[ d^2 \text{ for holes} = 2.75^2 + 8.25^2 + 13.75^2 + 19.25^2 = 635 \]

\[ \Sigma Ad^2 = 4 \times 0.22 \times \frac{3}{8} \times 635 = 210 \text{ in.}^4 \]

The net section modulus of the pair of splice plates is

\[ S_{net} = \frac{41.5}{2} \left( 2 \times \frac{1}{12} \times \frac{3}{8} \times 41.5^3 - 210 \right) = 205 \text{ in.}^4 \]

The net area of each pair of splice plates is

\[ A_{net} = 2 \times \frac{3}{8} \times 41.5 - 8 \times 2 \times \frac{3}{8} \times 0.22 = 31.13 - 1.32 = 29.81 \text{ in.}^2 \]
The splice plates must resist the direct flange load $P$, the torsional shear $V$, and the moment due to eccentricity of the torsional shear $M_v$. These are computed as follows:

\[
P = \frac{902}{2} = 451 \text{ kips}
\]

\[
V = \frac{53}{2} = 27 \text{ kips}
\]

\[
M_v = \frac{27 \times 3.25}{12} = 7 \text{ kip-ft}
\]

The maximum direct stress in the splice plates then is

\[
f_t = \frac{451}{29.81} + \frac{7 \times 12}{205} = 15.1 + 0.4 = 15.5 < 36 \text{ ksi}
\]

and the maximum shear stress on the gross area of a pair of plates is

\[
f_s = \frac{27}{31.13} = 0.87 < 19.8 \text{ ksi}
\]

**Check of Flange Splices for Fatigue**

The flange splice plates are then checked for fatigue under Service Loads. The range of live-load moment at the splice equals

\[M_{Lv} = 1.35[1,595-(-1,000)] = 3,503 \text{ kip-ft}
\]

And the range of average stress in the flanges, disregarding the relatively small effect of torsion, is

Top Flange: \[f_{nu} = \frac{3,503 \times 12(28.84+7.66)}{61,995} = 24.7 \text{ ksi}\]

Bot. Flange: \[f_{nu} = \frac{3,503 \times 12(28.75-7.66)}{61,995} = 14.3 \text{ ksi}\]

The corresponding range of stress in the gross section of the flange splice plates is

Top Flange: \[f_t = \frac{24.7(\gamma_t)(16.50-0.42)}{12 \times \gamma_{ts} + 2 \times 5.375 \times \gamma_t} = 18.9 < 27.5 \text{ ksi}\]

Bot. Flange: \[f_t = \frac{14.3 \times 44.90}{4 \times 41.5 \times \gamma_t} = 7.65 < 27.5 \text{ ksi}\]

The flange splice plates, therefore, are satisfactory.

**Flange Bolts**

The number of bolts required in the flange splice is determined by the capacity needed for transmitting the flange force under the Overload $D + \frac{5}{3}(L+J)$. The total moment on the section is 3,698 kip-ft (see Web Moments for Overload).

The average stress in the top flange is

\[f_b = \frac{3,698 \times 12(28.84+7.66)}{61,995} = 26.2 \text{ ksi}\]

And the flange force becomes

\[P_{top} = 26.2 \left( \frac{16.50-0.42}{2} \right) = 211 \text{ kips}\]

For this flange force, the number of bolts required is

\[\frac{211}{32.0} = 6.6 \text{ bolts}\]
Use 8 bolts.

The average stress in the bottom flange is

\[ P_{\text{Bot.}} = 15.1 \times 44.90 = 677 \text{ kips} \]

For this flange force, the number of bolts required is

\[ \frac{677}{32.0} = 21.2 \text{ bolts} \]

For detail purposes, 32 bolts are used. With this substantial margin of additional bolts, the torsional effects on the bolt pattern may be neglected.

**Stiffener Splice**

Next, the splice is designed for the ST7.5×25, longitudinal, bottom-flange stiffener. A splice of the stiffener is desirable to assure that the interruption of the stiffener at the field splices does not become a node for buckling. The splice is designed for the axial-load capacity of the ST7×25. This capacity equals the product of the allowable compression stress for the bottom flange and the area of the stiffener.

The allowable compression is a function of the torsional shear stress \( f_\tau \), the coefficient \( K_s \), and the buckling coefficient \( K \) furnished by the combination of the longitudinal stiffener and the bottom flange. As previously calculated for the field splice location, the torsional shear stress \( f_\tau = 1.1 \text{ ksi} \). The other parameters required for calculation of the allowable compression stress in the bottom flange are determined as follows:

\[
K = \sqrt[3]{\frac{I_s}{0.125t^4b}} = \sqrt[3]{\frac{243.2}{0.125(0.5)^4(45)}} = 7.0 > 4 \quad \text{Use 4.}
\]

\[
K_s = \frac{5.34 + 2.84 \left(\frac{243.2}{(45)(0.5)^4}\right)^{\frac{1}{3}}}{(1+1)^4} = 3.83
\]

\[
\Delta = \sqrt{1 - 3 \left(\frac{1.1}{36}\right)^2} = 0.9986
\]

With the use of the preceding results,

\[
R_1 = \frac{97.08}{\sqrt{\frac{1}{2} \left[ 0.9986 + \sqrt{(0.9986)^2 + 4 \left(\frac{1.1}{36}\right)^2 \left(\frac{4}{3.83}\right)^2} \right]}} = 194.2
\]

\[
R_2 = \frac{210.3}{\sqrt{\frac{1}{1.2} \left[ 0.9986 - 0.4 + \sqrt{(0.9986 - 0.4)^2 + 4 \left(\frac{1.1}{36}\right)^2 \left(\frac{4}{3.83}\right)^2} \right]}} = 420.4
\]

Because \( w \sqrt{F_c} / t = 45 \sqrt{36} / 0.5 = 540 > (R_2 = 420.4) \), the allowable compression stress in the bottom flange is

\[
F_0 = 26.210 \times 4 \left(\frac{0.5}{45}\right)^2 = \frac{(1,100)^2}{26.210(3.83)^2} \left(\frac{0.5}{45}\right)^2 = 12.9 \text{ ksi}
\]

With an allowable compression stress of 12.9 ksi for the bottom flange, the force on the stiffener is

\[
P_{st} = 12.9 \times 7.35 = 94.8 \text{ kips}
\]
The stiffener splice also must be designed to resist lateral bending. The lateral-bending moment is taken as that associated with the bottom-flange stress of 12.9 ksi and is computed with the theory presented in General Design Considerations under Lateral Bending Stresses under $DL_{1}$. The stress at the top of the ST 7.5 stiffener, with $\gamma_{b} = d_{\text{Bot. of steel}}$ for the net steel section at the splice, is

$$f_{b} = \frac{\gamma_{b} - \gamma_{t}}{\gamma_{b}} \times 12.9 = 8.4 \text{ ksi}$$

Hence, the lateral bending moment is

$$M_{l} = \frac{f_{b}td^{2}}{10R} = \frac{8.4 \times 5.64 \times 0.622 (12.31 \times 12)^{2}}{10 \times 410.38 \times 12} = 13.1 \text{ kip-in.}$$

**SPLICE OF ST 7.5 X 25**

With $\frac{3}{4}$-in.-dia. A325 bolts and an allowable load in single shear of $0.442 \times 21 = 9.3$ kips per bolt, the number of bolts required for direct load is

$$\frac{94.8}{1.3 \times 9.3} = 7.8 \text{ bolts}$$

Try 10 bolts, four in the stem splice and six in the flange splice.

**STIFFENER-FLANGE SPLICE**

The polar moment of inertia of the flange bolt group is

$$I = 2 \times 3 (1.75)^{2} + 2 \times 2 (2.5)^{2} = 43.38 \text{ in.}^{4}$$

The force on the outermost bolt due to lateral bending and direct load is computed and found to be within the allowable value:

Longitudinal force from direct load = $\frac{94.8}{1.3 \times 10} = 7.29 \text{ kips}$

Longitudinal component from moment = $\frac{12.9 \times 1.75}{1.3 \times 43.38} = 0.40 \text{ kips}$
Lateral component from moment $= \frac{12.9\times2.5}{1.3\times43.38} = 0.57$ kips

Resultant total bolt load $= \sqrt{(7.29 + 0.40)^2 + (0.57)^2} = 7.7 < 9.3$ kips

The area required for the splice plates for direct load is

$$A_p = \frac{94.8}{36} = 2.63 \text{ in.}^2$$

Try a $\frac{3}{4}\times6$-in. splice plate on top of the flange and a $\frac{3}{4}\times5$-in. plate on the stem, each with two longitudinal rows of bolts. The net area of the plates is

Flange: $6\times\frac{3}{4} - 2(\frac{3}{4}\times\frac{3}{4}) + 0.15(6\times\frac{3}{4}) = 1.93$
Stem: $5\times\frac{3}{4} - 2(\frac{3}{4}\times\frac{3}{4}) + 0.15(5\times\frac{3}{4}) = 1.50$

$$3.43 > 2.63 \text{ in.}^2$$

In this net section of the flange splice plate, the bolt holes remove the following percentage of the area:

$$\frac{2(\frac{3}{4})\frac{3}{4}}{6\times\frac{3}{4}} \times 100 = 29.2\%$$

Hence, the fraction of hole to be deducted is

$$\frac{29.2 - 15.0}{29.2} = 0.49$$

The moment of inertia of the splice plate thus is

$$I = \frac{1}{12} \times \frac{3}{8} (6)^3 - 0.49 \times \frac{3}{8} \times \frac{7}{8} (1.75)^2 = 6.26 \text{ in.}^4$$

The total stress in the flange splice then is

$$F_p = \frac{94.8}{3.43} + \frac{12.9 \times 3}{6.26} = 33.8 < 36 \text{ ksi}$$

The splice design therefore is satisfactory.

**COMPARISON WITH STRAIGHT BRIDGE**

The curved box-girder bridge of this example requires about 17% more structural steel than its straight counterpart of Chapter 7. About half of the additional steel is attributable to the top lateral bracing and to the change from a rigid frame support to a continuous support. A quarter of the additional steel is that from intermediate crossframes. The remainder results from use of lower allowable stresses, higher live-load impact factors and other provisions by which the Guide Specifications account for curvature.

Other effects of curvature include:

- A490 bolts instead of A325 bolts for field splice of main girder material
- 15% more shear connectors than those required for the straight bridge

**FINAL DESIGN**

Drawings of the curved box-girder bridge of the example are shown on the following sheets.
INTERIOR SUPPORT DIAPHRAGM

SECTION C-C

LATERAL BRACING
CONNECTION DETAILS

BOLTED FIELD SPICE DETAILS

ALTERNATE FIELD SPICE DETAILS
WELDED DESIGN

DESIGN EXAMPLE
Curved Box Girder
Two-Span Bridge
Framing Plan

Note: Material dimensions and welding same as End Diaphragm, except as shown.

Note: Fasteners for longitudinal stiffener splice are 1/4"d A325 bolts.

Note: Fasteners for bolted splice of main girder material are 1/4"d A490 bolts.
Introduction

This chapter presents two designs for short span bridge superstructures constructed entirely of structural steel. Both are designed for prefabrication in the shop in 8 foot sections. This width was chosen to allow ease of handling, transportation and erection.

All-steel designs require considerable fabrication and in short spans are not expected to have the most favorable first costs unless they are mass produced. However, they do have certain other advantages. Among these are the ease and rapidity with which they can be erected to form a complete superstructure; and their low maintenance costs when constructed of corrosion resistant steels.

Design I is for a pre-fabricated all steel sectional multi-cell girder bridge. Top and bottom flanges are continuous horizontal plate separated by closely spaced inclined webs, intermediate diaphragm at mid-span and vertical transverse plates at the end. Elastomeric pads are used for bearings.

Although illustrated for job fabrication in 8 ft sections, it can be made in width multiples of 2 feet without any major revision in design and details. Inclined webs of bent plate are used to reduce both the number of components and the number of welding operations.

Since the girder is a torsionally rigid closed box, it has advantageous load distributing qualities. Thus, the over-all depth for a 50 ft span is less than 27″, compared with the usually required 37″ for a conventional composite rolled beam bridge of identical span. The design is compact, neat in appearance and provides maximum clearance or minimum grades for the bridge approaches. It is a bridge with the major-
Design I

ASSUMPTIONS

This highway bridge is designed for HS-20 live loading. Vertical deflection, limited to $1\frac{1}{400}$ of the horizontal span, is the controlling design factor.

It is designed as an orthotropic steel plate deck bridge. The continuous top plate serves as the top flange of the box girder, the roadway deck and as transverse flexural member between webs. The method outlined in the AISC Design Manual for Orthotropic Steel Plate Bridge is used in designing the top plate.

Each 8' section is assumed to take two-thirds of a lane of live loading, based on a 12' traffic lane. This same coefficient is used in the stress and deflection calculations.

The final design was investigated for load distribution by Guyon-Massonnet theory and charts. Load distribution for the outside 8' section varies from 0.520 lane to 0.743 lane, with an average of 0.639 lane; this is close to the load distribution in the original design assumption.

While the $\frac{3}{16}''$ inside webs and $\frac{1}{4}''$ bottom plate are less than the thickness required by AASHO Specifications, the inside webs have both sides enclosed and the bottom plate has one side enclosed in the airtight box to protect against corrosion. (See Figures 1 and 2.)

Design Calculations

Assume:
Live Load distribution = $\frac{3}{4}$ Lane/8' width
Weight of structure = 45#/ft.$^2$
Weight of wearing surface = 25#/ft.$^2$

50 FT SIMPLE SPAN

\[
\begin{align*}
\text{N.A.} &= [36.00 \times 0.19 + 36.00 (0.38 + 12) + 24.00 (0.38 + 24 + 0.12)] \times \frac{1}{96.00} \\
&= [7 + 446 + 588] \times \frac{1}{96.00} = 1041 \frac{1}{96.00} = 10.85'' \\
\text{I}_{\text{NA}} &= 36.00 (10.85 - 0.19)^2 + 36.00 (12.38 - 10.85)^2 + 24.00 (24.50 - 10.85)^2 + \frac{1}{12} \times \frac{3}{16} \times 8 \times (24)^3 \\
&= 4090 + 80 + 4480 + 1730 - 10370 \text{ in.$^4$}
\end{align*}
\]

\[
\begin{align*}
\text{Impact} &= \frac{50}{L + 125} = .286 \\
M_{\text{LL}} &= 628 \times \frac{3}{4} \times 1.286 = 538 \\
M_{\text{L}} &= .070 \times 8 \times \frac{(50)^2}{8} = 175 \frac{\text{kl}}{\text{ft}}
\end{align*}
\]

\[
\begin{align*}
V_{\text{LL}} &= 58.5 \times \frac{3}{4} \times 1.286 = 50.2 \\
V_{\text{L}} &= 0.070 \times 8 \times \frac{50}{2} = 14.0 \frac{\text{kl}}{\text{ft}}
\end{align*}
\]

*See Appendix A, this chapter
\[ Z_{\text{top}} = \frac{10370}{955} = 955 \text{ in.}^3, \quad f_l = \frac{713 \times 12}{955} = 8.96 \text{ ksi} \]
\[ Z_{\text{fl.}} = \frac{10370}{753} = 753 \text{ in.}^3, \quad f_u = \frac{713 \times 12}{753} = 11.36 \text{ ksi} \]

\[ \text{Ave. } v = \frac{64.2}{36.00} = 1.78 \text{ ksi} \]

Wt. of 2 End Plates: \[ 24.5 \times \frac{1}{4} \times 2 \times 8' \times 3.4 = 333# / 50 \text{ ft.} \]

Int. Diaphragm: \[ 20 \times \frac{1}{4} \times 8 \times 3.4 = 136# / 50 \text{ ft.} \]

2" - \frac{7}{16}" bt. PL: \[ 2 \times \frac{7}{16} \times 4 \times 3.4 = 5.1# / \text{ft. / 8' width} \]

\frac{7}{16}" outside PL's \[ 24 \times \frac{7}{16} \times 3.4 \times \frac{1}{2} = 12.8# / 8' width \]

Railing PL's 12" x \frac{7}{8}" x 1'-10" x 32 x 3.4 x /50 x 32

\[ \frac{1}{4} \times 3" \text{ PL} \times 0.75 \times 3.4 / 32 \]

Main Mat'l:

Total Material

Allowable deflection = \[ \frac{1}{800} \text{ span} = \frac{50 \times 12}{800} = 0.75" \]

\[ \text{½ lane/8' section} \]

16 x \frac{7}{8} x 2 x 1.286 = 27.4k wheel load

4 \times \frac{1}{4} \times 2 \times 1.286 = 6.9k

27.4 + 27.4 + 6.9 = 61.7k

C.G. Load = \[ \frac{27.4 \times 14 + 6.9 \times 28}{61.7} = \frac{384 + 193}{61.7} = \frac{577}{61.7} = 9.4 \text{ ft.} \]

\[ R_A = \frac{6.9 \times 8.7 + 27.4 \times 22.7 + 27.4 \times 36.7}{50.0} = \frac{60 + 622 + 100.6}{50.0} = \frac{1688}{50.0} = 33.7k \]

\[ R_B = \frac{27.4 \times 13.3 + 27.4 \times 273 + 6.9 \times 41.3}{50.0} = \frac{364 + 748 + 285}{50.0} = \frac{1397}{50.0} = 28.0k \]

L.L. + I Deflection:

\[ M_1 = 33.7 \times 13.3 = + 448kI \]

\[ M_2 = 448 + 6.3 \times 14.0 = 448 + 89 = 537kI \]

\[ M_3 = 28.0 \times 8.7 = + 244kI \]

\[ M_4 = 244 + 21.1 \times 14.0 = 244 + 295 = + 539kI \]
\[ 448 \times 13.3 \times \frac{1}{2} = 2980 \times \frac{13.3 \times 8}{3} = 26,500 \]
\[ 448 \times 14.0 = 6270 \times (13.3 + 7.0) = 127,000 \]
\[ 90 \times 14.0 \times \frac{1}{2} = 630 \times (13.3 + 14.0 \times \frac{1}{2}) = 14,200 \]
\[ 294 \times 14.0 \times \frac{1}{2} = 2060 \times (27.3 + 14.0 \times \frac{1}{4}) = 66,000 \]
\[ \frac{397900}{50} = 7958 \]
\[ 244 \times 14.0 = 3420 \times (27.3 + 7.0) = 117,300 \]
\[ 244 \times 8.7 \times \frac{1}{2} = \frac{1060 \times (41.3 + 8.7 \times \frac{1}{3})}{16420} = \frac{46,900}{397,900} \]

Deflection at pt. 2 = \[8462 \times 27.3 \times 2980 (14.0 + 13.3 \times \frac{1}{2}) - 6270 \times 7.0 - 630 \times (14.0 \times \frac{1}{2})] \]
\[ \times \frac{29,000 \times 10,370}{1728} \]
\[ = \frac{[231,000 - 55,000 - 43,900 - 2,900] \times 1728}{29,000 \times 10,370} \]
\[ = \frac{129,200 \times 1728}{29,000 \times 10,370} = 0.744\text{"} \]

Deflection at 24.6' from left support = \[8467 \times 24.6 \times 2980 (24.6 - \frac{3}{4} \times 13.3) - 448 \times 11.3 \times \frac{11.3}{2} \]
\[ - 90 \times \frac{11.3}{14.0} \times 11.3 \times \frac{1}{2} \times \frac{11.3}{3} \times \frac{1728}{29,000 \times 10,370} \]
\[ = \frac{(208,000 - 46,800 - 28,600 - 1,600) \times 1728}{29,000 \times 10,370} = \frac{131,000 \times 1728}{29,000 \times 10,370} = 0.753\text{"} \]

Investigation of web (24 \times \frac{1}{4}) (Design Manual for High Strength Steels)

\[ \frac{b}{a} = 2/50 = 0.04 \]
\[ k = \frac{5.35 + 4(b/a)}{2} = 5.35 + 0.01 = 5.36 \]
\[ b/t = 24/0.188 = 128 \]
\[ \sqrt{k} = \frac{128}{\sqrt{5.36}} = 2.32 = 55.2 \]

From Table II since \( \frac{b}{t} \sqrt{k} > 41.5 \) (point c), use Formula 14 c
\[ \nu_s = \frac{19,660,000 \times k}{(128)^2} = \frac{19,660,000 \times 5.36}{(128)^2} = 6,440 \text{ psi} \]

For \( N = 1.80 \)

Allowable shearing stress = \[\frac{6440}{1.80} = 3,570 \text{ psi} > 1.78 \text{ ksi} \]

Web Buckling Due to Compression
\[ S_{w} = 1.8 S_{w} = \frac{\sqrt{5.3}}{4770} \left( \frac{h}{t} \right) = 1.8 \times 36,000 \times \sqrt{\frac{(36,000)^2}{4770}} \left( \frac{128}{\sqrt{24}} \right) \]
\[ S_{w} = 36,000 \]
\[ S_{w} = 64.80 - 6.830 \left( \frac{128}{4.9} \right) = 64.80 - 37.40 = 27.40 \text{ ksi} \]
\[ h/t = \frac{24}{1.1875} = 128 \]
\[ 8.96 \times \frac{10.85 - 0.375}{10.85} = 8.64 \text{ ksi} < \frac{27.40}{1.8 \times 0.7} = 21.75 \]
\[ k = 24 \]

Web Buckling Due to Shear
\[ V_{w} = 1.8 v_{w} - n \left( \frac{h}{t} \right) \]
Constant \( n = \frac{\sqrt{V_i^2}}{4770} \) \( S_r = 36000 \) \( \nu_r = 19,000 \)

\( = \frac{\sqrt{19,000^2}}{4770} = \frac{2,620,000}{4770} = 550 \)

\( V_r = 1.8(19,000) - 550\left(\frac{128}{\sqrt{49}}\right) = 34,200 - 14,400 = 19,800 \text{ psi} > 17,800 \text{ psi} \)

Floor Plate Design (Design Manual for Orthotropic Steel Plate Deck Bridge, AI SC)

12k wheel load \( 2g \times 2c - 22'' \times 12'' \)

Unit Pressure \( p \) (Incl. 30% Impact) = 59 psi

(a) Beam Moments

\[ M_o = 2 \left( \frac{11}{12} \right) \sum \left[ -0.5 \left( \frac{X}{a} \right) + 0.866 \left( \frac{X}{a} \right)^2 - 0.366 \left( \frac{X}{a} \right)^3 \right] \text{padx} \]

\[ = 2 \left( \frac{59}{12} \right)^2 \left[ -0.5 \left( \frac{11}{12} \right)^2 + 0.866 \left( \frac{11}{12} \right)^3 - 0.366 \left( \frac{11}{12} \right)^4 \right] \]

\[ = -890\#'' \]

\[ M_1 = -317\#'' \]

\[ V_o = R_o + \frac{-M + M_o}{a} = \frac{(59)(11)(6.5)}{12} + \frac{890 - 317}{12} = 351 + 49 = 400\#' \]

Bending Moment at edge of plate (\(3/16''\) from the center of support)

\[ M_l = -890 + (400)(.188) - \frac{(59)(.188)^2}{2} = 890 + 75 - 1 = -816\#'' \]

Moment at midspan \( M_c = 458\#'' \)

(b) Plate Moments and Stresses:

plate factor \( \psi \), Fig. 6.3b, Case 3

\( 2c/a = 12/12 = 1.0 \quad \psi_s = 0.87 \) (for moment at support)

\[ M_l = 0.87 \times -816 = -710 \# \text{ in./in.} \]

\( 3/8'' \) plate (A441)

\[ S = \frac{(0.375)^2}{6} = 0.0235 \text{ in.}^3/\text{in.} \]

\[ f_{\text{max}} = \frac{710}{0.0235} = 30,200 \text{ psi} \quad \frac{30,200}{27,000} = 1.12 \quad (\text{Flexibility of rib will tend to reduce moment}) \]

\[ \psi_c = 0.70 \) (for moment at mid-span) \quad M_c = 0.70 (458) = 320 \# \text{ in./in.} \]

\[ f = \frac{320}{0.0234} = 13,700 \text{ psi} \]
1/6 Division

\[ M_o = \left[ 0.0609 + 0.0840 + 0.0793 + 0.0568 + 0.0271 \right] \times 59 \times 2 \times 12 \]

\[ = -0.6162 \times 59 \times 2 \times 12 = -873" \]

1/10 Division

\[ M_o = \left[ 0.0417 + 0.0683 + 0.0819 + 0.0849 + 0.0793 + 0.0673 + 0.0512 + 0.0332 + 0.0154 \times \frac{\pi}{2} \right] \times 59 \times 1.2 \times 12 \]

\[ = -0.5181 \times 2 \times 59 \times 1.2 \times 12 = -880" \]

\[ V = \left[ 0.9263 + 0.8351 + 0.7307 + 0.6176 + 0.5000 + 0.3824 + 0.2693 + 0.1649 + 0.0737 \times \frac{\pi}{2} \right] \times 59 \times 1.2 \times 12 \]

\[ = 5.1323 \times 59 \times 1.2 \times 12 = 436" \]

(c) Maximum Deflection—

\[ t \ p \gtrsim (0.007) \times (12) \times 65 \times 59 = 0.328" \times \frac{\pi}{2} \] (p. 160)

(d) Effect of rib flexibility—

\[ P = 800# \]

\[ A = \frac{1}{10} \times 1 = \frac{1}{10} \times 0.19 \text{ in.}^2 \]

\[ PL = \frac{800 \times 0.24}{0.19 \times 29,000,000} = 0.0035" \]

\[ M_{ef} = \frac{6 \times E \times K \Delta}{L} = \frac{6 \times 29,000,000 \times \frac{0.0235}{12} \times \frac{\pi}{4} \times 0.0035}{12 \times 12 \times 4} = 74.6" \]

\[ \frac{74.6}{990} = 8\% \]

N.A. = \[ 36.00 \times 0.19 + 33.00 (0.38 + 11.00) + 24.00 (0.38 + 22.00 + 0.12) \times \frac{1}{93.00} \]

\[ = [7 + 376 + 540] \times \frac{1}{93.00} = \frac{923}{93.00} = 9.91" \]

\[ d_{un} = 36.00 \times (9.91 - 0.19)^2 + 33.00 (11.38 - 9.91)^2 + 24.00 (22.50 - 9.91)^2 + \frac{1}{10} \times \frac{1}{10} \times 8 \times (22)^3 \]

\[ = 3,400 + 70 + 3,800 + 1,330 = 8,600 \text{ in.}^4 \]

Deflection = \[ \frac{109,200 \times 1728}{29,000 \times 8600} = 0.757 \simeq 0.750" \]
\[ S_{120} = \frac{5600}{9.91} = 568 \text{ in.}^3 \]
\[ S_{96} = \frac{5600}{12.72} = 676 \text{ in.}^3 \]
\[ M_{(u+1)} = +428kI = 500(\frac{2}{3}) \times (1.286) \]
\[ M_{(u+1)} = -304kI = 373 \times (\frac{2}{3}) \times (1.222) \]

Assume total D.L. = 70 psf.
\[ M_{(u)} = -\frac{1}{3} \times 0.070 \times (50)^2 = -22kI \times 8' \text{ width} - 176kI \]
\[ M_{(u)} = +0.0700 \times 0.070 \times (50)^2 = +12kI \times 8' \text{ width} + 96kI \]

Total Design Moment = +428 + 96 = +524kI
-304 - 176 = -480kI

At 20' from outside support:
\[ f_{(u)} = \frac{524 \times 12}{868} = -7.25 \text{ ksi} \]
\[ f_{(u)} = \frac{524 \times 12}{676} = +9.30 \text{ ksi} \]

At Interior Support:
\[ f_{(u)} = \frac{480 \times 12}{868} = +6.65 \text{ ksi} \]
\[ f_{(u)} = \frac{480 \times 12}{676} = -8.52 \text{ ksi} \]

Ratio of \( \frac{W}{t} \) = 19.25 = 48 (Bottom)

---

From AISC Booklet
"Moment, Shears and Reactions Continuous Highway Bridge Tables"

---

HS 23 Loading
Max. Reaction at A 55.7kI/lane
Max. Reaction at B 68.6kI/lane
Max. shear in AB at B = -62.6kI
Max. Moment = +500.7kI in AB at X
-373.2kI at B

Impact Coeff. .286 (I)
.222 (VI)

Dist. X = 20.0 ft.
L.L. + I. Deflection —

With lane loading .640k/ft. and conc. load of 18k

\[ M_{20} = + .0700 \times .640 \times (50.0)^2 + .2064 \times 18 \times 50 = + 112 + 186 = + 298kl \]

\[ M_{20} = [0.1527 + (.1216 + .0409 \times .2) + \ldots] \times 32 \times 50.0 = .2986 \times 32 \times 50.0 = 478kl \]

\[ R_A = [(5.753 + .1233 \times .2) \ldots] \times 32 = 30.6k \]

\[ M_6 = 30.6 \times 6 = + 184kl \]

\[ M_{34} = 30.6 \times 34 = 8 \times 28 = 32 \times 14 = 1040 - 224 - 448 = 368kl \]

**II/9.8**
\[ (500 - \frac{1}{2} (500 - 368)) \times \frac{x}{14} \times x = 1826 \]
\[ x^2 - 106x + 387 = 0 \]
\[ x = 3.75 \text{ ft.} \]

Deflection = \[
[7162 \times 23.8 - \frac{184 \times 6}{2} \times 19.8 - 184 \times 14 \times (3.8 + 7.0) - 316 \times 14 \times \frac{1}{2} \times (3.8 + \frac{14.0}{3})]
- 482 \times 3.8 \times 1.9 \times \frac{1728}{29,000 \times 1}
= [170,450 - 10,920 - 27,800 - 18,800 - 3,480] \times \frac{1728}{29,000 \times 1}
= \frac{109,200 \times 1,728}{29,000 \times 1} = \frac{6,520}{0.75} = 8,700 \text{ in.}^4 \approx 8,600 \text{ in.}^4\]

Required \( I \) = \[
\frac{6,520}{0.75} = 8,700 \text{ in.}^4 \approx 8,600 \text{ in.}^4\]

Longitudinal Stiffeners
\( b/t = 24/\frac{1}{4} = 96 \)
\[
\frac{78,640,000}{(96)^2} = 8.54 \text{ ksi} \]
8.54 \( \times 1.80 = 4.74 \text{ ksi} \)

Use \( 3\frac{1}{2}" \times \frac{3}{4}" \) stiffeners \( 42 \times \frac{1}{4} = 10.5" \) (Design Manual for High Strength Steels)

\[
\begin{align*}
10.5 \times \frac{1}{4} &= 2.63 \times 0.125 = 0.33 \times 2.53 \\
3.5 \times \frac{3}{4} &= 1.10 \times 2.00 = 2.20 \times 2.53 \\
\frac{2.53}{3.73} &= 0.68 \text{ N.A.} \\
I &= \frac{1}{12} \times \frac{3}{4} \times (3.5)^3 + 1.10 (2.00 - 0.68)^2 + 2.63 (0.68 - 0.13)^2 \\
&= 1.1 + 1.9 + 0.8 = 3.8 \text{ in.}^4 \\
r &= \sqrt{\frac{3.8}{3.73}} = \sqrt{1.02} = 1.01 \\
I/r &= \frac{1.44}{1.01} = 143 \\
f &= 9830 \text{ psi (compression)} \\
Total \text{ Force} &= 9830 (3.73 + 2.63) = 9830 (6.36) = 62.5 \\
\text{Av. stress} &= \frac{62.5}{23(\frac{1}{4})} = 10.09 \text{ ksi allowable} > 8510 \text{ psi actual OK} \]

11/65

II/9.9
Appendix A

Load Distribution

Investigation of Load Distribution by the Guyon-Massonnet Theory

32—Longitudinal Beams at 1 ft. apart
Diaphragm at mid-span
Section of Long. Beam:

\[ \begin{align*}
1 \text{ Top Pl} & \quad 12 \times \frac{3}{8} \quad = \quad 4.50 \\
1 \text{ Bott. Pl} & \quad 12 \times \frac{1}{4} \quad = \quad 3.00 \\
2 \text{ Webs} & \quad 24 \times \frac{3}{4} \quad = \quad 4.50 \\
\end{align*} \]

\[ A = 12.00 \text{ in.}^2 \quad I = 1300 \text{ in.}^4 \]

\[ Z_{\text{top}} = 119 \text{ in.}^3 \]
\[ Z_{\text{tot.}} = 94 \text{ in.}^3 \]

Diaphragm:

Area \[ = \frac{1}{2} \times 20 = 5.00 \text{ in.}^2 \]
\[ J = \frac{1}{12} \times \frac{1}{4} \times (20)^4 = 167 \text{ in.}^4 \]
\[ S = 167/10 = 16.7 \text{ in.}^3 \]

Load Distribution:

\[ 2a = 50 \text{ ft.} \quad \text{Length of bridge} \]
\[ 2b = 32 \text{ ft.} \quad \text{Width of bridge} \]

\[ i = \frac{1}{12} \quad \text{=} \quad \frac{1300}{12} = 108 \text{ in.}^3/\text{in.} \]

Flexural

\[ j = \frac{300}{167} \quad = \quad \frac{0.56 \text{ in.}^3/\text{in.}}{1} \]

\[ \theta = \frac{b}{2a} \quad \text{=} \quad \frac{16}{50} \quad \sqrt{\frac{108}{0.56}} = 0.32 \times 3.73 = 1.19 \quad \text{(Same as Guyon 1946)} \]

Torsion

\[ J_s = \frac{1}{2} \times \left( \frac{1}{2} \right)^2 \times (20) = 0.104 \text{ in.}^4 \]

\[ l_s = \frac{4A^2}{2\Sigma} = \frac{4 \times (12 \times 24)^2}{2 \times \frac{24}{12} \times \frac{12}{.094} + \frac{12}{.25}} = \frac{332,000}{910 + 32 + 48} = \frac{332,000}{590} = 563 \text{ in.}^4 \]

\[ G_{\text{ia}} = \frac{563}{12} = 46.9 \text{ in.}^3/\text{in.} \]

\[ G_{\text{ia}} = \frac{104}{300} = 0.0003 \text{ in.}^3/\text{in.} \]

\[ G_{\text{ia}} + G_{\text{io}} = 46.9 \text{ in.}^3/\text{in.} \]

\[ a = \frac{G(\omega - jo)}{2E \sqrt{v}} = \frac{46.9}{\sqrt{108 \times 0.56}} \times \frac{G}{2E} \quad \text{For steel } G = .384 \text{ E (shear modulus)} \]

\[ = \frac{46.9}{2 \times 7.8} \times .384 = 1.15 > 1 \]
| Load at Section | Position on Section | \(-b\) | \(-3b/4\) | \(-b/2\) | \(-b/4\) | 0 | \(+b/4\) | \(+b/2\) | \(+3b/4\) | \(+b\) | \(\Sigma\) |
|-----------------|---------------------|------|--------|--------|--------|---|------|------|------|------|-----|------|
| 0               |                     | 0.43 | 0.60  | 0.92  | 1.37  | 1.70| 1.37 | 0.92 | 0.60 | 0.43 | .987|
| \(b/4\)         |                     | 0.23 | 0.33  | 0.53  | 0.90  | 1.38| 1.73 | 1.48 | 1.08 | 0.82 | .999|
| \(b/2\)         |                     | 0.12 | 0.18  | 0.30  | 0.57  | 0.93| 1.47 | 1.80 | 1.78 | 1.54 | .988|
| \(3b/4\)        |                     | 0.07 | 0.10  | 0.21  | 0.33  | 0.60| 1.09 | 1.93 | 2.50 | 2.73 | 1.015|
| \(b\)           |                     | 0.03 | 0.06  | 0.12  | 0.21  | 0.43| 0.82 | 1.53 | 2.73 | 4.47 | .997|

Check with Simpson's rule:

Load at 0:

\[
\frac{1}{3 \times 8} \left( \frac{0.43 + 4(0.60 + 1.37 + 1.37 + 0.60) + 2(0.92 + 1.70 + 0.92) + 0.43}{24} \right) = \frac{1}{24} [23.70] = 0.987
\]

Load at \(\frac{b}{4}\):

\[
\frac{1}{3 \times 8} \left( \frac{0.23 + 4(0.33 + 0.90 + 1.73 + 1.08) + 2(0.53 + 1.38 + 1.48) + 0.82}{24} \right) = \frac{1}{24} [23.99] = 0.999
\]

Load at \(\frac{b}{2}\):

\[
\frac{1}{3 \times 8} \left( \frac{0.12 + 4(0.18 + 0.57 + 1.47 + 1.78) + 2(0.30 + 0.93 + 1.80) + 1.54}{24} \right) = \frac{1}{24} [23.72] = 0.988
\]

Load at \(\frac{3b}{4}\):

\[
\frac{1}{3 \times 8} \left( \frac{0.07 + 4(0.10 + 0.33 + 1.09 + 2.50) + 2(0.21 + 0.60 + 1.93) + 2.73}{24} \right) = \frac{1}{24} [23.72] = 1.015
\]

Load at \(b\):

\[
\frac{1}{3 \times 8} \left( \frac{0.03 + 4(0.06 + 0.21 + 0.82 + 2.73) + 2(0.12 + 0.43 + 1.53) + 4.47}{24} \right) = \frac{1}{24} [23.92] = 0.997
\]
### Position on Section

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$-b$</th>
<th>$-3b/4$</th>
<th>$-b/2$</th>
<th>$-b/4$</th>
<th>$0$</th>
<th>$+b/4$</th>
<th>$+b/2$</th>
<th>$+3b/4$</th>
<th>$+b$</th>
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</thead>
<tbody>
<tr>
<td>Load at $b$</td>
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<td>0.01</td>
<td>0.02</td>
<td>0.03</td>
<td>0.05</td>
<td>0.11</td>
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#### Value of $\lambda K_i$

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<thead>
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<th>$\lambda K_i$</th>
<th>$\Sigma \lambda K_i$</th>
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</thead>
<tbody>
<tr>
<td>3b/4</td>
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<tr>
<td>b/2</td>
<td>0.50</td>
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</table>

#### $\Sigma \lambda K_i$

| $\lambda K_i$ | 0.09 | 0.14 | 0.23 | 0.42 | 0.73 | 1.22 | 1.76 | 2.19 | 2.57 |

**Ave. wheel load**

$$\text{Ave. wheel load} = \frac{2.57 \times 2 + 2.19 \times 4 + 1.76 \times 2}{8} = \frac{17.42}{8} = 2.18 \text{ P}$$

$$P = \frac{2.18}{(4 \text{ B}_{-1})}$$

### Position on Section

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$-b$</th>
<th>$-3b/4$</th>
<th>$-b/2$</th>
<th>$-b/4$</th>
<th>$0$</th>
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#### Value of $\lambda K_i$

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<th>$\Sigma \lambda K_i$</th>
</tr>
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<tr>
<td>b/4</td>
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<tr>
<td>0</td>
<td>0.25</td>
</tr>
</tbody>
</table>

#### $\Sigma \lambda K_i$

| $\lambda K_i$ | 0.23 | 0.32 | 0.52 |

**Ave. wheel load**

$$\text{Ave. wheel load} = \frac{0.23 + 0.32 \times 2 + 0.52}{4} = \frac{1.39}{4} = 0.35 \text{ P}$$

$$P = \frac{0.35}{4} = 0.088 \text{ P}.$$  

**With 2 lanes**

$$\text{With 2 lanes} = 0.545 \text{ P} + 0.088 \text{ P} + 0.633 \text{ P}$$

**Load Distribution for 8' Exterior Section**

$$\frac{(2.57 + 2.33) \text{ P}}{4} = 0.70 \text{ P}$$

$$\frac{(1.76 + 52) \text{ P}}{4} = 0.57 \text{ P}$$

$$0.633 \text{ P}$$
Value of $K_i$ (Full Torsion Curve $a = 1, \theta = 1.19$)

<table>
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<th>Load at Section</th>
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<th>$-3b/4$</th>
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<tr>
<td>$b/4$</td>
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<td>.25</td>
<td>.44</td>
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</tbody>
</table>

Ave. Wheel Load: $\frac{16 + 2(2.65) + .45}{4} = 0.283P$  
Ave. Wheel Load: $\frac{2.81 + 2.32(2) + 1.63}{4} = 2.27$  
Two Lanes: $\frac{.568}{4} = .071P$  

$\frac{2.8 + .45}{4} = .743P$  
$\frac{1.63 + .45}{4} = .39P$
Design II

GENERAL

In this design, the shape, thickness and spacing of the floor plate ribs and the thickness of the floor plate are selected to take full advantage of the material at critical points along the span. Combined longitudinal compressive stresses in the floor plate, due to the bending in the rib sections and stresses imposed on it as part of the girders, are close to the allowable limit of the material when combined with the transverse stresses in the floor plate spanning across the rib webs. Tensile stresses in the bottom of the ribs are near the allowable capacity of the material. The depth of these ribs is such that the bottom of the ribs are very close to the neutral axis of the composite girder sections, so that compressive stresses in the ribs over the floorbeams are not critical. Girder flanges are sized as required within the allowable stress limits of the material.

Design computations were prepared for the four deck systems listed below. The computations for design B are shown in this chapter. Results of designs A, C, and D are represented in the data summary shown in Figure 3. This summary also indicates the major design features of designs A, C, and D, and the estimated quantities of structural steel required for each design.

A. A design was prepared for a structure not meeting the minimum material thickness requirements of AASHO Specifications. This USS COR-TEN Steel design utilizes 12 ga. thickness deck ribs and stiffened web plates for the girders, varying from 6 ga. thickness for 40 foot spans to 7/16 inch thickness for 80 foot spans.

B. A design was prepared for a structure meeting the minimum material thickness requirements of AASHO Specifications for main members, except that the deck ribs are of 7/16 inch material (the minimum thickness recommended by AISC for closed ribs in orthotropic designs). COR-TEN Steel was used for this design. All webs were 9/16 inch plates, stiffened as required.

C. A third COR-TEN Steel design was prepared, similar to B above, except that no web stiffeners are employed. Although this is not the lightest weight design, it may be the most economical because it eliminates stiffeners and their inherent shop fabrication costs.

D. A design was prepared for a 50 foot span using A36 grade steel throughout with unstiffened girder webs. This design provides a basis for relative first cost comparison with the COR-TEN design described under C above.

ASSUMPTIONS

Floor beams span continuously through the girder webs and cantilever beyond the outside girders to support the safety curb and bridge railing. For calculating moments and shears, the floor beams were assumed to be on unyielding supports. Fatigue stresses were considered on the basis of 2,000,000 cycles of load. However, in lieu of a detailed grid analysis of the floor system, the floor beams are selected to provide a minimum transverse stiffness equivalent to that furnished by concrete slab and stringer designs, in order to insure the applicability of the AASHO distribution factors.

Additionally, the live load was apportioned to each girder by assuming the floor beams continuous over fixed supports, and positioning the truck wheels to produce maximum reactions. This gave a larger load to the interior girders while the AASHA method gave a larger load to the exterior girders. The interior girders are designed for 0.843 times the standard truck loading and the exterior girders are designed for 0.625 times the standard truck loading.

Where the girder spacing is less than 1/2 of the girder span, the entire cross-sectional area of the steel deck may be assumed to participate as the top flange of the girders provided it is adequately stiffened to prevent buckling. This has been confirmed by test measurements of stresses in existing European steel plate deck bridges and the assumption is used in this design. Adequate lateral support against flange buckling is assumed to be furnished by the floor beams spaced at 10 ft. o.c. reducing the max. L/r value to 120/2.58 = 46.5.

The safety curb is designed to support wheel loads without exceeding 75 percent of the yield strength of the material. The same allowable stress increase is allowed in the floor beams when the wheels are located on the curb and bridge railing loads are applied. Fatigue stresses are not considered critical for this loading condition. Bridge railings are designed for the loads specified in AASHO 1.2.11 (1).
DESIGN FOR 50 FOOT SPAN

Requirements:
(a) Roadway—28'-0" curb to curb + 21'-6" safety curbs = 31'-0" clear between railings.
(b) Span 50'-0" c. to c. bearings (simple span)
(c) Live Load—HS20 Impact Allowed
(d) Deck Surfacing—20 psf

Material:
Deck, Curbs, Ribs, Floor Beams, Girders—USS COR-TEN
Railings & Posts—ASTM A 36 Galvanized
Field Bolts—ASTM A 325
Bearing—Elastomeric Neoprene Pads

Design of Deck:
Spans
Deck plate—73/6" c. to c. rib webs.
Deck ribs—10'-0" c. to c. floor beams. For moments and shears assume five spans continuous over unyielding supports (effect of deflections of floor beams and girders on rib moments and shears is negligible).

Live load distribution
Deck plate—For contact area between deck plate and tire use recommendations outlined in AISC Design Manual for Orthotropic Steel Bridges, Sect. 3.4.1.2. For effective width of deck plate supporting wheel concentrations refer to Sect. 6.2.1.2 of the above Manual.

Deck Ribs—AASHTO 1.3.6 specifies that for open steel grid floors the wheel loads shall be distributed, normal to the main bars, a width equal to 1/4 inches per ton of axle load plus twice the distance center to center of main bars. For 16 ton axle and 73/6" rib web spacing this width is \((16 \times 1\frac{1}{4}) + (2 \times 7\frac{3}{6}) = 20 + 14\frac{1}{2} = 34\frac{3}{4}" \) or 2.35 ribs.

(d) Deck Surfacing—20 psf.
Deck Plate

Wheel load = 12,000#  I = 30%  
Contact Area = 2g × 2c = 22" × 12" = 264 in²  

Tire pressure = \frac{12,000# \times 1.30}{264} = 59 \text{ psi}  

M_{\text{MAX}} (1" strip) = \frac{1}{10} WL^2 = \frac{59 \times (7.38)^2}{10} = 322 \text{ ft-lb}  

2c/a = 12/7.38 = 1.63  

\psi_s = 0.96  

Av. mom. across effective PL width = 0.96 \times 322 = 309 \text{ ft-lb/inch}  

For \frac{3}{16}" PL; S = \frac{(.3125)^2}{6} = 0.0163 \text{ in}^2/\text{in.}  

Max. Transverse PL stress = 309/.0163 = 18,960 \text{ psi}  

---

**Diagram of (2) DECK RIBS: L. L. MOMS. & SHEARS.**

MAX. POS. L. L. MOM. = 16 x 10 x .121 x 340 = 5.34 k\text{ft} AT A.

MAX. NEG. L. L. MOM. = (16 x 10 x .121) + (4 x 10 x .005) x 19.8 k\text{ft} AT B.

MAX. POS. L. L. MOM. AT x 0.8 DR. SPAN = 16 x 10 x .183 = 29.6 k\text{ft} AT C.

MAX. L. L. SHEAR = 16 x 1.034 + 16.55 k AT D.
Deck PL ½" - Formed Rib PL ¾"

![Diagram of deck section properties with dimensions and calculated values]

**Section Properties:**
- Weight: 27.9 lb/L.F.
- Area: 8.19 Sq."
- I = 54.6 in^4
- Sm = 26.8 in^3
- SMy = 10.4 in^3
- r = 2.58 in.

**DL (2.35 ribs):**
\[
DL = 2.35 \times 27.9 = 65.5 \text{ lb/L.F.}
\]
\[
+ \text{surfacing} = 2.35 \times \frac{14.75}{12} \times 20 = 57.8 \text{ lb/L.F.}
\]
\[
\text{DL} = 123.3 \text{ lb/L.F. (say 125 lb/L.F.)}
\]

**Max. Pos. Moment @ A**
\[
M_{DL} + M_{LL+H} = 0.125 \times 0.077 \times (10.0)^2 + 34.0 \times 1.30 = 45.2 \text{ k"}
\]

**Max. Stresses**
- Compr. (Top) = \( \frac{45.2 \times 12}{2.35 \times 26.8} = 8.62 \text{ ksl} \)
- Tens. (Bot.) = \( \frac{45.2 \times 12}{2.35 \times 10.4} = 22.2 \text{ ksl} \)

**Max. Neg. Moment @ B**
\[
M_{DL} + M_{LL+H} = 0.125 \times 0.077 \times (10.0)^2 + 19.6 \times 1.30 = 26.7 \text{ k"}
\]

**Max. Stresses**
- Tens. (Top) = \( \frac{26.7 \times 12}{2.35 \times 26.8} = 5.08 \text{ ksl} \)
- Compr. (Bot.) = \( \frac{26.7 \times 12}{2.35 \times 10.4} = 13.1 \text{ ksl} \)

**Max. Pos. Moment @ C**
\[
M_{DL} + M_{LL+H} = 0.125 \times 0.033 \times (10.0)^2 + 29.2 \times 1.30 = 38 \text{ k"}
\]

**Max. Stresses**
- Compr. (Top) = \( \frac{38.4 \times 12}{2.35 \times 26.8} = 7.31 \text{ ksl} \)
- Tens. (Bot.) = \( \frac{38.4 \times 12}{2.35 \times 10.4} = 18.8 \text{ ksl} \)
Max. Shear @ D
\( V_{DL} + V_{LL+I} = 0.125 \times 5.0 + 16.55 \times 1.30 = 22.15k \)
Assume shear on 3 webs only
Shear per web = \( 22.15/3 = 7.38k/\text{web} \)
Web A = \( 1875 \times 6.44/\cos 14^\circ = 1.24\) in\(^2\)
D/t = \( 6.44/1875 \times \cos 14^\circ = 35.4 \)
Allow. V = 15.0 ksi
\( V = 7.38 \times 1.24 = 5.95\) ksi < 15.0

Provide \( \frac{5}{16}'' \) Brg. Stiff. over Floor Beam Wabs
Max. Rib reaction = \( 7.38 \times 2 + 2/3 \times .63 = 15.18k \)
Brg. under stiff = \( 15.18k/0.3125 \times 2.90 = 16.8\) ksi < 40.0

Deck Rib Splices
Use full penetration butt welds for Rib splices located approximately 2.5' from floor beam where max. stresses are below the allow. fatigue stress of approx. 16,000 psi.

Curbs
Loads — 500#/L.F. horizontal at top of curb
Check curb section for supporting a 16k wheel load plus impact without exceeding 75% of yield stress (37.5 ksi).

\[
\text{I} = 122 \text{ in}^4 \\
Z_{\text{top}} = 34.3 \text{ in}^3 \\
Z_{\text{bot}} = 21.2 \text{ in}^3
\]

Section ok for horiz. load by observation.
\( M_{UL+I} = 34.0 \times 1.30 = 44.2 \)
\( M_{UL} = 0.035 \times 0.077 \times (10.0)^2 = 0.3 \)
\( = 44.5k\)

Max. stresses
Top (Compr.) = \( \frac{44.5 \times 12}{34.3} = 15.6\) ksi < 37.5
Bot. (Tens.) = \( \frac{44.5 \times 12}{21.2} = 25.2\) ksi < 37.5

Railing and Posts (A 36 steel)
Top Rail — Vert. load = 100#/L.F.
Mom. = \( 0.10 \times (10.0)^2/8 = 1.25k' \)
Top Rail—Horiz. load = 150#/L.F.
Mom. = \( 0.15 \times (10.0)^2/8 = 1.88k' \)
Tubing 4 \times 2 \times .250#/Z_{XX} = 2.345 Z_{YY} = 1.532
Max. stress = \( \frac{1.532}{2.345} + \frac{1.88 \times 12}{2.345} = 19.38\) ksi < 20.0

Intermediate Rail
Horiz. load = 300#/L.F.
Mom. = \( 0.30 \times (10.0)^2/8 = 3.75k' \)
Z reqd = \( \frac{3.75 \times 12}{20} = 2.25 \text{ in}^3 \)
Use Tubing 4 \times 2 \times .250

II/9.18
Posts
Mom. @ top of curb = 1.50 × 2.25 ÷ 3.0 × .92 = 6.14k
Z reqd = \( \frac{6.14 \times 12}{20} \) = 3.69 in³
Use Tubing 4 × 4 × .250; Z = 3.994 in³
H. S. Bolts reqd @ top of curb (%" single shear)
N = \( \frac{1.5 \times 3.33 + 3.0 \times 2.0}{1.08 \times 5.96} \) = 1.71
Use 2 Bolts

Floor Beams

Dead Loads
Railing & Posts = 22#/ft × 10 + 75#/post = 300#
Curb = 35#/ft × 10 × 1.15 (contingency) = 400#
Deck (Including surfacing) = 49#/ft² × 10 × 1.14 = 558#/ft
Fl Bm. wgt. = 26#/ft
Use 600#/ft

Dead Load Moments & Shears
\( M_a = -0.3 \times 3.7 - 0.44 \times 2.8 - 0.6 \times (2.0)^2/2 = -3.54k \)
\( V_A = -1.94k \)
\( M_{Ax} = 0.6 \times .080 \times (8.0)^2 - .475 \times 3.54 = +1.40k' \)
\( M_b = -06 \times .100 \times (8.0)^2 + .1875 \times 3.54 = -3.18k' \)
\( V_{Ab} = +2.45k \ V_{Aa} = -2.35 \)
\( M_{Aw} = 0.6 \times .025 \times (8.0)^2 + .1875 \times 3.54 = +1.62k' \)
\( V_{Aw} = +2.40k \)
Loads on Fl. Bm. Cantilever

\[
P = 0.10k/l \times 10 = 1.0k \\
H_1 = 0.15k/l \times 10 = 1.5k \\
H_2 = 0.30k/l \times 10 = 3.0k \\
H_3 = 0.50k/l \times 10 = 5.0k \\
W = 16.55k \\
W' = 16.55k
\]

Loading (1) DL + (LL (W) + 1) + P + H_2 + H_3
Loading (2) DL + (LL (W') + 1) + P + H_1 + H_2

Max. Mom. @ A:

Loading (1) At basic allow. stresses

\[
M_{BL} = -3.5 \\
M_{UL} = -16.6 \\
M_i = 0.30 + 16.6 = -5.0 \\
M_r + H_2 + H_3 = -29.0 \\
= -54.1k
\]

\[
V = -1.94 \\
V = -16.55 \\
V = -4.97 \\
V = -1.00 \\
V = -24.46k
\]

\[
Z \text{ reqd} = \frac{54.1 \times 12}{27.0} = 24.1 \text{ in}^2
\]

Loading (2) At 75% yield stress

\[
M_{BL} = -3.5 \\
M_{UL} = -41.4 \\
M_i = 0.30 \times 41.4 = -12.4 \\
M_r + H_1 + H_2 = -19.0 \\
= -76.3k \\
V = -24.46k
\]

\[
Z \text{ reqd} = \frac{76.3 \times 12}{37.5} = 24.4 \text{ in}^2
\]

\[
M_{48} = 8.0 \times W (0.2042 - 0.0206 + 0.0086) = 1.538 W \\
W = 16.0 \times 1.034 = 16.55k
\]

\[
M_{BL} = +1.4 \\
M_i = 1.538 \times 16.55 = +25.4 \\
M_i = 0.30 \times 25.4 = +7.6 \\
= +34.4k
\]
For full penetration butt welded splice at A span AB with 2,000,000 loading cycles PL=1.4/34.4=.041

\[ F_r = \frac{16000}{1 - (0.8 \times 0.041)} = 16000 \times \frac{0.967}{1} = 16,500 \text{ psi} \]

\[ Z \text{ reqd} = \frac{34.4 \times 12}{16.55} = 24.9 \text{ in}^3 \]

Max. Mom. @ B

\[ M_s = 8.0 \times W (-0.100 - 0.00705 - 0.0750 + 0.0250) = -1,488 \text{ W} \]

\[ M_{sl} = -1,488 \times 16.55 = -24.6 \]

\[ M_i = 0.30 \times (-24.6) = -7.4 \]

\[ = -35.2k' \]

If Floor Beams are butt welded to webs of girders

\[ F_r = \frac{16000}{1 - (0.8 \times \frac{3.2}{35.2})} = 16000 \times \frac{0.927}{1} = 17,250 \text{ psi} \]

\[ Z \text{ reqd} = \frac{35.2 \times 12}{17.25} = 24.5 \text{ in}^3 \]

\[ M_{sl} = \text{(Not critical by observation)} \]

Max. Shear

\[ V_{sa} = 2.35 + (16.55 \times 1.328 \times 1.30) = 30.91k \]

A concrete slab and steel stringer bridge with 8'-0" stringer spacing requires a 7" slab. The expression for a floor beam of equal stiffness may be written as follows:

\[ K \frac{E_{sl}}{L_s} = K \frac{E_{lc}}{L_c} \]

which can be reduced to

\[ L_s = \frac{E_{lc} \cdot L_c}{E_i} \]

Therefore, for a 10'-0" floor beam spacing

\[ L_s = \frac{10 \times 12 \times (7)^3}{12 \times 10} = 316 \text{ in}^4 \text{ (Minimum floor beam)} \]

For Floor Beams Use 16 B 26

\[ l = 298.1 \text{ in}^4 \]

\[ Z = 38.1 \text{ in}^3 > 24.9 \]

Max. web shear = \[ \frac{30.91}{15.65 \times 0.250} = 7.90 \text{k}\]

Floor Beam Bolted Field Splice

16 B 26

\[ l = 298.1 \text{ in}^4 \]

Less 4-7/8" holes = \[ -70.7 \]

Net \[ I = 227.4 \text{ in}^4 \]

\[ Z = \frac{227.4}{7.83} = 29.04 \text{ in}^3 \]

Allow \[ M = \frac{29.04 \times 27}{12} = 65.3k' \]
Max. Mom. @ splice = 34.4k

Allow Girder Mom. = \( \frac{38.1 \times 27}{12} = 85.7k \)

75% allow. = 64.3k

Splice Design Mom. = 64.3k

Design shear = \( .75 \times .25 \times 15.65 \times 15.0 = 44.02k \)

Av. Flange stress = \( \frac{64.3 \times 12 \times 7.65}{298.1} = 19.80 \text{kips} \)

Fig. force = 19.8 \times 5.5 \times .345 = 37.6k

\( \frac{3}{8} \)" H.S. Bolts—Double shear = 11.92k

Flange Bolts reqd = \( \frac{37.6}{11.92} = 3.15 \)

Mom. of Inertia of bolts

\[ 2 \times (2.75)^2 = 15.1 \]

\[ 2 \times (5.5)^2 = 60.5 \]

\[ 8 \times (7.65)^2 = 468.2 \]

\[ 543.8 \]

\[ \text{Stress/Fig. Bolt} = \frac{64.3 \times 12 \times 7.65}{543.8} = 10.85k < 11.82 \]

Stress/web Bolt

From Mom. = \( \frac{64.3 \times 12 \times 5.5}{543.8} = 7.80k \)

From Shear = 44.02/5 = 8.80k

Max. stress = \( \sqrt{(7.80)^2 + (8.80)^2} = 11.77k < 11.92 \)

Min. Splice Material o.k. by observation

Ea. Flange Splice—\{ 1-PL 5\( \frac{1}{2} \) \times \( \frac{5}{8} \) \}, 4-\( \frac{3}{4} \)" H.S. Bolts

Web Splice—2-PLs 5\( \frac{1}{2} \) \times \( \frac{5}{8} \) \times 1-\( \frac{1}{2} \)"—5-\( \frac{3}{4} \)" H.S. Bolts

\( \text{(b) GIRDER CALCULATIONS:} \)

\[ \text{Exterior Girder Properties} \]

Top Fig. A = 36.75

Web A = 9.38

Bot. Fig. A = 8.13

Totl. Area = 54.26 in\(^2\)

\[ I = 6990 \text{ in}^4 \]

\[ Z_t = 822 \text{ in}^3 \]

\[ Z_t = 308 \text{ in}^3 \]
Dead Loads
- Railing = .030 k/ft \times 11.7/8.0 = .044
- Curb = .035 k/ft \times 10.8/8.0 = .047
- Deck PL = .0128 \times 10 \times 5.0/8.0 = .080
- Ribs = .0121 \times 6 \times 4.0/8.0 = .036
- .0121 \times 1 \times 8.9/8.0 = .013
- Fl. Bins = .026 \times 11.8/1 \times 10 \times 5.9/8 = .023
- Flange & Web = .060
- Surfacing = .020 \times 10 \times 5.0/8.0 = .125

Total DL = .428 k/ft

Location of Wheels for Max. Ext. Gdr. LL
Simple Spans = A = 1.0 W + 2/8 W = 1.25 W
or .625 Lanes
Cont. spans = A = W(1.000 + .164 + .022) = 1.186 W
or .593 Lanes
AASHTO = A = W \left( \frac{8.0}{4.0 + 8.4} \right) = 1.333 W
or .667 Lanes

Max. Moment
- Mo_L = .43 \times (50)^2/8 = 134.3
- M_L = .667 \times 627.9 = 418.8
- M_I = .286 \times 418.8 = 119.8

672.9 k

Max. Shear
- V_L = .43 \times (50)^2/2 = 10.7
- V_L = .667 \times 58.5 = 39.0
- V_I = .286 \times 39.0 = 11.2

Max. Bot. Flg. stress = \frac{672.9 \times 12}{308} = 26.2 ksf < 27.0

Max. Top Flg. stress

Gdr. stress = \frac{672.9 \times 12}{822} = 9.82

Long. Rib compr. = 7.31
Max. Long. = 17.13
Max. Transv. = 18.96

Max. Compr. = \sqrt{(17.13)^2 + (18.96)^2} = 25.6 ksf < 27.0

Web Stresses
For \frac{1}{2} in PL: D/t = (30 - 50)/.3125 = 94.4 > 58

\frac{t}{D} = \frac{7500}{7500} = \frac{1}{7500}

Allow. f_v = \left( \frac{7500}{D/t} \right)^2

Allow. f_v = \left( \frac{7500}{94.3} \right)^2 = 6320 \text{ psi}

Allow. V = 9.38 \times 6.32 = 59.3 k
Interior Girder Properties
Top Fig. A = 51.38
Web A = 9.38
Bot. Fig. A = 10.00
Tot. Area = 70.76 in²
I = 8570 in⁴
Z₁ = 1106 in³
Z₂ = 364 in³
Dead Load
Deck PL = 0.128 x 8.0 = 1.02
Ribs = 0.121 x 6 = 0.73
Fl. Bm. = 0.26 x 8.0/10 = 0.21
Flg. & Web = 0.066
Surfacing = 0.20 x 8 = 1.60
Total Gdr. DL = 0.422 say .43 k/ft

ASSUMED INTERIOR GIRDER SECTION.

Location of Wheels for Max. Int. Gdr. LL
Simple Spans—B = 4/8 W + 6/8 W + 2/8 W = 1.50 W
or 75 Lanes
Cont. Spans—B = W (.725 + .8505 + .2605 – .1500)
= 1.686 W or .843 Lanes
AASHO — B = W (8.0/5.5) = 1.454 W
or .727 Lanes
Max. Mom.
M₀ = .43 x (50)²/8 = 134.3
M₁ = .843 x 627.9 = 529.3
M₂ = .286 x 529.3 = 151.4
= 815.0 k

Max. Shear
V₀ = .43 x 50/2 = 10.7
V₀ = .843 x 58.5 = 49.3
V₁ = .286 x 49.3 = 14.1
= 74.1 k

Interior Girder
Max. Bot. Fig. stress = \frac{815.0 \times 12}{364} = 26.8kal < 27.0
Max. Top Fig. stress
Gdr. stress = \frac{815.0 \times 12}{3106} = 8.85
Long. Rib compr. = 7.31
Max. Long. = 16.16
Max. Transv. = 18.96 k
Max. Compr. = \sqrt{(16.16)^2 + (18.96)^2} = 25.0 kal < 27.0

Web Stresses
For 30 x ½ PL Allow. V = 59.3 k
(see Ext. Gdr. Calculations)
For 30 x ¾ PL Allow. V = 98.9 k
Live Load Deflections

\[
\begin{align*}
\text{Dist.} & \quad 0' & 5' & 10' \\
V_{OL} & = 10.7 & 8.6 & 6.4 \\
V_{UL} & = 49.3 & 43.3 & 37.2 \\
V_l & = 14.1 & 12.7 & 11.2 \\
\end{align*}
\]

\begin{align*}
\text{MAX SHEARS} \\
74.1^k & \quad 64.6^k & \quad 54.8^k \\
\text{LOADING FOR MAXIMUM MOM. AND DEFL.} \\
W/4 & \quad 8.67' & \quad 14.0' & \quad 14.0' & \quad 13.33' \\
50.0 & \quad & & & \\
R=1.02W \quad & \quad & \quad & \quad & \quad \\
R=1.23W \\
\end{align*}

\[
\begin{align*}
\frac{\text{Max. Defl. (@ A)}}{\text{EI}} &= \frac{4780W}{284W} \times 1728 = \frac{9259.800W}{284W} \\
\text{Max. } \frac{W}{2} & x 13.33 \times 15.34 = -1677 \\
\text{Max. } \frac{W}{2} & x 10.89 \times 5.44 = -972 \\
\text{Max. } \frac{W}{2} & x 10.89 \times 3.63 = -50 \\
\end{align*}
\]

\[
\begin{align*}
\text{Max. } \frac{W}{2} & = 29,000kW \text{, LL Defl. } = \frac{284W}{2} \\
\text{Total I for Bridge } &= 2(6990 + 8570) = 31,120 \text{ in}^4 \\
\text{Avg. LL & I Defl. } &= \frac{284 \times 16.0 \times 4 \times 1.286}{31,120} = 0.750'' \\
\text{Max. Allow. LL Defl. } &= \frac{50 \times 12}{800} = 0.750'' \\
\end{align*}
\]

DL Deflections

\[
\begin{align*}
\text{For Ext. Girders } &= \frac{5 \times 0.43 \times (50)^4 \times 1728}{384 \times 29000 \times 6990} = 0.298'' \\
\text{For Int. Girders } &= \frac{5 \times 0.43 \times (50)^4 \times 1728}{384 \times 29000 \times 8570} = 0.244'' \\
\end{align*}
\]

\text{Estimate of Cost:}

\begin{align*}
\text{Bridge Railing and posts—} & \quad 102 \text{ Lin. Ft.} \quad @ \$5.50 = \$ \quad 561 \\
\text{Bridge Deck & Girders (COR-TEN)—} & \quad 54,880 \text{ Lbs.} \quad @ \$0.22 = \$12,074 \\
\text{Total Cost } &= \$12,635
\end{align*}
HALF SECTIONAL PLAN

HALF PLAN

NOTES:

1. DESIGN SPECIFICATIONS-ASHD Standard Specifications For Highway Bridges (1986) and AWS Specifications For Welded Highway and Railway Bridges (1983)

2. LIVE LOAD-NO-50-44-14119 LOADING


4. ELASTOMERIC BEARING DIMENSIONS ARE FOR 50-DIAMETER HANGER bars; SURFACE STEEL SHALL BE 1/8" THICK; ELASTOMERIC ELEMENTS AND ADHESIVE ON TOP AND BOTTOM SURFACES OF ELASTOMERIC PADS

5. FOR BRIDGE/RAILING SEE USA DESIGN A SODIUM-FILLED THERMAL RUBBER MOLD, BARRINGTON, IL

FIGURE 1
ALL STEEL SUPERSTRUCTURE 50 FT. SIMPLE SPAN

9/68

11/9/27
PART PLAN - BRIDGE DECK - 50 FT SPAN

HALF SECTION NEAR MID SPAN

TYPICAL CROSS SECTION

SECTION A-A

ELEVATION B-B

SUMMARY - BRIDGE DECK DESIGN DATA

<table>
<thead>
<tr>
<th>TYPE OF STEEL</th>
<th>U.S.S. COR TEN</th>
<th>A572</th>
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<tbody>
<tr>
<td>SPAN LENGTH C-BRACING</td>
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<td>50 FT</td>
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<td>RIB PLATE THICKNESS</td>
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<td>5/16&quot;</td>
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<td>FORM RIB PLATE THICKNESS</td>
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<td>3/16&quot;</td>
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<td>EXTerior Girders</td>
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<td>TOTAL</td>
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<td>STEEL WET - LBS/OSFT</td>
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<td>35.1</td>
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*Design based on maximum design requirements for minimum material thickness.

FIGURE 3
SIMPLE SPAN
ALL STEEL SUPERSTRUCTURE

NOTE FOR DETAILS SEE FIGURE 4
Note:

Designs presented in this chapter are in accordance with the Eighth Edition of the Standard Specifications for Highway Bridges of the American Association of State Highway and Transportation Officials (AASHTO) dated 1961. They should be reviewed for adequacy in conforming to the latest edition of the AASHTO Specifications and subsequent interims.

Cost information in this chapter should be reviewed in light of present material and labor costs.
Steel Substructures

Steel Abutments

The abutment drawings shown in this chapter evolved from studies which attempted to develop an ideal design that would be inexpensive, simple and have a long life. In addition it was considered desirable to be able to use the design as a full height (20' grade to grade) or a stub abutment.

It was evident that the wing wall treatment should be such as would help keep the cost down. Further, the basic design details for the back wall and wings should apply throughout the range of heights.

To assure long life certain features were incorporated into the design that exceed normal design requirements. To take into account mechanical or vandalism damage, the corrugated sheets are 18 gage rather than the 28 gage design requirement. The 6 x 6 angles on the wing are twice the design length for added stability not contemplated in the design. No provision has been made for special protection of the buried faces below finished ground other than the galvanizing on the corrugated sheets. If soil conditions warrant such special protection a protective coating can, of course, be applied.

The abutment, as shown, includes a vertical bearing H section under each stringer. The H section is selected to have sufficient capacity for cantilever action. A smaller H section extending from below the top of the stringer seat to the roadway surface is to be welded to the back of the larger H section. A split-tee section, shaped to roadway crown, is placed across the tops of the upper H sections to form the top of the backwall. This split-tee section was selected to resist wheel loads.

The abutment accommodates a twenty-four foot roadway with 10 foot wide shoulders. A standard material with 3 inch by 1 inch corruga-

tions spans vertically between 4 inch I-beam members which are placed horizontally between the vertical H sections and the vertical corner angles.

A series of nine abutments was established based on the above scheme. Both bearing piles and WF shapes were designed for each of the nine abutments, the size and weight varying as necessary to resist the load. WF shapes were selected because of lighter sections. In place prices of $0.15 per pound for structural steel and $0.26 per pound for galvanized corrugated sheets were used to develop Figure 1.

Figure 1 shows that above the eight to ten foot height range, the slope of the curve is such that doubling the abutment height more than triples the cost. This leads to the problem of the combined effect of span length on superstructure and substructure costs.

In order to investigate the combined effect, costs were derived for the steel semi-orthotropic superstructure in Chapter 9 of this volume for spans from 40 ft. on up. Using minimum openings of 40', 50' and 60' with full height abutments, the spans were increased and the abutment heights decreased to get the combined cost effect plotted in Figure 2.

The curves indicate optimum structure economy where the abutments are from 10' to 12' high and the spans 23' to 28' longer than minimum opening. The medium abutment included in this chapter is 8' to 9' high. Consideration of the curves shows very slight additional cost. From $100 on the 40' curve to $1,000 on the 60', the additional structure cost permits opening up the respective spans by 5' and 10'. This provides a better appearance for the structure than the optimum span arrangement and is completely or partially offset by the savings in roadway quantities. In a specific problem the roadway
I section connecting tops of piles

L = 8 \times 12 = 96''

Use 9[13.4 Cor.Ten. \ r = 0.67

\frac{L}{r} = \frac{96}{0.67} = 143 < 200

Extend 9[13.4 9'' beyond web of exterior pile.

\text{LL} + \text{DL} = 55.6' + 46.3' (\text{No Impact, AASHO p. 17})

55,600# \div 6,000 (\text{AASHO p. 58}) = 9.33\text{'' req'd.}

\text{BP36} A = 10.6\text{''} \quad S = 29.9

f_s = \frac{55,600}{10.6} + \frac{42,300 \times 12}{29.9} = 5250 + 17,000 = 22,250\#/\text{''}

Use 10BP42 A = 12.35\text{''} \quad S = 43.4

f_s = \frac{55,600}{12.35} + \frac{42,300 \times 12}{43.4} = 4500 + 11,700 = 16,200\#/\text{''}

For f_s = 20,000 \quad M: 20,000 - 4500 = 15,500 = \frac{M \times 12}{43.4}

M = 56,000\#

\frac{M}{8} = 7,000\#'/

100 \times (14 + x) = 1400 + 100x

100 \times (5.5 + x) = 550 + 100x

500 \times (6.67 + x) = 3330 + 500x

5280 + 700x = 7000 \quad x = 2.45'

Max. spill-through height for this pile = 4 + 2.45 = 6.5'

\therefore \text{From 3' to 6.5' @ no increase in cost.}
\[ M = \frac{1}{2} \times 174 \times 10^2 = 8700\#' \]

\[ S \text{ req'd.} = \frac{8700 \times 12}{27,000} = 3.87 \text{ in.}^3 \quad 4 \times 4M13.0 \quad S = 5.2 \text{ in.}^3 \]

Requires moment connection to BWF20.

Data from Superstructure:

<table>
<thead>
<tr>
<th></th>
<th>40' Span</th>
<th>50' Span</th>
<th>60' Span</th>
</tr>
</thead>
<tbody>
<tr>
<td>D.L.</td>
<td>4.8</td>
<td>51.3</td>
<td>6.3</td>
</tr>
<tr>
<td>L.L.</td>
<td>46.5</td>
<td>5.0</td>
<td>41.8</td>
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<tr>
<td>Imp.</td>
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<td>11.0</td>
<td>14.1</td>
</tr>
<tr>
<td></td>
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<td>52.8</td>
<td>69.7</td>
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<table>
<thead>
<tr>
<th></th>
<th>70' Span</th>
<th>80' Span</th>
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<tbody>
<tr>
<td></td>
<td>Int.</td>
<td>Ext.</td>
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<tr>
<td>D.L.</td>
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<td>62.4</td>
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<tr>
<td>L.L.</td>
<td>52.6</td>
<td>10.5</td>
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<tr>
<td>Imp.</td>
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<td>19.6</td>
</tr>
<tr>
<td></td>
<td>75.9</td>
<td>62.7</td>
</tr>
</tbody>
</table>

Abut. 4' Fill 12WF36 as piles

40' Span: \( M = 42,300\# \quad N = 51,300\# \)

\[ f_s = \frac{51,300}{10.59} \times \frac{42,300 \times 12}{45.9} = 4850 + 11,050 = 15,900\#/\square \]

50' Span: \( f_s = 16,350\#/\square \)

60' Span: \( f_s = \frac{59,400}{10.59} + 11,050 = 5620 + 11,050 = 16,670\#/\square \)

70' Span: \( f_s = \frac{62,400}{10.59} + 11,050 = 5900 + 11,050 = 16,950\#/\square \)

80' Span: \( f_s = \frac{65,500}{10.59} + 11,050 = 6200 + 11,050 = 17,250\#/\square \)
Side Pile:
\[ 6 \times 40 = 240 \quad \frac{240}{2} \times 6 = 720\# \]

M: 720\# \times 6' = 4320\#/ft. wall

S for 6' wall \[ \frac{4320 \times 12 \times 6}{20,000} = 15.6 \text{ in.}^3 \]

Use 8BP36 \( S = 29.9 \)

12' Wings 14BP73, 28' long
8' Fill Height \( f_1 = \frac{20,000}{\text{ft.}} \) Pile at Capacity

Capacity of 14BP89 \( A = 26.19\% \quad S = 131.2 \text{ in.}^3 \)

\[ \frac{55,600}{26.19} = 2120 \quad 20,000 - 2120 = 17,880\#/\text{ft.} \]

\[ M = 195,000 \]

\[ \frac{24,400}{8} \]

\[ 100 \times (19 + x) = 1,900 + 100x \]

\[ 100 \times (10.5 + x) = 1,050 + 100x \]

\[ 2000 \times (8.33 + x) = 16660 + 2000x \]

\[ = 19,610 + 2200x = 24,400 \]

\[ x = 2.2' \]

Height Range 8' to \( (8 + 2.2) = 8' \) to 10.2'

Try 14WF68 \( A = 21.76\% \quad S = 103.0 \text{ in.}^3 \)

\[ f_1 = \frac{55,600}{21.76} + \frac{157,000 \times 12}{103} = 2560 + 18,300 = 20,860\#/\text{ft.} \]

Try 18WF64 \( A = 18.80\% \quad S = 117.0 \text{ in.}^3 \)

Limit 20,000 - \( \frac{55.600}{18.80} = 2960 \) = 17,040 = \( M \times \frac{12}{117.0} \) \( M = 166,000 \quad M = 166,000 \quad \frac{M}{8} = 20,700\#/\text{ft.} \]

\[ 19,610 + 2200x = 20,700 \quad 2200x = 1090 \quad x = 0.5 \quad 8.5' \]

Try 18WF77 \( A = 22.63\% \quad S = 141.7 \text{ in.}^3 \)

Limit 20,000 - \( \frac{55.600}{22.63} = 2460 \) = 17,540 = \( M \times \frac{12}{141.7} \) \( M = 207,000 \quad \frac{M}{8} = 25,800\#/\text{ft.} \]

19,610 + 2200x = 25,800 \quad 2200x = 6170 \quad x = 2.8 + 8' = 10.8'

See Data from Superstructure—Minimum Abutment

12' Wings 8' Fill

Using 18WF64 as piles \( A = 18.80\% \quad S = 117.0 \text{ in.}^3 \)

40' Span: \( M = 157,000\# \quad N = 51,300\# \)

\[ f_1 = \frac{51,300}{18.80} + \frac{157,000 \times 12}{117.0} = 2730 + 16,100 = 18,830\#/\text{ft.} \]
Side wall:—

40# × 2.37' = 95#/fur over 11' ±
150# × 2.75' = 412#/fur over 7' ±
260# × 2.87' = 745#/fur over 3' ±

Wall Elevation

Load Diagrams

\[ M_{\text{MAX}} = 0.1283 \times \frac{800}{2} \times 12' \times 12' = 7400#' \]

\[ S_{\text{req'd.}} = \frac{7400 \times 12}{27,000} = 3.3 \text{ in.}^3 \]

\[ 4 \times 4M13.0 \quad S = 5.20 \text{ in.}^3 \]

Estimate for Medium Abutment

1 - ST10WF48 = 48# × 44' = 2120
4 - 8WF20 = 80# × 4' = 320
4 - L 6 × 6 × \frac{7}{16} = 70# × 20' = 1400
4 - 4 × 4M13.0 = 52# × 44' = 2290
3 - 4 × 4M13.0 = 39# × 24' = 936
1 - 9L13.4 = 13.4# × 28' = 375
2 - PL 19 × \frac{3}{4} = 96.9# × 1.33' = 129
2 - PL 19 × \frac{3}{4} = 96.9# × 1.67' = 162
12 - Blocking PL @ 7" = 84

Basic Structural = 7816#/ @ $0.15 = $1173

Corrugated PL:
44 × 9' = 396
8 × 6' = 64
8 × 7' = 56
8 × 4' = 32

\[ 548\text{#/f ur} @ 2.84#/\text{fur} = 1557#/ @ $0.26 = \$405 \]

Steel Piles:
4 - 18WF6c × 28' = 2170#/ @ 0.15 = \$1075

Cost per abutment

\[ \$2653 \]

\[ \text{Say } \$2660 \]

per abutment
Design Examples—Steel Bearing Piles

Steel bearing pile design is discussed, and procedures and formulas are given in the United States Steel Corporation’s “Highway Structures Design Manual,” Volume I, Chapter 10. The following examples illustrate the design of steel bearing pile foundations most used for highway structures. Five examples are given:

1. Stub Abutment for Short Span Steel Bridges
2. Pile Bent
3. Cantilever Abutment (design procedure also typical for retaining wall)
4. Bridge Pier on Land
5. Bridge Pier in Water (with Tremie Concrete Seal)

Various soil conditions are assumed to illustrate the design of friction piles in fine grained material, friction piles in coarse grained material, and piles bearing on rock.

A single row of piles is used in Examples 1 and 2; larger pile groups are used in Examples 3, 4 and 5.

The design forces for stub abutments are dependent upon the arrangement of spans, span lengths, type of bearing device for the roadway stringers, temperature variation and type of soil. Abutments with shallow backwalls generally have the capability to move as the roadway stringers expand or contract with temperature changes. Since these movements are generally small for short span bridges, the longitudinal force induced in the pile due to movement is small and within the allowable lateral pile load. Thus it is desirable to “fix” the roadway stringers to both abutments for relatively short spans and carry all longitudinal forces due to wind, traction and temperature changes to the abutments. As the span length of the superstructure increases, the temperature movement to be accommodated by the abutment and piles increase. Excessive lateral forces and excessive bending stresses may be induced in the piles. When the length between abutments exceeds about 250’ to 300’, it is desirable to provide expansion bearings on both abutments with traction and longitudinal wind forces carried by the intermediate pile bents. The only longitudinal force carried by the abutment is due to dead load friction in the expansion devices. Example 1 assumes that the thermal movement of the stringers exceeds the permissible movement of the abutment and piles and, that expansion type bearings are used on both abutments.

Since pile bents are relatively flexible, movement due to temperature changes will cause only small loads and moment in the piles. Where fixed bearings are used on both abutments, the only longitudinal force on pile bents is caused by temperature changes. Example 2 illustrates this condition.

Where expansion bearings are used on the abutments and the roadway stringers are fixed to each pile bent, the pile bents must be designed for traction, longitudinal wind and temperature changes.

If a series of continuous spans are supported by pile bents, longitudinal forces due to traction, wind and temperature changes must be carried by the pile bents.
The following general method of design is recommended for Example 1:
1. Determine the vertical and horizontal load per pile. If one pile is placed under each stringer, the dead load and live load to each pile is equal to the stringer reaction plus a portion of the weight of the pile cap.
2. Choose the pile size and determine the depth of penetration required to develop the axial load in the pile.
3. Determine the allowable lateral load per pile and compare to the actual lateral load per pile. The allowable lateral load per pile generally is controlled by the permissible lateral movement of the pile at the ground surface. If the actual lateral load exceeds the allowable; increase the pile size, increase the number of piles, or batter piles.
4. Check settlement of the piles if critical to design of the structure.

The following general method of design is recommended for Example 2:
1. Determine the vertical and horizontal load per pile. If one pile is placed under each stringer, the dead load and live load to each pile is equal to the stringer reaction plus a portion of the weight of the pile cap.

Determine the horizontal loads due to wind, traction and centrifugal force and resolve into components parallel to the row of piles (H_x) and normal to the row of piles (H_y) as shown in Figure A. Determine the vertical load per pile by taking moments about the top of piles and applying the following equation:

\[ Q_m = \frac{F_v}{r} + \frac{M_v x}{2x^2} \]  

(Eq. A)

where \( Q_m \) = vertical load on any given pile \( m \)
\( F_v \) = vertical load due to dead load and live load
\( M_v \) = moment with respect to the \( Y \) axis through the centroid of the pile group
\( x \) = distance of pile from \( Y \) axis
\( r \) = number of piles

(For definition of above terms, see Figure A)

If piles are battered parallel to the row of piles, determine the amount of horizontal load carried by the batter and subtract this from the total horizontal load parallel to the row of piles. The remaining horizontal load is divided equally among the piles and is carried by lateral bearing of the piles against the soil.

2. Tentatively choose the pile size and determine the depth of penetration required to develop the axial load in the pile. If the pile extends above the ground surface, combined stresses due to axial load and bending may govern the pile size.
3. Determine the depth below ground surface at which the piles may be considered fixed.
4. Determine the allowable compressive stress in the pile based upon the appropriate column formula.
5. Determine the longitudinal force on the piles due to temperature change in the superstructure.
6. Determine the maximum stresses in the pile due to axial load plus bending.
7. Check settlement of the piles if critical to design of the structure.
In Examples 3, 4 and 5, groups of piles are embedded in rigid concrete footings. The following steps are recommended for the design of such a foundation:

1. Determine loads on the pile group at the elevation of the top of piles. Vertical load, moments about the two perpendicular axes passing through the centroid of the pile group, and horizontal shears parallel to these two axes are required. The vertical load, moments and shears often are calculated at the top of footing as part of the pier design. The designer then transfers these to the elevation of the top of piles by increasing the vertical load by the weight of footing plus earth cover, and changing each moment by the pertinent horizontal shear multiplied by the depth from the top of footing to the top of piles.

2. Tentatively choose the pile size and allowable load. If friction piles are used, determine the length of embedment required to develop the allowable load. If the soil quality is sufficient to develop high pile capacities, greatest economy is obtained with a smaller number of large piles.

3. Tentatively choose a footing size and pile pattern based upon past designs and the relationship of moments to vertical loads. For example, if moments on the pile group are small, the number of piles will be approximately equal to the vertical load divided by the allowable pile load; if moments on the pile group are large, the number of piles may be greater than two times the vertical load divided by the allowable pile load, and it may be necessary to use a large spacing between piles or to concentrate piles near the periphery of the footing so as to increase resistance to overturning moments.

4. Determine the maximum pile load for the trial pile group based upon the following assumptions:
   (a) The footing is perfectly rigid.
   (b) The tops of piles are hinged to the footing so that no bending moment is transferred from the footing to a pile.
   (c) Piles are short elastic columns so that deformations and the stress distribution are linear.

These assumptions permit the use of the following elastic equation for the calculation of pile loads: (See Figure B)

\[
Q_m = \frac{F_v}{r} \pm \frac{M_{xy}}{\Sigma x^2} \pm \frac{M_{xy}}{\Sigma y^2}
\]

(Eq. B)

**FIGURE B—DEFINITION OF TERMS USED FOR CALCULATION OF PILE LOADS FOR PILE GROUP SUBJECTED TO VERTICAL LOAD PLUS MOMENT**
where $Q_m = \text{vertical load on any given pile } m$

$F_v = \text{total vertical load acting at the centroid of the pile group}$

$r = \text{number of piles}$

$M_x = \text{moment with respect to the } X \text{ axis through the centroid of the pile group}$

$M_y = \text{moment with respect to the } Y \text{ axis through the centroid of the pile group}$

$x = \text{distance of pile from } Y \text{ axis}$

$y = \text{distance of pile from } X \text{ axis}$

(For definition of above terms, see Figure B)

5. If the maximum pile load determined in Step 4 differs from the allowable pile load, change the number of piles and/or the arrangement of the piles, and if necessary change the size of the footing. In some cases, it may be necessary to change the size of pile. Recalculate the maximum pile load and adjust the pile pattern until the maximum pile load is approximately equal to the allowable pile load.

6. Determine the lateral load capacity of the pile group and compare to the actual horizontal loads applied to the pile group.

7. If the actual horizontal loads exceed the lateral load capacity of the pile group, batter sufficient piles so that the lateral load capacity of the pile group plus the horizontal component of the battered piles equals or exceeds the actual horizontal loads.

8. When designing friction piles in fine grained materials, determine if the allowable axial load per pile is reduced due to group action.

9. Check settlement of the pile group if critical to design of the structure.

Equations, figures and tables referred to in the examples may be found in the United States Steel Corporation’s “Highway Structures Design Manual,” Volume I, Chapter 10.

**Example 1—Stub Abutment for Short Span Steel Bridges**

This type of abutment is applicable for short span bridges with a shallow backwall. A single row of vertical piles is used with one pile beneath each stringer. The pile design generally is governed by the lateral displacement of the piles. If horizontal loads are large or the soil quality is poor, it may be necessary to batter some piles, to increase the pile size or to increase the number of piles. Piles for this abutment are designed as friction piles in coarse grained soil (Part A) and as friction piles in fine grained soil (Part B).
STRINGER REACTIONS
The maximum stringer reactions are assumed to be:

Dead load = 36 kips
Live load = 53 kips

In accordance with AASHO, Section 1.2.12, impact is not applied to piles below the ground surface.

Since the bearings are an expansion type, the maximum horizontal force that can be applied to the abutment by the stringers is equal to the horizontal force necessary to overcome friction in the bearings. This friction force varies with the type of expansion bearing. For this example, assume that the friction force is 10% of the dead load reaction.

PART A—SINGLE ROW OF PILES IN COARSE GRAINED MATERIAL
Abutment Dead Load (Interior Pile)

Determine vertical dead load force and dead load moment for the interior piles spaced at 9'3" center to center.

<table>
<thead>
<tr>
<th>Loading</th>
<th>$F_v$ (kips)</th>
<th>Arm about Φ Brg. (ft)</th>
<th>Moment about Φ Brg (kip-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Left</td>
<td>Right</td>
</tr>
<tr>
<td>1) 2.0 x 2.83 x 0.15 x 9.25</td>
<td>7.85</td>
<td>0.33</td>
<td>—</td>
</tr>
<tr>
<td>2) 1.0 x 3.0 x 0.15 x 9.25</td>
<td>-4.16</td>
<td>1.25</td>
<td>—</td>
</tr>
<tr>
<td>3) 1.5 x 0.5 x 0.15 x 9.25</td>
<td>1.05</td>
<td>2.00</td>
<td>—</td>
</tr>
<tr>
<td>4) 1.0 x 0.5 x 0.15 x 9.25</td>
<td>0.35</td>
<td>1.92</td>
<td>—</td>
</tr>
<tr>
<td>Total</td>
<td>13.41</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

EARTH PRESSURE

Assume soil properties of the abutment backfill to be the same as the original soil properties.

Active earth pressure,

$$P_a = K_a \gamma h$$

$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi} = \frac{1 - \sin 30^o}{1 + \sin 30^o} = \frac{1 - 0.500}{1 + 0.500} = 0.333$$

$$P_a = 110 \text{ pcf} \times 0.333 \times 4.0 = 147 \text{ psf}$$

$$P_a = 0.147 \frac{(4.0)}{2} = 0.294 \text{ ksf}$$

Surcharge Load

Section 1.2.19 of the AASHTO specifications states that live load surcharge is not considered where an adequately designed reinforced concrete approach slab is provided. However, the weight of the slab is supported by the backfill and is considered as dead load surcharge. This load is equated to an equivalent height of soil.

$$h_s = \frac{\text{wt. of approach slab}}{\text{wt. of soil}} = \frac{1.0 \times 0.15}{0.11} = 1.36 \text{ ft}$$

$$\therefore P_s = 110 \times 0.333 \times 1.36 = 49.8 \text{ psf} \approx 0.050 \text{ ksf}$$

and $P_s = 0.050 \times 4.0 = 0.200 \text{ ksf}$

Earth Pressure for Interior Piles at 9'-3' Center to Center

Forces about Top of Piles

<table>
<thead>
<tr>
<th>Force</th>
<th>$H$ (kips)</th>
<th>Arm (ft)</th>
<th>$M$ (kip-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth Pressure</td>
<td>-0.294 x 9.25</td>
<td>2.72</td>
<td>0.33</td>
</tr>
<tr>
<td>Surcharge</td>
<td>-0.200 x 9.25</td>
<td>1.85</td>
<td>1.0</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>4.57</td>
<td></td>
</tr>
</tbody>
</table>

Passive soil resistance in front of the abutment should not be considered.

FORCES AT TOP OF INTERIOR PILES

Loads for AASHTO Groups I and III are tabulated. The design of stub abutments is governed by Group I loading for axial pile capacity and Group III loading for lateral pile capacity.
Group I—Dead Load, Live Load and Earth Pressure

<table>
<thead>
<tr>
<th>Loading</th>
<th>$F_v$ (kips)</th>
<th>Transverse Direction</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$H$ (kips)</td>
</tr>
<tr>
<td>D.L. Superstructure</td>
<td>36.0</td>
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<td>D.L. Abutment</td>
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<tr>
<td>L.L.</td>
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</tr>
<tr>
<td>Earth Pressure</td>
<td></td>
<td>4.57</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>102.4</strong></td>
<td><strong>4.57</strong></td>
</tr>
</tbody>
</table>

Group III—Dead Load, Live Load, Earth Pressure and Friction

<table>
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<tr>
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<tr>
<td></td>
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<td>$H$ (kips)</td>
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<tr>
<td>D.L. Superstructure</td>
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<td>D.L. Abutment</td>
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<td>L.L.</td>
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<tr>
<td>Earth Pressure</td>
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<tr>
<td>Friction</td>
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<td><strong>Total</strong></td>
<td><strong>102.4</strong></td>
<td><strong>8.17</strong></td>
</tr>
</tbody>
</table>

PILE DESIGN

1. Size

Piles of relatively low capacity generally are sufficient to support pile cap abutments. However, for a given load per pile, a pile with greater perimeter and greater end area will develop the load with less depth of penetration.

For example, compare an HP 10×42 pile with an HP 8×36 pile. The HP 10×42 pile has approximately 25% more perimeter and 56% more end area with an increase of only 17% in weight. The HP 10×42 pile also offers greater resistance to lateral loads.

Use HP 10×42 piles for this design example.

**Pile Properties**

- $A_s=12.4$ in.$^2=0.086$ ft$^2$
- $d=9.72$ in. $=0.810$ ft
- $b_f=10.078$ in. $=0.839$ ft
- $P=2(d+b_f)=3.30$ ft
- $A=db_f=0.680$ ft$^2$
- $S_s=43.4$ in.$^3$
- $l_s=211$ in.$^4$

2. Depth of penetration

The ultimate bearing capacity of a pile ($Q_u$) is equal to the sum of the end bearing ($Q_{ue}$) plus the skin friction ($Q_{us}$).

$$Q_u = Q_{ue} + Q_{us}$$

a. End Bearing Component

Assume that the depth of pile penetration is at least 10 times the section
depth \( b \) of the pile and the ground water level is at least a distance of 1.5 \( b \) below the pile tip.

\[ Q_{us} = (\frac{1}{2} \gamma dN_\gamma + K_b \gamma_s D N_\phi) A \]  
\[ \text{(Eq. 2)} \]

Using Fig. 8 and \( \phi = 30^\circ \) determine values for \( N_\gamma, N_\phi \) and \( K_b, \)

\[ N_\gamma = 65 \quad N_\phi = 82 \quad \text{and} \quad K_b = 0.40 \]

\[ Q_{us} = [(\frac{1}{2} \times 0.110 \times 0.810 \times 65) + (0.40 \times 0.110 \times 82D)]0.680 \]

\[ = (2.93 + 3.61D)0.68 = 1.99 + 2.46D \]

b. Skin Friction Component

\[ Q_{us} = \frac{1}{2} p K_b \gamma_s D^2 (\tan \delta) \]  
\[ = \frac{1}{2} \times 3.30 \times 0.40 \times 0.110 D^2 \times \tan 20^\circ \]

\[ = 0.026D^2 \]  
\[ \text{(Eq. 4)} \]

c. Required Penetration

Group I loading governs, \( Q_m = 102.4 \) k/pile

\[ F_s \times Q_m = Q_u = Q_{us} + Q_{us} \]

Use a safety factor = 2.5

\[ 2.5 \times 102.4 = 1.99 + 2.46D + 0.026D^2 \]

\[ 256.0 - 1.99 = 2.46D + 0.026D^2 \]

\[ D^2 + 94.6D + (47.3)^2 = 9770 + (47.3)^2 \]

\[ (D + 47.3)^2 = 12007 \]

\[ D + 47.3 = 109.6 \]

\[ D = 62.3; \text{ use } 63.0 \text{ ft} \]

The depth of pile penetration should be verified by either a load test or careful observance of the resistance to driving.

Check assumption that \( D \geq 10b \)

\[ 63.0 > 10 \times 0.810 = 8.10 \text{ ft} \]

If \( D \) had been less than \( 10b \), depth of penetration would be recalculated using Eq. 3. Also, since the ground water level is \( > 1.5d \) below the pile tip, it will not influence the depth of pile penetration.

**Lateral Load Capacity**

Estimates of lateral load capacities are calculated using a simplified method by Broms. Since the piles are embedded only 12 inches into the concrete cap and the cap is relatively free to rotate, consider the piles as being “free-headed.” Also consider that the soil is disturbed down to the bottom of the pile cap and neglect passive soil pressure acting on the pile cap.

**Determine if Piles are “Long,” “Short” or “Intermediate”**

From Fig. 16, estimate the modulus of subgrade reaction, \( n_h \), for a loose to medium dense sand to be 28 k/ft².

\[ \eta = \frac{n_h}{\sqrt{E_I}} \]  
\[ \text{(Eq. 21)} \]

\[ \eta = \sqrt{\frac{28.0}{29 \times 10^3 \times 211}} = 0.000659 = 0.230 \text{ ft}^{-1} \]

\[ \eta D = 0.230 \times 63.0 = 14.49 \]

Since \( \eta D \) is greater than 4.0, consider the pile as being “long.”

II/11.8  

7/73
Calculate Effective Height of Lateral Load

Group III loading governs

\[ H = 8.17 \text{ kips} \]

\[ M = 4.21 \text{ kip-ft or } -4.21 \text{ kip-ft} \]

\[ e = 1.00 - \frac{4.21}{8.17} = 1.00 - 0.52 \]

\[ e = 0.48 \text{ ft} \]

Find \( P_u \) from Fig. 18 by substituting values for the following quantities:

\[
\frac{P_u}{K_s b^2 \gamma_s} = \frac{M_{\text{plastic}}}{K_s b^2 \gamma_s} \frac{e}{b} \quad \text{(Eq. 23)}
\]

Since the pile is loaded both axially and laterally, the reduced plastic moment capacity is dependent upon the ratio of axial load to the plastic axial capacity of the pile section.

Determine if \( Q_m/Q_v \) is equal to or less than 0.15

\[
\frac{Q_m}{Q_v} = \frac{102.4}{12.4 \times 36.0} = 0.229 > 0.15
\]

\[ M_s = 1.18 M_{\text{plastic}} (1 - Q_m/Q_v) \quad \text{(Eq. 35)} \]

\[ = 1.18 M_{\text{plastic}} (1 - 0.229) = 0.910 M_{\text{plastic}} \]

\[ M_{\text{plastic}} = n_s f_s s \quad \text{(Eq. 24)} \]

Since the pile is loaded in the direction of maximum moment resistance of the pile, \( n_s \) is equal to 1.1.

\[ M_s = 0.910 \times 1.1 \times 36.0 \times 43.4 = 1564 \text{ kip-in.} = 130.3 \text{ kip-ft} \]

\[ K_s = \frac{1 + \sin \phi}{1 - \sin \phi} = \frac{1 + 0.500}{0.500} = 3.00 \]

\[ M_s \frac{130.3}{K_s b^2 \gamma_s} = \frac{3.00 (0.839)^2 \times 0.110}{797} \]

\[ e = \frac{0.48}{0.839} = 0.572 \]

From Fig. 18:

\[
\frac{P_u}{K_s b^2 \gamma_s} = 130
\]

\[ P_u = 130 \times 3.00 (0.839)^2 \times 0.110 \]

\[ = 25.3 \text{ kips} \]

Using a safety factor of 2.5, the allowable lateral load is:

\[ P_u = \frac{25.3}{2.5} = 10.1 \text{ kips} \]

For Group III loading, a stress increase of 25% is allowed.

\[ H = 8.17 \text{ kips} < 10.1 \times 1.25 = 12.6 \text{ kips} \]

Thus, the HP 10 × 42 pile section is adequate to carry the lateral design load.
LATERAL DISPLACEMENT

Estimates of lateral displacements are calculated using a simplified method by Broms. The lateral load capacity generally is limited by the permissible lateral displacement due to the working load. The lateral displacements, for permanent and temporary lateral forces, at the ground surface are estimated using Fig. 19 and substituting values for the following quantities:

\[
\frac{\Delta a(EI)^{2/5}(n_s)^{2/5}}{P_s D} = \eta D; \quad \frac{e}{D}
\]

(Eq. 25)

\[
\eta D = 14.49 \quad \text{and} \quad \frac{e}{D} = \frac{0.48}{63.0} = 0.0076
\]

For a "long" pile, the lateral displacement is independent of the pile embedment. Since \( \eta D = 14.5 \) corresponding to \( D = 63 \) ft is beyond the range of Fig. 19, calculate the lateral displacement for \( \eta D' = 10 \).

For \( \eta D' = 10 \),

\[
D' = \frac{10}{0.230} = 43.5 \text{ ft} \quad \text{and} \quad \frac{e}{D'} = \frac{0.48}{43.5} = 0.0110
\]

From Fig. 19:

\[
\Delta a = \frac{(EI)^{2/5}(n_s)^{2/5}}{P_s D'} = 0.31
\]

\[
\Delta a = \frac{0.31(43.5)(12)P_s}{(29 \times 10^3 \times 211)^{2/5}} = \frac{161.8P_s}{600 \times 3.80}
\]

\[
= \frac{0.0710 P_s}{}
\]

**Group I Loading—Earth Pressure**

\[ P_s = H = 4.57 \text{ kips} \]

\[ \Delta a = 0.0710 \times 4.57 \]

\[ = 0.324" = \frac{7}{2}\text{ in.} \]

**Group III Loading—Earth Pressure and Friction**

\[ P_s = H = 8.17 \text{ kips} \]

\[ \Delta a = 0.0710 \times 8.17 \]

\[ = 0.580" = \frac{7}{2}\text{ in.} \]

Thus a permanent lateral displacement of \( \frac{7}{2}\text{ in.} \) will occur with an additional lateral movement of \( \frac{7}{2}\text{ in.} \) occurring with the application of the friction force. Where accurate determinations of these values are required, field tests should be made.

SETTLEMENT AT TOP OF PILE

Settlement will result from settlement of the pile tip plus the elastic deformation of the pile. The settlement of the pile tip results from that portion of the load which reaches the tip while the elastic deformation is based on the average of the load at the top and tip of the pile.

In the analysis for the depth of penetration, the ultimate end bearing component is:

\[
Q_s = 1.99 + 2.46D = 1.99 + (2.46 \times 63.0) = 157.0 \text{ kips}
\]

\[
\% \text{ of ultimate load at the pile tip} = \frac{157.0}{2.5 \times 102.4} = 0.613 = 61.3\%
\]

The approximate working load at the pile tip:

\[
Q_w = 102.4 \times 0.613 = 62.8 \text{ kips}
\]
Settlement of Pile Tip
Estimate from Fig. 11 using the average settlement curve.
load/ultimate load = 62.8/2.5 x 102.4 = 0.245
\[ \Delta_r = 0.08 \text{ in.} \]

Elastic Deformation
Load @ Top of piles = 102.4 kips
Load @ Tip of piles = 62.8
Average pile load = 82.6
\[ \Delta_t = \frac{QD}{A_r E} = \frac{82.6 \times 63.0 \times 12}{12.4 \times 29 \times 10^3} = 0.174 \text{ in.} \]
\[ \therefore \text{Total Settlement at Top of Pile} = 0.256' \approx \frac{1}{4} \text{ in.} \]

PART B—SINGLE ROW OF PILES IN FINE-GRAINED MATERIAL
EARTH PRESSURE

Active earth pressure, $P_a = K_s \gamma h$

\[
K_s = \frac{1 - \sin \phi}{1 + \sin \phi} = \frac{1 - \sin 33^\circ}{1 + \sin 33^\circ} = 1 - 0.545
\]

$K_s = 0.294$

$P_a = 0.294 \times 0.110 \times 4.0 = 0.129 \text{ ksf}$

$P_a = 0.129(4.0)/2 = 0.258 \text{ k/ft}$

Surcharge Load

Section 1.2.19 of the AASHO Specifications states that live load surcharge is not considered where an adequately designed reinforced concrete approach slab is provided. However, the weight of the slab is supported by the backfill and considered as dead load surcharge. This load is equated to an equivalent height of soil.

\[
h_s = \frac{\text{wt. of approach slab}}{\text{wt. of soil}} = \frac{1.0 \times 0.15}{0.110} = 1.36 \text{ ft}
\]

\[
P_s = 0.110 \times 0.294 \times 1.36 = 0.044 \text{ ksf}
\]

$P_s = 0.044 \times 4.0 = 0.176 \text{ k/ft}$

Earth Pressure for Interior Piles (9 ft-3 in. Center to Center)

Forces about Top of Piles

<table>
<thead>
<tr>
<th>Force</th>
<th>$H$ (kips)</th>
<th>Arm (ft)</th>
<th>$M$ (kip-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth Pressure—0.258 × 9.25</td>
<td>2.40</td>
<td>0.33</td>
<td>0.79</td>
</tr>
<tr>
<td>Surcharge—0.176 × 9.25</td>
<td>1.63</td>
<td>1.00</td>
<td>1.63</td>
</tr>
<tr>
<td>Total</td>
<td>4.03</td>
<td></td>
<td>2.42</td>
</tr>
</tbody>
</table>

Passive soil resistance in front of the abutment should not be considered.

STRINGER REACTIONS—Refer to Part A.

ABUTMENT DEAD LOAD (Interior Pile)—Refer to Part A.

FORCES AT TOP OF INTERIOR PILES

Loads for AASHO Group I and III are tabulated. The design of stub abutments is governed by Group I loading for axial pile capacity and Group III loading for lateral pile capacity.

Group I—Dead Load, Live Load and Earth Pressure

<table>
<thead>
<tr>
<th>Loading</th>
<th>$F_v$ (kips)</th>
<th>Transverse Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$H$ (kips)</td>
<td>$M$ (kip-ft)</td>
</tr>
<tr>
<td>D.L. Superstructure</td>
<td>36.0</td>
<td>—</td>
</tr>
<tr>
<td>D.L. Abutment</td>
<td>13.4</td>
<td>—</td>
</tr>
<tr>
<td>L.L.</td>
<td>53.0</td>
<td>—10.57</td>
</tr>
<tr>
<td>Earth Pressure</td>
<td>—</td>
<td>4.03 + 2.42</td>
</tr>
<tr>
<td>Total</td>
<td>102.4</td>
<td>4.03 − 8.15</td>
</tr>
</tbody>
</table>
Group III—Dead Load, Live Load, Earth Pressure and Friction

<table>
<thead>
<tr>
<th>Loading</th>
<th>( F_v ) (kips)</th>
<th>Transverse Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>D.L. Superstructure</td>
<td>36.0</td>
<td>(-)</td>
</tr>
<tr>
<td>D.L. Abutment</td>
<td>13.4</td>
<td>(-)</td>
</tr>
<tr>
<td>L.L.</td>
<td>53.0</td>
<td>(-)</td>
</tr>
<tr>
<td>Earth Pressure</td>
<td>(-)</td>
<td>4.03</td>
</tr>
<tr>
<td>Friction</td>
<td>(-)</td>
<td>3.60</td>
</tr>
<tr>
<td>Total</td>
<td>102.4</td>
<td>7.63</td>
</tr>
</tbody>
</table>

PILE DESIGN

1. Size—For discussion refer to Part A. Use HP 10 \(\times\) 42 Pile.

2. Depth of Penetration

For H-piles, the end bearing component \((Q_u)\) of the ultimate bearing capacity \((Q_u)\) of a pile is negligible and the ultimate bearing capacity is equal to the skin friction component \((Q_s)\). The lesser of the \(Q_u\) values found by equations 10, 11 and 12 is used to determine the depth of pile penetration.

   a. Cohesion around the net perimeter

   \[
   Q_u = 2(d + b_f) cD = 2(0.810 + 0.839)1.0D = 3.30D \quad (\text{Eq. 10})
   \]

   b. Adhesion around the entire perimeter

   \[
   Q_u = 2(d + 2b_f)c_s D \quad (\text{Eq. 11})
   \]

   From Fig. 9, for \(c = 1.0\), \(c_s = 0.73\)

   \[
   c_s = 0.73 \times 1.0 = 0.73
   \]

   \[
   Q_u = 2(0.810 + 2 \times 0.839)0.73D = 3.63D \]

   c. Adhesion to the flange and cohesion across the web opening

   \[
   Q_u = 2(db + b_f c_s)D = 2[(0.810 \times 1.0) + (0.839 \times 0.73)]D = 2.84D \quad \geq 2.84D \quad \text{governs}
   \]

   Use safety factor = 2.5

   Group I loading governs, \(Q_m = 102.4\) k/pile

   \[
   Q_u = 2.5 Q_m = 2.5 \times 102.4 = 2.84D
   \]

   \[
   D = 90.1 \text{ ft} \quad \text{Use 90.0 ft}
   \]

The depth of pile penetration should be verified by either a load test, driving records, or pile capacities and performances of existing pile installations at nearby sites.
LATERAL LOAD CAPACITY

Estimates of lateral load capacities are calculated using a simplified method by Broms. Since the piles are embedded only 12 inches into the concrete cap and the cap is relatively free to rotate, consider the piles as being “free-headed.” Also consider that the soil is disturbed down to the bottom of the pile cap and neglect passive soil pressure acting on the pile cap.

Determine if Piles are “Long” or “Short”

\[
\beta = \sqrt[4]{\frac{K}{4EI}} \quad \text{(Eq. 26)}
\]

\[
K = \frac{160mc}{L} \quad \text{(Eq. 27)}
\]

\[
\therefore \beta = \sqrt[4]{\frac{160mc}{4EI}}
\]

for \( c = 1.0 \) use \( \bar{m} = 0.34 \)

\[
\beta = \sqrt[4]{\frac{160 \times 0.34 \times 1.0 \times 144}{4 \times 29 \times 10^3 \times 211}} = \sqrt[4]{0.000320}
\]

\[
= 0.134 \text{ ft}^{-1}
\]

\[
\beta D = 0.134 \times 90.0 = 12.06
\]

Since \( \beta D \) is greater than 2.25, the pile is considered as being “long.”

Calculate Effective Height of Lateral Load

Group III loading governs

\[
H = 7.63 \text{ kips}
\]

\[
M = -4.55 \text{ kip-ft}
\]

\[
e = 1.00 - \frac{4.55}{7.63} = 1.00 - 0.60
\]

\[
e = 0.40 \text{ ft}
\]

Determine \( P_u \) from Fig. 21 by substituting values for the following quantities:

\[
P_{uk} \quad M_s \quad \frac{e}{b} \quad \text{(Eq. 29)}
\]

From Part A, \( M_s = 137.1 \text{ kip-ft} \)

\[
M_s = \frac{130.3}{cb^3} = \frac{1.0(0.839)^3}{1.0(0.839)^3} = 220
\]

\[
\frac{e}{b} = \frac{0.40}{0.839} = 0.48
\]

From Fig. 21: \( P_{uk} = 51 \)

\[
P_u = 51 \times 1.0(0.839)^3 = 35.9 \text{ kips}
\]

Using a safety factor = 2.5

\[
P_s = \frac{35.9}{2.5} = 14.4 \text{ kips}
\]

For Group III loading, a stress increase of 25% is allowed.

\[
H = 7.63 < 14.4 \times 1.25 = 18.0 \text{ kips}
\]

Thus, the HP 10x42 pile section is adequate to carry the lateral design load.
LATERAL DISPLACEMENT

Estimates of lateral displacements are calculated using a simplified method by Broms. The lateral displacement of long piles \((\beta D > 2.25)\) is independent of the pile embedment. Lateral displacement at the ground surface, at working lateral loads less than one-half the ultimate values, may be approximated using Fig. 22 and substituting values for the following quantities:

\[
\frac{\Delta_s k \delta D}{P_a}; \quad \beta D; \quad \frac{e}{D}
\]

(Eq. 30)

\[
\beta D = 12.06 \quad \text{and} \quad \frac{e}{D} = \frac{0.40}{90.0} = 0.0044
\]

For a long pile, the lateral displacement is independent of the pile embedment. Since \(\beta D = 12.06\) corresponding to \(D = 90\) ft is beyond the range of Fig. 22, calculate the lateral displacement for \(\beta D' = 5\).

\[
D' = \frac{5.0}{0.134} = 37.3' \quad \text{and} \quad \frac{e}{D} = \frac{0.40}{37.3} = 0.0107
\]

From Fig. 22:

\[
\frac{\Delta_s k \delta D}{P_a} = 10.4
\]

\[
k = \frac{160mc}{b} = \frac{160 \times 0.34 \times 1.0}{0.839} = 64.8 \text{ kcf}
\]

\[
\Delta_s = \frac{10.4P_a}{64.8 \times 0.839 \times 37.3} = 0.0051P_a
\]

Group I Loading—Earth Pressure

\[P_a = H = 4.03 \text{ kips}
\]

\[\Delta_s = 0.0051 \times 4.03 = 0.0206' \times 12 = 0.247' = \frac{1}{4} \text{ in.}
\]

Group III Loading—Earth Pressure and Friction

\[P_a = H = 7.63 \text{ kips}
\]

\[\Delta_s = 0.0051 \times 7.63 = 0.0389' \times 12 = 0.467' = \frac{3}{8} \text{ in.}
\]

Lateral displacement in fine-grained soils increase with time due to consolidation and creep of the soil. The approximate long term displacement may be found by reduction of the \(k\) value to \(\frac{1}{4}\) that for a clay with a cohesive strength between 0.5 and 1.5. For long term displacement use \(H\) resulting from earth pressure and surcharge.

\[
k = \frac{160mc}{b} \times \frac{1}{4} = 64.8 \times \frac{1}{4} = 16.2 \text{ kcf}
\]

\[
\beta = \sqrt[4]{\frac{16.2 \times 0.839 \times 144}{4 \times 29 \times 10^2 \times 211}} = \sqrt[4]{0.000080} = 0.095 \text{ ft}^{-1}
\]

\[
\beta D = 0.095 \times 90.0 = 8.55
\]

Use Fig. 22 and \(\beta D = 5.0\)

\[
D' = \frac{5.0}{0.059} = 84.75 \quad e = \frac{0.40}{84.75} = 0.0047
\]

\[
\frac{\Delta_s k \delta D}{P_a} = 10.3
\]

\[
\Delta_s = \frac{10.3P_a}{16.2 \times 0.839 \times 52.6} = 0.0144P_a
\]
Long term displacement due to earth pressure

\[ P_e = H = 4.03 \text{ kips} \]

\[ \Delta_e = 0.0144 \times 4.03 \]
\[ = 0.0580 \times 12 \text{ in.} = 0.696 \text{ in.} \]

Thus a lateral displacement of \( \frac{3}{4} \) in. will occur at the time of initial earth pressure loading. Due to consolidation and creep of the soil, the lateral displacement due to earth pressure will increase to \( \frac{1}{4} \) in. An additional \( \frac{1}{4} \) in. lateral movement will occur with application of the friction force. Where accurate determination of these values is required, field tests should be made.

SETTLEMENT AT TOP OF PILE

Consolidation Settlement

Settlement is computed by a consolidation analysis for a soil layer starting at the tip of the pile and extending upward a distance of \( D/3 \). The assumed load distribution for computing settlement is given in Fig. 13. The amount of settlement is determined by the following equation:

\[ \Delta_s = \frac{C_s D/3}{1 + e_s} \log \left( \frac{q_e + \Delta_s}{q_e} \right) \]

where 
- \( q_e \) = Existing Effective Overburden pressure at a height \( D/6 \) above the pile tip.
- \( \Delta_s \) = Pressure due to \( Q \) at a height \( D/6 \) above the pile tip.
- \( e_s \) = Void Ratio
- \( C_s \) = Compression Index

\[
\begin{align*}
\text{D} & = 90.0' \\
\frac{3}{4} D & = 60.0' \\
\frac{1}{2} D & = 30.0' \\
\frac{1}{8} D & = 15.0' \\
21.20' \text{ Dia.} & \quad 38.52' \text{ Dia.} \\
\end{align*}
\]

Determine the existing overburden pressure at a height of \( D/6 \) above the pile tip. The height of overburden can be approximated by the distance from the original ground surface to the point where the pressure is to be found.

\[ H = (4.0 + 90.0 - 15.0) = 79.0 \text{ ft} \]

\[ q_e = 78.5 \times 0.12 = 9.48 \text{ ksf} \]
Determine change in pressure due to pile load at a height of $D/6$ above the pile tip.

$$\Delta_s = \frac{Q}{A_H}$$

where $A_H$ = area at section located at a height $D/6$ above the pile tip.

$$\Delta_s = \frac{102.4}{\pi (32.89)^2} = 0.12 \text{ ksf}$$

Since the piles are spaced 9'-3" on centers, the pressure distribution for the piles will overlap almost 100%. Therefore, the total change in pressure will be approximately $2 \times \Delta_s = 2 \times 0.12 = 0.24 \text{ ksf}$

The approximate settlement is:

$$\Delta_r = \frac{0.08(30.0)}{1 + 1.0} \left( \log \frac{9.42 + 0.24}{9.42} \right) = 1.20 (\log 1.025)$$

$$= 1.00 (0.01072) = 0.0107 = 0.128 \text{ in.}$$

$$\Delta_r = \frac{1}{2} \text{ in.}$$

**Settlement due to Elastic Deformation**

The average pile load, $Q' = \frac{102.4}{2} = 51.2 \text{ kips}$

$$\Delta_s = \frac{Q' D}{A_s E} = \frac{51.2 \times 90.0 \times 12}{12.4 \times 29 \times 10^3} = 0.154 \text{ in.}$$

Total settlement $= 0.128 + 0.154 = 0.282 \leq \frac{1}{2} \text{ in.}$

**Example 2—Pile Bents**

Pile bents provide an economical method of support for short span steel bridges. One pile is placed under each roadway stringer, and the tops of piles are tied together by steel members or a concrete cap. This gives a rigid frame to carry horizontal loads applied parallel to the pile bent.

Roadway stringers are attached to the pile bents by bearing devices that permit rotation of the stringers, but prohibit longitudinal movement of the stringers with respect to the bents. When roadway stringers expand or contract due to temperature changes, the top of the pile bents move with the stringers. Since pile bents are relatively flexible, this movement causes only small loads and moments in the piles.

An interior pile bent of a four span continuous unit will be designed. The ends of the four span continuous unit are supported by abutments that longitudinally are much stiffer than the pile bents. Therefore, it will be assumed that all longitudinal force due to traction and wind are transferred to the abutments.

Pile bents may be built with or without bracing. Bracing reduces the bending stresses in the piles. This bent will be analyzed without bracing and with bracing as shown on the elevation view of the bent.
If the superstructure of this example had been longer (over 300 feet) or if a series of continuous spans had been used, the roadway stringers would still be fixed to each pile bent but expansion bearings would be used at the abutments. Each pile bent would then be designed for longitudinal traction and wind forces in addition to temperature forces. Larger piles would then be required for the bents with fixed bearings. An analysis of a pile bent subjected to longitudinal traction and wind forces would be similar to that shown in this example for temperature forces.

**LOADS**

Determine total vertical and horizontal loads at the top of the pile bent for Groups I, II and III loading. Loading combinations are given in AASHO Specification, Section 1.2.22.

Expansion and contraction of the superstructure will place additional longitudinal loads on the piles. This additional horizontal force is dependent upon the pile section properties and cannot be determined until the pile section is chosen, and the point of fixity is determined. The longitudinal load due to temperature change does not affect the axial load on the pile, but does affect the bending stress in the pile.
### Group Ia—Dead Load+Live Load

<table>
<thead>
<tr>
<th>Loading</th>
<th>( F_v ) (kips)</th>
<th>Transverse Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dead Load</td>
<td>308</td>
<td>(-)</td>
</tr>
<tr>
<td>Live Load</td>
<td>228</td>
<td>(-)</td>
</tr>
<tr>
<td>Total</td>
<td>536</td>
<td>(-)</td>
</tr>
</tbody>
</table>

### Group Ib—Dead Load+Live Load+Impact

<table>
<thead>
<tr>
<th>Loading</th>
<th>( F_v ) (kips)</th>
<th>Transverse Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dead Load</td>
<td>308</td>
<td>(-)</td>
</tr>
<tr>
<td>Live Load</td>
<td>228</td>
<td>(-)</td>
</tr>
<tr>
<td>Impact</td>
<td>65</td>
<td>(-)</td>
</tr>
<tr>
<td>Total</td>
<td>601</td>
<td>(-)</td>
</tr>
</tbody>
</table>

### Group II—Dead Load+Wind

<table>
<thead>
<tr>
<th>Loading</th>
<th>( F_v ) (kips)</th>
<th>Transverse Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dead Load</td>
<td>308</td>
<td>(-)</td>
</tr>
<tr>
<td>Wind on Structure</td>
<td>(-)</td>
<td>10.0</td>
</tr>
<tr>
<td>Total</td>
<td>308</td>
<td>10.0</td>
</tr>
</tbody>
</table>

### Group III—Dead Load+Live Load+Impact+Traction+0.3 Wind+Wind on Live Load

<table>
<thead>
<tr>
<th>Loading</th>
<th>( F_v ) (kips)</th>
<th>Transverse Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dead Load</td>
<td>308</td>
<td>(-)</td>
</tr>
<tr>
<td>Live Load</td>
<td>328</td>
<td>(-)</td>
</tr>
<tr>
<td>Impact</td>
<td>65</td>
<td>(-)</td>
</tr>
<tr>
<td>Traction</td>
<td>(-)</td>
<td>(-)</td>
</tr>
<tr>
<td>Wind on Live Load</td>
<td>(-)</td>
<td>5.0</td>
</tr>
<tr>
<td>0.3 Wind on Structure</td>
<td>(-)</td>
<td>3.0</td>
</tr>
<tr>
<td>Total</td>
<td>601</td>
<td>8.0</td>
</tr>
</tbody>
</table>

### PILE LOADS

Calculate the vertical pile loads at the top of the pile bent:

\[
Q_m = \frac{F_v}{r} \pm \frac{M_{vp}}{2r^2} \quad \text{(Eq. A)}
\]
where \( r = 4 \)
\[
\Sigma x^2 = 2(1)[(4.625)^2 + (13.875)^2]
\]
\[
= 427.8 \text{ pile-ft}^2
\]

**Group Ia Loading**

Vertical component of \( Q = \frac{536}{4} = 134.0 \text{ kips} \)

Since the end piles are battered 1 1/2 horizontal : 12 vertical, the maximum axial pile load is
\[
Q = 134.0 \cdot \frac{\sqrt{(12.0)^2 + (1.5)^2}}{12} = 134.0 \cdot \frac{12.09}{12} = 135.0 \text{ kips}
\]

**Group Ib Loading**

Vertical component of \( Q = \frac{601}{4} = 150.3 \text{ kips} \)

The maximum axial pile load is:
\[
Q = 150.3 \cdot \frac{12.09}{12} = 151.4 \text{ kips}
\]

**Group II Loading**

Maximum vertical component of \( Q = \frac{308}{4} + \frac{20.0 \cdot 13.87}{427} = 77.0 + 0.7 = 77.7 \text{ kips} \)

Minimum vertical component of \( Q = 77.0 - 0.7 = 76.3 \text{ kips} \)

The maximum axial pile load is:
\[
Q = 77.7 \cdot \frac{12.09}{12} = 78.3 \text{ kips}
\]

**Group III Loading**

Maximum vertical component of \( Q = \frac{601}{4} + \frac{51.0 \cdot 13.87}{427} = 150.3 + 1.7 = 152.0 \text{ kips} \)

Minimum vertical component of \( Q = 150.3 - 1.7 = 148.6 \text{ kips} \)

The maximum axial pile load is:
\[
Q = 152.0 \cdot \frac{12.09}{12} = 153.1 \text{ kips}
\]

Determine the approximate pile size for an allowable bearing stress of 9.0 ksi.

**Group Ia Loading**

Pile area = \( \frac{135.0}{9.0} = 15.0 \text{ in.}^2 \)

Try an HP 12×53 pile
Pile Properties

\[ A_r = 15.6 \text{ in.}^2 \]

\[ I_x = 394 \text{ in.}^4 \quad I_y = 127 \text{ in.}^4 \]

\[ r_x = 5.03 \text{ in.} \quad r_y = 2.86 \text{ in.} \]

\[ S_x = 66.9 \text{ in.}^3 \quad S_y = 21.1 \text{ in.}^3 \]

**DETERMINE THE DEPTH OF FIXITY**

Assume that the very loose organic silt strata offers no lateral resistance to pile movement. Therefore, the depth of fixity will begin at the top of the medium dense silty sand strata. The depth \( D \) is the point where the pile is considered restrained against rotation and is given by the following equation:

\[ D = 1.8 \sqrt{\frac{EI}{n_h}} \]  
(Eq. 36)

**Longitudinal Direction**

\[ D = 1.8 \sqrt{\frac{29 \times 10^3 \times 394}{28 \times 144}} = 1.8 \sqrt{2834} = 1.8(4.90) \]

\[ = 8.82 \text{ ft} \quad \text{Use 10.00 ft} \]

Embedment length of the pile = 38 ft > 3D = 30.0 ft; therefore, "fixity" can be assumed.

**Transverse Direction**

\[ D = 1.8 \sqrt{\frac{29 \times 10^3 \times 127}{28 \times 144}} = 1.8 \sqrt{613} = 1.8(3.91) \]

\[ = 7.04 \text{ ft} \]

**DETERMINE EFFECTIVE L/r OF PILES AND ALLOWABLE COMPRESSIVE STRESS**

**Longitudinal Direction**

In the longitudinal direction, the top of the pile bent is free to rotate and translate. For this condition, Table C1.8.1 (AISC Specification) recommends an effective length of 2.1l.

\[
\text{Effective length, } l = 2.1(23.82) \\
= 50.02 \text{ ft} \\
\text{Effective } l/r_x = \frac{50.02 \times 12}{5.03} = 119.3
\]

The allowable compressive stress is:

\[ F_x = 16,000 - 0.38 \left( \frac{l}{r_x} \right)^2 \]

\[ F_x = 16,000 - 0.38(119.3)^2 = 10,592 \text{ psi} = 10.59 \text{ ksi} \]
Transverse Direction—Unbraced

In the transverse direction, the top of the pile bent is fixed against rotation but free to translate. For this condition, Table C1.8.1 (AISC Specification) recommends an effective length of $1.2l$.

$$\text{Effective length, } l = 1.2(22.04) = 26.45 \text{ ft}$$

$$\text{Effective } l/r_a = \frac{26.45 \times 12}{2.86} = 111.0$$

The allowable compressive stress is:

$$F_a = 16,000 - 0.38\left(\frac{l}{r}\right)^2$$

$$F_a = 16,000 - 0.38(111.0)^2 = 11,319 \text{ psi} \geq 11.32 \text{ ksi}$$

Transverse Direction—Braced

$$\text{Effective length, } l = 1.2(12.04) = 14.45 \text{ ft}$$

$$\text{Effective } l/r_a = \frac{14.45 \times 12}{2.86} = 60.62$$

Allowable $F_a = 16,000 - 0.38(60.62)^2 = 14,604 \text{ psi} = 14.60 \text{ ksi}$

The compressive stress due to axial load is:

**Group I₂ Loading**

$$f_a = \frac{151.4}{15.6} = 9.71 \text{ ksi} < 10.59 \text{ ksi}$$

Additional stresses due to bending must also be considered.

**DETERMINE THE LONGITUDINAL FORCE DUE TO TEMPERATURE CHANGE**

Stringers under the roadway will expand or contract as the temperature changes. Each pile bent will move longitudinally an amount equal to the temperature movement of the stringers. The resulting longitudinal force $H_y$ in each pile is calculated by:

$$H_y = \frac{3EI\Delta}{L^3}$$
where \( E = \) modulus of elasticity of steel
\( I = \) moment of inertia of the pile
\( \Delta = \) movement of roadway stringer due to temperature change
\( L = \) length from top of pile to point of fixity

Design for a temperature drop of 78°

\[ \Delta = \text{coefficient of thermal expansion} \times \text{length} \times \text{temp. change} \]
\[ = 0.00000065 \times 50 \times 12 \times 78 = 0.304 \text{ in.} \]

\[ H_y = \frac{3EI\Delta}{L^2} = \frac{3 \times 29 \times 10^3 \times 394 \times 0.304}{(23.82)^2 \times 1728} = 0.45 \text{ kips} \]

CHECK STRESSES IN PILES DUE TO AXIAL LOAD PLUS BENDING—UNBRACED PILE BENT

In the transverse direction, the piles are considered fixed at the point of fixity and fixed at the top of bent with a point of inflection at mid-height.

Therefore, maximum \( M_y = \frac{22.04}{2} \) \( H_z = 11.02 \) \( H_z \) at top of pile and at point of fixity.

Check stresses at the top of pile. Buckling must be considered at the top of pile while the pile is laterally supported at the point of fixity.

Check stresses by the AISC Interaction Formula

\[ \frac{f_s}{F_s'} + \frac{C_{my}f_{by}}{(1 - \frac{f_s}{F_{ey}})F_{by}} + \frac{C_{my}f_{by}}{(1 - \frac{f_s}{F_{ey}})F_{by}} \leq 1 \]  
(Eq. 1.6-1a, p. 5-22, 1970 AISC Specs.)

where \( F_s' = \) axial stress that would be permitted if axial force alone existed
\( F_s = \) compressive bending stress that would be permitted if bending moments alone existed

\[ F_s' = \frac{12\pi^2E}{23\left(K\frac{l_b}{r_b}\right)^2} = \frac{149,000,000}{\left(K\frac{l_b}{r_b}\right)^2} \]

\( l_b = \) actual unbraced length in the plane of bending
\( r_b = \) corresponding radius of gyration
\( K = \) effective length factor in the plane of bending
\( f_s = \) computed axial stress
\( f_b = \) computed compressive bending stress at the point under consideration
\( C_m = 0.85 \) for compression members in frames subject to joint translation
Group II

\[ F_v = 308 \text{ kips} \quad H_s = 10.0 \text{ kips} \]

\[ Q_{\text{max}} = 78.3 \text{ k/pile} \]

\( H_s \) carried by batter = \((77.7 - 76.3) \frac{1.5}{12} = 0.18 \text{ kips} \)

Remaining \( H_s = \frac{10.00 - 0.18}{4} = 2.46 \text{ k/pile} \)

\[ M_s = 11.02 \times 2.46 = 27.1 \text{ kips} \]

\[ f_a = \frac{78.3}{15.6} = 5.02 \text{ ksi} \quad f_{by} = \frac{27.1 \times 12}{21.1} = 15.4 \text{ ksi} \]

\[ F'_{sy} = \left( \frac{K l_{by}}{r_{by}} \right)^2 = \frac{149,000,000}{111^2} = 12,090 \text{ psi} = 12.1 \text{ ksi} \]

Check \( \frac{f_a}{F_a} + \frac{C_m f_{by}}{\left(1 - \frac{f_a}{F'_{sy}}\right)F_s} < 1 \)

\[ 5.02 \frac{0.85 \times 15.4}{1.25 \times 10.59 + \frac{0.85 \times 15.4}{1.25 \times 12.1}} = 0.379 + 0.782 = 1.161 \]

No Good

Obviously, HP12 × 53 piles without bracing are not sufficient. However, other loading cases will be investigated to illustrate the procedure for design without bracing.

Group III

\[ F_v = 601 \text{ kips} \quad H_s = 8.00 \text{ kips} \]

\[ Q_{\text{max}} = 153.1 \text{ kips} \]

\( H_s \) carried by batter = \((152.0 - 148.6) \frac{1.5}{12} = 0.43 \text{ kips} \)

Remaining \( H_s = \frac{8.00 - 0.43}{4} = 1.89 \text{ k/pile} \)

\[ M_s = 11.02 \times 1.89 = 20.9 \text{ kip-ft} \]

\[ f_a = \frac{153.1}{15.6} = 9.81 \text{ ksi} \quad f_{by} = \frac{20.9 \times 12}{21.1} = 11.89 \text{ ksi} \]

Check \( \frac{f_a}{F_a} + \frac{C_m f_{by}}{\left(1 - \frac{f_a}{F'_{sy}}\right)F_s} < 1 \)

\[ 9.81 \frac{0.85 \times 11.89}{10.59 \times 1.25 + \frac{0.85 \times 11.89}{10.59 \times 1.25 \times 12.1}} = 0.740 + 1.151 = 1.891 \]

No Good

Group VI = Group III plus a longitudinal force due to temperature of 0.45 k/pile.

The maximum moment due to a longitudinal force occurs at the point of fixity. However, stresses at the point of fixity are not critical because the pile is laterally supported and buckling need not be considered.

Assuming full lateral support at midway between the top of the bearing stratum and the point of fixity, check stresses and consider buckling.
\[ Q_a = 153.1 \text{ kips} \]
\[ M_y = \left(11.02 - \frac{7.04}{2}\right)1.89 = 14.18 \text{ kip-ft} \]
\[ M_z = \left(15 + \frac{7.04}{2}\right)0.45 = 8.33 \text{ kip-ft} \]
\[ f_a = \frac{153.1}{15.6} = 9.81 \]
\[ f_{by} = \frac{14.18 \times 12}{21.1} = 8.06 \quad f_{bz} = \frac{8.33 \times 12}{66.9} = 1.49 \]
\[ F'_{yz} = \frac{149,000,000}{(K \frac{L_{by}}{r_{by}})^2} = \frac{149,000,000}{(119.3)^2} = 10,470 \text{ psi} = 10.47 \text{ ksi} \]

Check \[ \frac{f_a}{F'_{yz}} + \frac{C_{mf} f_{bz}}{1 - \frac{f_a}{F'_{yz}}} F_{bz} + \frac{C_{mf} f_{by}}{1 - \frac{f_a}{F'_{yz}}} F_{by} < 1.0 \]
\[ \frac{9.81}{10.59 \times 1.40} + \frac{0.85 \times 1.49}{20.0 \times 1.40} + \frac{0.85 \times 8.06}{12.1 \times 1.04} \frac{20.0 \times 1.40}{1.04} \]
\[ 0.661 + 0.136 + 0.581 = 1.378 \quad \text{No Good} \]

Since the piles are considerably overstressed without bracing, provide bracing and recheck stresses.

**CHECK STRESSES IN PILES DUE TO AXIAL LOAD PLUS BENDING—BRACED PILE BENT**

In the transverse direction, assume a point of inflection midway between the point of fixity and the elevation of the bottom bracing strut.

Therefore, maximum \( M_y = \frac{12.04}{2} H_y = 6.02 \) \( H_y \) at the point of fixity and at the elevation of the bottom bracing strut. Check stresses in the pile at the bottom bracing strut. Buckling must be considered at the elevation of the bottom bracing strut while the pile is laterally supported at the point of fixity.

**Group II**

\[ F_x = 30.8 \text{ kips} \quad H_y = 10.0 \text{ kips} \]

\[ Q_{\text{max}} = 78.3 \text{ k/lp} \]

\[ H_y \text{ carried by batter} = (77.7 - 76.3)\frac{1.5}{12} = 0.18 \text{ kips} \]

Remaining \( H_y = \frac{10.00 - 0.18}{4} = 2.46 \text{ k/lp} \)

\[ M_y = 6.02 \times 2.46 = 14.8 \text{ kip-ft} \]

\[ f_a = \frac{78.3}{15.6} = 5.02 \text{ ksi} \quad f_{by} = \frac{14.8 \times 12}{21.1} = 8.40 \text{ ksi} \]

\[ F'_{by} = \frac{149,000,000}{(K \frac{L_{by}}{r_{by}})^2} = 40,547 \text{ psi} = 40.55 \text{ ksi} \]
\[
\text{Check } \frac{f_a}{F_a} + \frac{C_m f_{by}}{(1 - \frac{f_a}{F_{ey}})F_b} < 1
\]

\[
\frac{5.02}{10.59 \times 1.25} + \frac{0.85 \times 8.40}{(1 - \frac{5.02}{40.55 \times 1.25})20 \times 1.25} = 0.379 + 0.317 = 0.696 < 1
\]

**Group III**

\[
F_v = 6.01 \text{ kips} \quad H_s = 8.00 \text{ kips}
\]

\[
Q_{\text{max}} = 153.1 \text{ kips}
\]

\[
H_s \text{ carried by batter} = \frac{8.00 - 0.43}{4} = 1.89 \text{ k/pile}
\]

\[
M_v = 6.02 \times 1.89 = 11.4 \text{ kip-ft}
\]

\[
f_s = \frac{153.1}{15.6} = 9.81 \text{ ksi} \quad f_{by} = \frac{11.4 \times 12}{21.1} = 6.48 \text{ ksi}
\]

\[
\text{Check } \frac{f_a}{F_a} + \frac{C_m f_{by}}{(1 - \frac{f_a}{F_{ey}})F_b} < 1
\]

\[
\frac{9.81}{10.59 \times 1.25} + \frac{0.85 \times 6.48}{(1 - \frac{9.81}{40.55 \times 1.25})20 \times 1.25} = 0.741 + 0.273 = 1.014
\]

Slightly greater than 1 OK

**Group VI = Group III plus a longitudinal force due to temperature of 0.45 k/pile.**

Since the moments due to longitudinal force are relatively small, combined stresses in the pile at the elevation of the bottom bracing strut govern.

\[
Q_{\text{max}} = 153.1
\]

\[
M_v = 6.02 \times 1.89 = 11.4 \text{ kip-ft}
\]

\[
M_s = 10.0 \times 0.45 = 4.50
\]

\[
f_s = \frac{153.1}{15.6} = 9.81 \text{ ksi} \quad f_{by} = \frac{11.4 \times 12}{21.1} = 6.48 \text{ ksi}
\]

\[
f_{bx} = \frac{4.50 \times 12}{66.9} = 0.81 \text{ ksi}
\]

\[
F_{sx} = \frac{149,000,000}{(K \frac{t_{by}}{r_{by}})^2} = \frac{149,000,000}{119.3^2} = 10,470 \text{ psi} = 10.47 \text{ ksi}
\]

\[
\text{Check } \frac{f_a}{F_a} + \frac{C_m f_{bx}}{(1 - \frac{f_a}{F_{sx}})F_{bx}} + \frac{C_m f_{by}}{(1 - \frac{f_a}{F_{ey}})F_{by}} < 1
\]

\[
\frac{9.81}{10.59 \times 1.40} + \frac{0.85 \times 6.48}{(1 - \frac{9.81}{40.55 \times 1.40})\times 1.40} + \frac{0.85 \times 0.70}{(1 - \frac{9.81}{10.47 \times 1.40})\times 1.40} = 0.662 > 0.238 > 0.074 = 0.974 < 1 \text{ OK}
\]

Therefore, HP 12×53 piles with bracing are sufficient.
Bracing Design

Weld single angles to each flange of piles.

Group II

\( H_x \) carried by batter = 0.18 kips
\( H_x \) carried by bracing = 10.00 - 0.18 = 9.82 kips

Bracing in compression

Axial compressive force, \( F = 9.82 \times \frac{10.94}{10.50} = 10.23 \) kips

Section 1.7.12, AASHO, limits the \( l/r \) for bracing members to 140.

Min. \( r_s = \frac{10.94 \times 12}{140} = 0.94 \)

Try 2-L 5 x 5 x \( \frac{5}{16} \) in.
Area = 6.06 sq in.
\( r_s = 0.994 \)

\( f_a = \frac{10.23 \text{ kips}}{6.06} = 1.69 \text{ ksi} \)

\( l/r = \frac{10.94 \times 12}{0.994} = 132 \)

\( F_a = 16,000 - 0.30(132)^2 = 10,770 \text{ psi} = 10.77 \text{ ksi} \)

\(.:.\) Angles are adequate in compression

Bracing in tension

Axial tension force, \( F = 10.23 \) kips

Section 1.7.15, AASHO, limits the effective area to the net area of the connected leg plus one-half of the area of the outstanding leg.

Effective Area = \( [3.03 - \frac{1}{2}(4.69 \times 0.313)]^2 \)
= 4.60 sq in.

\( f_t = \frac{10.23}{4.60} = 2.22 \text{ ksi} < 20.0 \text{ ksi} \)

Use 2-L's 5 x 5 x \( \frac{5}{16} \) in. for all bracing
Example 3—Cantilever Abutment

This design is typical for abutments that must extend several feet above the finished ground line. Two or more rows of piles at relatively close spacing are required. The size of the footing and the pile spacing is adjusted to give approximately equal loads to all piles. Piles are battered to carry a large portion of the horizontal load caused by earth pressure.

Piles in this example are designed for rock bearing. An abutment on friction piles is designed exactly the same except the additional steps of calculating pile penetration and pile settlement are included.

The design of pile foundations under retaining walls is similar to this example except that the only loads considered are the weight of the wall, weight of the earth above the footing, and earth pressure.

STRINGER REACTIONS
The maximum stringer reaction are:

\[ DL = 126 \text{ k/} \text{stringer} \]
\[ LL = 56 \text{ k/} \text{stringer} \]

In accordance with AASHO, Section 1.2.12, impact is not applied to piles below the ground surface. Since the bearings are an expansion type, the maximum horizontal force that can be applied to the abutment by the stringers is equal to the horizontal force necessary to overcome friction in the bearings. This friction force varies with the type of expansion bearings used. For this example, assume that the friction force is 10% of the dead load reaction.

\[ F = 126.0 \text{ k} \times 0.10 = 12.6 \text{ k/} \text{stringer} \]
Determine reaction for 1.0 ft. of abutment width by dividing the stringer reaction by the stringer spacing.

Dead Load = 126/8.50 = 14.8 k/ft
Live Load = 56/8.50 = 6.6 k/ft
Friction = 12.6/8.50 = 1.48 k/ft

EARTH PRESSURE
Assume the soil properties of the abutment backfill to be the same as the original soil properties.

Active earth pressure, $P_a = K_a \gamma H$

\[
K_a = \frac{1 - \sin \phi}{1 + \sin \phi} = \frac{1 - \sin 32^\circ}{1 + \sin 32^\circ} = \frac{1 - 0.531}{1 + 0.531} = 0.306
\]

\[
P_a = 0.110 \text{ kcf} \times 0.306 \times 24.17 = 0.814 \text{ ksf}
\]

\[
P_a = 0.814 \times \frac{24.17}{2} = 9.84 \text{ kips}
\]
Surcharge

Section 1.2.19 of the AASHO Specifications states that live load surcharge is not considered where an adequately designed reinforced concrete approach slab is provided. However, the weight of the slab is supported by the backfill and is considered as dead load surcharge. This load is equated to an equivalent height of soil.

\[ h_s = \frac{\text{wt. of approach slab}}{\text{wt. of soil}} = \frac{0.150 \times 1.33}{0.110} = 1.82 \text{ ft} \]

\[ P_s = 1.82 \times 0.110 \times 0.306 = 0.061 \text{ ksf} \]

\[ P_s = 0.061 \times 24.17 = 1.48 \text{ kips} \]

Passive soil resistance in front of the abutment should not be considered.

### DEAD LOAD PER FOOT OF ABUTMENT ABOUT PT. “A”

<table>
<thead>
<tr>
<th></th>
<th>( F_v ) (kips)</th>
<th>Arm about Pt. “A” (ft)</th>
<th>Moment about Pt. “A” (kip-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.0 \times 12.0 \times 0.15</td>
<td>5.40</td>
<td>6.00</td>
</tr>
<tr>
<td>2</td>
<td>2.67 \times 14.0 \times 0.15</td>
<td>5.61</td>
<td>3.58</td>
</tr>
<tr>
<td>3</td>
<td>0.96 \times \frac{1}{2} \times 11.5 \times 0.15</td>
<td>0.83</td>
<td>5.24</td>
</tr>
<tr>
<td>4</td>
<td>2.5 \times 1.67 \times \frac{1}{2} \times 0.15</td>
<td>0.31</td>
<td>5.50</td>
</tr>
<tr>
<td>5</td>
<td>1.5 \times 4.33 \times 0.15</td>
<td>0.97</td>
<td>4.42</td>
</tr>
<tr>
<td>6</td>
<td>1.67 \times 7.0 \times 0.15</td>
<td>1.75</td>
<td>5.75</td>
</tr>
<tr>
<td>7</td>
<td>1.67 \times 14.0 \times \frac{1}{2} \times 0.110</td>
<td>1.29</td>
<td>6.00</td>
</tr>
<tr>
<td>8</td>
<td>0.71 \times \frac{1}{2} \times 11.5 \times 0.110</td>
<td>0.45</td>
<td>6.12</td>
</tr>
<tr>
<td>9</td>
<td>23.2 \times 5.42 \times 0.110</td>
<td>13.83</td>
<td>9.29</td>
</tr>
<tr>
<td>10</td>
<td>2.0 \times 2.25 \times 0.110</td>
<td>0.50</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td>30.94</td>
<td></td>
<td>212.5</td>
</tr>
</tbody>
</table>

Determine the total vertical and horizontal loads about Pt. “A” (top of piles) for the following load cases.

### Case I—Dead Load + Earth Pressure

<table>
<thead>
<tr>
<th>Loading</th>
<th>( F_v ) (kips)</th>
<th>( H ) (kip)</th>
<th>Arm about Pt. “A” (ft)</th>
<th>Moment about Pt. “A” (kip-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dead Load of Abutment</td>
<td>30.9</td>
<td>—</td>
<td>6.88</td>
<td>+212.5 (( \cap ))</td>
</tr>
<tr>
<td>Dead Load of Superstructure</td>
<td>14.8</td>
<td>—</td>
<td>3.58</td>
<td>+ 53.0 (( \cap ))</td>
</tr>
<tr>
<td>Earth Pressure</td>
<td>—</td>
<td>9.84</td>
<td>7.06</td>
<td>− 69.5 (( \cap ))</td>
</tr>
<tr>
<td>Surcharge</td>
<td>—</td>
<td>1.48</td>
<td>11.08</td>
<td>− 16.4 (( \cap ))</td>
</tr>
<tr>
<td>Total—Case I</td>
<td>45.7</td>
<td>11.32</td>
<td></td>
<td>+179.6 (( \cap ))</td>
</tr>
</tbody>
</table>

Location of resultant, \( e = \frac{179.6}{45.7} = 3.93 \text{ ft} \)
### Case II—Dead Load + Live Load + Earth Pressure

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td>45.7</td>
<td>11.32</td>
<td>3.93</td>
<td>+179.6</td>
</tr>
<tr>
<td>Live Load</td>
<td>6.6</td>
<td>-</td>
<td>3.58</td>
<td>+23.6</td>
</tr>
<tr>
<td>Total—Case II</td>
<td>52.3</td>
<td>11.32</td>
<td></td>
<td>+203.2</td>
</tr>
</tbody>
</table>

Location of resultant, $e = \frac{203.2}{52.3} = 3.89$ ft

### Case III—Dead Load + Earth Pressure + Friction

<table>
<thead>
<tr>
<th>Loading</th>
<th>$F_v$ (kips)</th>
<th>$F_h$ (kips)</th>
<th>Arm about Pt. &quot;A&quot; (ft)</th>
<th>Moment about Pt. &quot;A&quot; (kip-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td>45.7</td>
<td>11.32</td>
<td>3.93</td>
<td>+179.6</td>
</tr>
<tr>
<td>Friction</td>
<td>-</td>
<td>1.48</td>
<td>17.5</td>
<td>-25.9</td>
</tr>
<tr>
<td>Total—Case III</td>
<td>45.7</td>
<td>12.80</td>
<td></td>
<td>+153.7</td>
</tr>
</tbody>
</table>

Location of resultant, $e = \frac{153.7}{45.7} = 3.36$ ft

Normally, Case II is critical when determining maximum pile load. Either Case I or Case II is critical when determining the required pile batter.

**PILE LOAD**

Assume a footing size, pile spacing and pile size. A footing width equal to about one half of the abutment height is reasonable for a first trial. Piles in the front row usually are spaced at 3 ft-0 in. to 4 ft-0 in., in the rear row at 6 ft-0 in. to 10 ft-0 in. When needed, a third row may be placed 3 ft-0 in. to 4 ft-0 in. behind the front row and spaced at a multiple of the front row spacing.

10 in. or 12 in. piles are sufficient under most abutment and retaining walls. This example will be based upon HP 12 x 53 piles loaded to 9 kips per square inch, or 140 kips per pile.

Assume the following footing and pile pattern.
Determine the centroid of the pile group measured from the edge of footing (Pt. “A”).

Row 1: \[
\frac{1 \text{ pile}}{3.20 \text{ ft}} = 0.313 \text{ piles/ft} \times 1.5 \text{ ft} = 0.469
\]

Row 2: \[
\frac{1 \text{ pile}}{8.00 \text{ ft}} = 0.125 \text{ piles/ft} \times 10.5 \text{ ft} = 1.310
\]

Centroid of pile group = \[
\frac{1.779}{0.438} = 4.06 \text{ ft from Pt. “A”}
\]

**Moment of Inertia of pile group**

\[
\Sigma x^2 = 0.313(2.56)^2 + 0.125(6.44)^2
\]

\[
= 7.23 \text{ pile-ft}^2
\]

Determine the vertical load carried by each pile:

**Case I**

Moment about centroid of pile group = 45.7(4.06 – 3.93)

\[
= 5.94 \text{ kip-ft}
\]

Vertical component of pile load, \( Q_m = \frac{F_r}{r} \pm \frac{M_x}{\Sigma x^2} \)

Row 1: \[
Q_m = \frac{45.7}{0.438} + \frac{5.94 \times 2.56}{7.23} = 104.3 + 2.1 = 106.4 \text{ kips}
\]

Row 2: \[
Q_m = 104.3 - \frac{5.94 \times 6.44}{7.23} = 104.3 - 5.3 = 99.0 \text{ kips}
\]

**Case II**

Moment about centroid of pile group = 52.3(4.06 – 3.89)

\[
= 8.89 \text{ kip-ft}
\]

Row 1: \[
Q_m = \frac{52.3}{0.438} + \frac{8.89 \times 2.56}{7.23} = 119.4 + 3.1 = 122.5 \text{ kips}
\]

Row 2: \[
Q_m = 119.4 - \frac{8.89 \times 6.44}{7.23} = 119.4 - 7.9 = 111.5 \text{ kips}
\]

**Case III**

Moment about centroid of pile group = 45.7(4.06 – 3.36)

\[
= 32.0 \text{ kip-ft}
\]

Row 1: \[
Q_m = \frac{45.7}{0.438} + \frac{32.0 \times 2.56}{7.23} = 104.3 + 11.3 = 115.6 \text{ kips}
\]

Row 2: \[
Q_m = 104.3 - \frac{32.0 \times 6.44}{7.23} = 104.3 - 28.5 = 75.8 \text{ kips}
\]

**LATERAL CAPACITY**

Determine the horizontal load per pile.

**Case I**

\[
P_e = \frac{H}{\text{Piles per foot}} = \frac{11.32}{0.438} = 25.8 \text{ k/pile}
\]
From Fig. 25, for an HP 12 × 53 pile embedded at least 18 feet in medium dense coarse-grained material above the water table, the ultimate lateral load capacity divided by a safety factor of 2.5 is equal to 15.0. Also 7.0 k will cause a lateral movement of \( \frac{1}{4} \) in. Since these capacities are much less than the computed lateral load of 25.8 k/pile batter the front two rows of pile. Try a 3 horizontal : 12 vertical batter.

\[
\text{Horizontal load per ft carried by batter} = \frac{\text{Vertical pile load}}{\text{Pile spacing}} \times \text{Pile batter}
\]

\[
H = 11.32 \text{ k/ft}
\]

Horizontal load taken by Row 1 battered piles = \( \frac{106.4}{3.2} \times \frac{3}{12} = 8.31 \text{ kips} \)

Lateral load to each pile = \( \frac{\text{Applied horiz. - horiz. load carried by batter}}{\text{piles per foot}} \)

\[
\frac{11.32 - 8.31}{0.438} = 6.9 \text{ k/pile} < 7.0 \text{ k/pile}
\]

**Case III**

\[
H = 12.80 \text{ k/ft}
\]

Horizontal load taken by Row 1 battered piles = \( \frac{115.6}{3.2} \times \frac{3}{12} = 9.03 \text{ kips} \)

Lateral load to each pile = \( \frac{12.80 - 9.03}{0.438} \)

\[
= 8.65 \text{ k/pile} < 7.0 \times 1.25 = 8.75
\]

Therefore, by battering the front row of piles, lateral movement of the abutment will be about \( \frac{1}{4} \) inch.

Check HP 12 × 53 for maximum axial load including the effect of batter.

**Group II (Governs)**

Vertical component of pile load = 122.5 kips

Horiz. component of pile load = \( 122.5 \times 3 \times \frac{1}{12} = 30.6 \)

\[\text{Axial pile load} = \sqrt{(122.5)^2 + (30.6)^2} \]

\[= 126.3 \text{ kips} \]

Allowable pile load = 140 kips

HP 12 × 53 piles spaced as shown are adequate.

**Example 4—Bridge Pier on Land**

The design of piles for this type of foundation is governed primarily by the axial capacity of the piles. For this bridge pier the piles are designed as friction piles in coarse-grained soil (Part A) and as friction piles in fine-grained soil (Part B).

**FORCES AT TOP OF FOOTING**

Determine forces at the top of the footing. These forces are used to design the column section and then to estimate the footing size and pile pattern. Loads for AASHO Groups I, II and III are tabulated. The design of many piers is governed by Group III loading.
PART A—PILE GROUP IN COARSE-GRAINED MATERIAL

Group I—Dead Load + Live Load + Impact

<table>
<thead>
<tr>
<th>Loading</th>
<th>(F_v) (kips)</th>
<th>Transverse Direction</th>
<th>Longitudinal Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(H_x) (kips)</td>
<td>(M_y) (kip-ft)</td>
</tr>
<tr>
<td>D.L. of Superstructure</td>
<td>966</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>D.L. of Pier</td>
<td>510</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>L.L.</td>
<td>206</td>
<td>—</td>
<td>680</td>
</tr>
<tr>
<td>Impact</td>
<td>45</td>
<td>—</td>
<td>150</td>
</tr>
<tr>
<td>Total Group I</td>
<td>1727</td>
<td>—</td>
<td>830</td>
</tr>
</tbody>
</table>
Group II—Dead Load + Wind

<table>
<thead>
<tr>
<th>Loading</th>
<th>$F_x$ (kips)</th>
<th>Transverse Direction</th>
<th>Longitudinal Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$H_x$ (kips)</td>
<td>$M_y$ (kip-ft)</td>
<td>$H_y$ (kips)</td>
</tr>
<tr>
<td>D.L. of Superstructure</td>
<td>966</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>D.L. of Pier</td>
<td>510</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Wind on Superstructure</td>
<td>—</td>
<td>63.0</td>
<td>3930</td>
</tr>
<tr>
<td>Wind on Pier</td>
<td>—</td>
<td>8.5</td>
<td>231</td>
</tr>
<tr>
<td>Total Group II</td>
<td>1476</td>
<td>71.5</td>
<td>4161</td>
</tr>
</tbody>
</table>

Group III—Dead Load + Live Load + Impact + Traction + 0.3 Wind + Wind on Live Load + Centrifugal Force + Friction

<table>
<thead>
<tr>
<th>Loading</th>
<th>$F_x$ (kips)</th>
<th>Transverse Direction</th>
<th>Longitudinal Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$H_y$ (kips)</td>
<td>$M_y$ (kip-ft)</td>
<td>$H_y$ (kips)</td>
</tr>
<tr>
<td>D.L. of Superstructure</td>
<td>966</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>D.L. of Pier</td>
<td>510</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>L.L.</td>
<td>206</td>
<td>—</td>
<td>680</td>
</tr>
<tr>
<td>Impact</td>
<td>45</td>
<td>—</td>
<td>150</td>
</tr>
<tr>
<td>0.3 Wind on Superstructure</td>
<td>—</td>
<td>18.9</td>
<td>1180</td>
</tr>
<tr>
<td>0.3 Wind on Pier</td>
<td>—</td>
<td>2.6</td>
<td>69</td>
</tr>
<tr>
<td>Wind on Live Load</td>
<td>—</td>
<td>10.0</td>
<td>716</td>
</tr>
<tr>
<td>Centrifugal Force</td>
<td>—</td>
<td>42.0</td>
<td>3000</td>
</tr>
<tr>
<td>Friction</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Traction</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Total Group III</td>
<td>1727</td>
<td>73.5</td>
<td>5795</td>
</tr>
</tbody>
</table>

PILE SIZE

High capacities piles concentrated near the periphery of a footing provide the most economical foundation when large loads and large moments must be supported.

However, high capacity piles should be used only if the soil is capable of supporting the high loads delivered by each pile.

Try HP 14 x 73 piles with an allowable steel stress of 9.0 ksi.

Allowable load per pile, $Q = 21.5 \times 9.0 = 193.5$ kips

**Pile Properties**

- $A_s = 21.5$ psf = 0.149 ft$^2$
- $d = 13.64$ in. = 1.13 ft
- $b_f = 14.586$ in. = 1.21 ft
- $p = 2(d + b_f) = 4.68$ ft
- $A = db_f = 1.37$ ft$^2$
DEPTH OF PENETRATION

Determine the depth of penetration required to develop an allowable load of 193.5 kips on an HP 14\times73 pile.

Piles driven in a group in coarse-grained material can be designed based upon the capacity of a single pile.

The ultimate bearing capacity of a pile penetrating different strata of soil is equal to the sum of the skin friction in each strata plus the end bearing at the tip of the pile.

\[ Q_u = 2Q_{us} + Q_{ue} \]

**Skin Friction Components**

Layer 1: Using Fig. 8, for \( \phi = 30^\circ \), \( N_\gamma = 65 \), \( N_q = 82 \) and \( K_b = 0.40 \)

\[ Q_{us} = \frac{1}{2} pK_b \gamma_d D_1 \tan \delta \quad \text{(Eq. 4)} \]
\[ = \frac{1}{2} \times 4.68 \times 0.40 \times 0.110 (15.0)^3 \tan 20^\circ \]
\[ = 8.4 \text{ kips} \]

Layer 2: Using Fig. 8, for \( \phi = 36^\circ \), \( N_\gamma = 230 \), \( N_q = 205 \) and \( K_b = 0.52 \)

\[ Q_{us} = \left[ \gamma_d D_1 + \frac{1}{2} \gamma_d D_1 D_p K_b \tan \delta \right] \quad \text{(Eq. 4a)} \]
\[ = \left[ (0.110 \times 15.0) + (\frac{1}{2} \times 0.120D_1)D_4 (4.68 \times 0.52) \right] \tan 25^\circ \]
\[ = [1.650 + 0.060D_1]1.135D_4 \]
\[ = 1.873D_4 + 0.0681D_4^2 \]

**End Bearing Component**

\[ Q_{ue} = \left[ \frac{1}{2} \gamma_d dN_\gamma + K_b N_q (\gamma_d D_1 + \gamma_d D_4) \right] A \quad \text{(Eq. 2a)} \]
\[ = \left[ (\frac{1}{2} \times 0.120 \times 1.13 \times 230) + 0.52 \times 205 (0.110 \times 15.0 + 0.120 \times D_4) \right] 1.37 \]
\[ = [15.6 + 106.6(1.65 + 0.120D_4)] 1.37 \]
\[ = [15.6 + 175.9 + 12.79D_4] 1.37 \]
\[ = 262.4 + 17.52D_4 \]

**Required Penetration**

\[ Q_u = F_s \times Q_u = 2Q_{us} + Q_{ue} \]

Using safety factor = 2.5

\[ Q_u = 2.5 \times 193.5 = 3.4 + 1.873D_4 + 0.0681D_4^2 + 262.4 + 17.52D_4 \]
\[ = 483.8 + 19.39D_4 + 0.0681D_4^2 \]
\[ D_4^4 + 284.7D_4 + (142.4)^2 = 3127 + (142.4)^2 \]
\[ (D_4 + 142.4)^2 = 234.05 \]
\[ D_4 + 142.4 = 153.0 \]
\[ D_4 = 10.6; \text{use 11.0 ft} \]

The depth of pile penetration should be verified by either a load test or careful observance of the resistance to driving.

Check assumption that total pile penetration > 10\( d \)

\[ 15.0 + 11.0 - 26.0 \geq 10 \times 1.13 = 11.3 \text{ ft} \]

Since the depth of ground water is greater than 1.5\( d = 1.7 \text{ ft} \) below the pile tip, it will not influence the depth of pile penetration.

**FOOTING SIZE AND PILE PATTERN**

Estimate the required number of piles by assuming that the vertical load will cause one-half of the total pile load.
No. of piles = \( \frac{1727}{193.5} \times 2 \approx 18 \) piles

Assume the footing dimensions and pile pattern shown. The piles are concentrated near the periphery of the footing to resist overturning moments.

\[
\Sigma x^2 = 5 \times 2(10.0)^2 + 2 \times 3(5.0)^2 = 1150 \text{ pile-ft}^2 \\
\Sigma y^2 = 5 \times 2(8.0)^2 + 2 \times 2(4.0)^2 = 704 \text{ pile-ft}^2
\]

**FORCES AT TOP OF PILES**

Transfer the forces and moments acting at the top of the footing down to the top of the piles. To these forces add the weight of the footing and the soil on top of the footing. Since the footing and piles are below the ground surface, impact is deducted from the loading.

**Group I—Dead Load+Live Load**

<table>
<thead>
<tr>
<th>Loading</th>
<th>( F_y ) (kips)</th>
<th>Transverse Direction</th>
<th>Longitudinal Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( H_x ) (kips)</td>
<td>( M_y ) (kip-ft)</td>
<td>( H_y ) (kips)</td>
</tr>
<tr>
<td>Forces at top of footing</td>
<td>1727</td>
<td>830</td>
<td>—</td>
</tr>
<tr>
<td>Impact</td>
<td>-45</td>
<td>-150</td>
<td>—</td>
</tr>
<tr>
<td>Wt. of footing</td>
<td>393</td>
<td>-</td>
<td>—</td>
</tr>
<tr>
<td>Wt. of soil</td>
<td>81</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Total Group I</td>
<td>2156</td>
<td>680</td>
<td>—</td>
</tr>
</tbody>
</table>
Group II — Dead Load + Wind

<table>
<thead>
<tr>
<th>Loading</th>
<th>$F_v$ (kips)</th>
<th>Transverse Direction</th>
<th>Longitudinal Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$H_x$ (kips)</td>
<td>$M_x$ (kip-ft)</td>
<td>$H_y$ (kips)</td>
</tr>
<tr>
<td>Forces at top of footing</td>
<td>1476</td>
<td>71.5</td>
<td>4161</td>
</tr>
<tr>
<td>Wt. of footing</td>
<td>393</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Wt. of soil</td>
<td>81</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$H_x(5.0)=71.5 \times 5.0$</td>
<td>—</td>
<td>—</td>
<td>358</td>
</tr>
<tr>
<td>$H_y(5.0)=43.3 \times 5.0$</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Total Group II</td>
<td>1950</td>
<td>71.5</td>
<td>4519</td>
</tr>
</tbody>
</table>

Group III — Dead Load + Live Load + Impact + Traction + 0.3 Wind + Wind on Live Load + Centrifugal Force + Friction

<table>
<thead>
<tr>
<th>Loading</th>
<th>$F_v$ (kips)</th>
<th>Transverse Direction</th>
<th>Longitudinal Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$H_x$ (kips)</td>
<td>$M_x$ (kip-ft)</td>
<td>$H_y$ (kips)</td>
</tr>
<tr>
<td>Forces at top of footing</td>
<td>1727</td>
<td>73.5</td>
<td>5795</td>
</tr>
<tr>
<td>Impact</td>
<td>-45</td>
<td>—</td>
<td>-150</td>
</tr>
<tr>
<td>Wt. of footing</td>
<td>393</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Wt. of soil</td>
<td>81</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$H_x(5.0)=73.5 \times 5.0$</td>
<td>—</td>
<td>—</td>
<td>368</td>
</tr>
<tr>
<td>$H_y(5.0)=92.7 \times 5.0$</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Total Group III</td>
<td>2156</td>
<td>73.5</td>
<td>6013</td>
</tr>
</tbody>
</table>

By inspection Group III loading with an allowable stress increase of 25% will govern.

PILE LOADING

$$Q_n = \frac{F_v}{m} = \frac{M_x}{\Sigma x^2} \pm \frac{M_y}{\Sigma y^2}$$  \hspace{1cm} (Eq. B)

Max. pile load, $Q_n = \frac{2156+6013 \times 10.0}{18} \pm \frac{5479 \times 8.0}{704}$

$$= 119.8 + 52.3 \pm 62.3$$

$$= 234.4 \text{ kips}$$

Min. pile load, $Q_n = 119.8 - 52.3 - 62.3$

$$= +5.2 \text{ kips} \hspace{1cm} \text{No uplift occurs}$$

The pile pattern and footing size selected are satisfactory.

DETERMINE IF BATTERED PILES ARE REQUIRED

The limited data available indicates that the lateral capacity of a pile group is approximately equal to the capacity of a single free-headed pile times the number of piles in the group. Also in a multistrata condition the surface layer governs the lateral load. Assume a lateral movement of $\frac{1}{4}$ inch at the ground surface is permissible. From Fig. 25, the allowable lateral load for a single free-headed HP 14×73 pile in
sand with $\phi$ equal to 30° is 9.0 kips per pile perpendicular to the pile flange and $\frac{3}{4} \times 9.0$ or 6.0 kips per pile parallel to the pile flange.

The allowable lateral load parallel to the $x$-axis is:
$$18 \times 6.0 \times 1.25 = 135 \text{ kips} > 73.5 \text{ kips}$$

The allowable lateral load parallel to the $y$-axis is:
$$18 \times 9.0 \times 1.25 = 202 \text{ kips} > 92.7 \text{ kips}$$

Therefore, battered piles are not required.

**SETTLEMENT AT TOP OF PILES**

The settlement of a pile group is greater than the settlement of an individual pile for the same unit pile loading. Fig. 12 relates the ratio between the settlement of a pile group and the settlement of a single pile to the width of the foundation.

$$\text{Avg. width of foundation} = \frac{23.0 + 19.0}{2} = 21.0 \text{ ft}$$

From Fig. 12:

$$\frac{\text{Foundation Settlement}}{\text{Single Pile Settlement}} = 7.6$$

Calculate the settlement due to dead load plus live load. Secondary loads such as wind, traction, friction and centrifugal force act for only short periods of time and cause unequal pile loads, resulting in a slight rotation of the footing.

$$\therefore \text{Pile reaction for } DL, \ Q_n = \frac{2156 - 206}{18} = 108.3 \text{ k/pile}$$

and Pile reaction for $LL$, $Q_n = \frac{206}{18} = 11.4 \text{ k/pile}$

Using the average curve in Fig. 12, determine the single pile settlement for dead and live loads.

Pile dead load

$$\frac{108.3}{483.8} = 0.223 \text{ and } \Delta = 0.06 \text{ in.}$$

Pile live load

$$\frac{11.4}{483.8} = 0.023 \text{ and } \Delta = 0$$

**Settlement of Pile Group at Pile Tip**

Dead Load: $\Delta_T = 0.06 \times 7.6 = 0.456 \text{ in.}$

Live Load: $\Delta_T = 0$

Total = 0.456 in. $\approx \frac{3}{8} \text{ in.}$

**Elastic Deformation**

In the analysis for the depth of penetration, the ultimate end bearing component is:

$$Q_u = 262.4 + 17.52D_1 = 262.4 + 17.52(10.6)$$

$$= 448.0 \text{ kips}$$

$$\% \text{ of ultimate load to pile tip} = \frac{448.0}{483.8}$$

$$= 92.6 \%$$

The approximate dead and live loads reaching the pile tip are:

$$\text{D.L.: } Q = 108.3 \times 0.926 = 100.3 \text{ kips}$$

$$\text{L.L.: } Q = 11.4 \times 0.926 = 10.6 \text{ kips}$$
The loads carried by skin friction are:
D.L.: \[ Q = 108.3 - 100.3 = 8.0 \text{kips} \]
L.L.: \[ Q = 11.4 - 10.6 = 0.8 \text{kips} \]

The average pile loads causing elastic deformation are:
D.L.: \[ Q = 100.3 - \frac{8.0}{2} = 96.3 \text{kips} \]
L.L.: \[ Q = 10.6 - \frac{0.8}{2} = 10.2 \text{kips} \]

The deformation due to dead load is:
\[ \Delta = \frac{96.3 \times 26.0 \times 12.0}{21.5 \times 29 \times 10^3} = 0.048 \text{ in.} \]

The deformation due to live load is:
\[ \Delta = \frac{10.2 \times 26.0 \times 12.0}{21.5 \times 29 \times 10^3} = 0.005 \text{ in.} \]
Total \( \Delta = 0.053 \text{ in.} \approx \frac{1}{4} \text{ in.} \)

The total settlement at top of piles \( \approx \frac{1}{4} \text{ in.} \)

**FORCES AT TOP OF FOOTING**
Refer to Part A.

**PILE SIZE**
High capacity piles concentrated near the periphery of a footing provide an economical foundation when large loads and large moments must be supported.

High capacity piles should be used only if the soil is capable of supporting the high loads delivered by each pile. High pile loads are more difficult to develop in fine-grained material than in coarse-grained material.

A “medium capacity” pile—an HP 12 \times 53 stressed to 9 ksi will be used in this design example.

Allowable load for a single pile, \( Q = 15.6 \times 9.0 = 140.4 \text{kips} \).

**Pile Properties**
\[ A_r = 15.6 \text{ in.}^2 = 0.108 \text{ ft}^2 \]
\[ d = 11.78 \text{ in.} = 0.982 \text{ ft} \]
\[ b_f = 12.046 \text{ in.} = 1.004 \text{ ft} \]
\[ p = 2(d + b_f) = 3.97 \text{ ft} \]
\[ A = db_f = 0.986 \text{ ft}^2 \]

**DEPTH OF PENETRATION**
Determine the depth of penetration required to develop an allowable load of 140 kips on an HP 12 \times 53 pile. Assume that single pile action controls the required depth of penetration. After the pile pattern has been established, determine if group action governs.

**Determine Penetration for a Single Pile**
For H-piles in fine-grained material, the end bearing capacity is negligible. Therefore, the ultimate bearing capacity is equal to the skin friction component \( Q_{us} \). The lesser of the \( Q_{us} \) values found by equations 10, 11 and 12 determines the depth of penetration.

**Cohesion Value**
\[ \text{Use Avg. } c = \frac{1.0 + 3.0}{2} = 2.0 \text{ ksf} \]
PART B—PILE GROUPING IN FINE-GRAINED MATERIAL

Adhesion Value

Using Fig. 9, for \( c = 2.0, \frac{c_a}{c} = 0.35 \)

\[ \therefore c_a = 0.35 \times 2.0 = 0.70 \text{ ksf} \]

a. Cohesion around the net perimeter

\[ Q_{uw} = 2(d + b_x) c_D \]
\[ = 2(0.982 + 1.004)2.0D \]
\[ = 7.94D \] \hspace{1cm} (Eq. 10)

b. Adhesion around the entire perimeter

\[ Q_{uw} = 2(d + 2b_x) c_a D \]
\[ = 2[(0.982 + (2 \times 1.004))0.70D \]
\[ = 4.19D \hspace{1cm} \text{Governs} \] \hspace{1cm} (Eq. 11)

c. Adhesion to the flange and cohesion across the web opening

\[ Q_{uw} = 2(d_c + b_x) c_a D \]
\[ = 2[(0.982 \times 2.0) + (1.004 \times 0.70)]D \]
\[ = 5.33D \] \hspace{1cm} (Eq. 12)
Adhesion around the entire perimeter governs. Using a safety factor = 2.5,

\[ D = \frac{140 \text{ k} \times 2.5}{4.19} \]

\[ D = 83.5 \text{ ft}; \text{ Use 84.0 ft} \]

This depth of pile penetration should be verified by a pile load test.
Check for group action after determining the footing size, number of piles and pile spacing.

**FOOTING SIZE AND PILE PATTERN**

Estimate the required number of piles by assuming that the vertical load will cause one-half of the total pile load.

\[ \text{No. of piles} = \frac{1727}{140.4} \times 2 = 25 \text{ piles} \]

Assume the footing dimensions and pile pattern shown.

![Diagram of footing and pile pattern]

**Moment of Inertia of Pile Group**

\[ \Sigma x^2 = 2 \times 5(10.0)^2 + 2 \times 5(6.0)^2 + 2 \times 3(2.0)^2 = 1384 \text{ pile-ft}^2 \]

\[ \Sigma y^2 = 2 \times 6(8.0)^2 + 2 \times 4(4.0)^2 = 896 \text{ pile-ft}^2 \]

**FORCES AT TOP OF PILES**

Transfer the forces and moments acting at the top of the footing down to the top of the piles. To these forces, add the weight of the footing and the soil on top of the footing. Since the footing and piles are below the ground surface, impact is deducted from the loading.

**Group I—Dead Load + Live Load**

<table>
<thead>
<tr>
<th>Loading</th>
<th>( F_v ) (kips)</th>
<th>Transverse Direction</th>
<th>Longitudinal Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( H_x ) (kips)</td>
<td>( M_v ) (kip-ft)</td>
<td>( H_y ) (kips)</td>
</tr>
<tr>
<td>Forces at top of footing</td>
<td>1727</td>
<td>830</td>
<td>—</td>
</tr>
<tr>
<td>Impact</td>
<td>-45</td>
<td>-150</td>
<td>—</td>
</tr>
<tr>
<td>Wt. of footing</td>
<td>393</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Wt. of soil</td>
<td>81</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Total Group I</td>
<td>2156</td>
<td>680</td>
<td>—</td>
</tr>
</tbody>
</table>
Group II—Dead Load + Wind

<table>
<thead>
<tr>
<th>Loading</th>
<th>( F_v ) (kips)</th>
<th>Transverse Direction</th>
<th>Longitudinal Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forces at top of footing</td>
<td>1476</td>
<td>71.5</td>
<td>4161</td>
</tr>
<tr>
<td>Wt. of footing</td>
<td>393</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Wt. of soil</td>
<td>81</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>( H_s(5.0) = 71.5 \times 5.0 )</td>
<td>--</td>
<td>--</td>
<td>358</td>
</tr>
<tr>
<td>( H_s(5.0) = 43.3 \times 5.0 )</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Total Group II</td>
<td>1950</td>
<td>71.5</td>
<td>4519</td>
</tr>
</tbody>
</table>

Group III—Dead Load + Live Load + Friction + Traction + 0.3 Wind + Wind on Live Load + Centrifugal Force

<table>
<thead>
<tr>
<th>Loading</th>
<th>( F_v ) (kips)</th>
<th>Transverse Direction</th>
<th>Longitudinal Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forces at top of footing</td>
<td>1727</td>
<td>73.5</td>
<td>5795</td>
</tr>
<tr>
<td>Impact</td>
<td>-45</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Wt. of footing</td>
<td>393</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Wt. of soil</td>
<td>81</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>( H_s(5.0) = 73.5 \times 5.0 )</td>
<td>--</td>
<td>--</td>
<td>388</td>
</tr>
<tr>
<td>( H_s(5.0) = 92.7 \times 5.0 )</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Total Group III</td>
<td>2156</td>
<td>73.5</td>
<td>6018</td>
</tr>
</tbody>
</table>

By inspection Group III loading with an allowable stress increase of 25% will govern.

**PILE LOADING**

\[
Q_a = \frac{F_v}{r} \pm \frac{M_x}{2x^2} \pm \frac{M_y}{2y^2}
\]  

(Eq. B)

Max. pile load, \( Q_a = \frac{2156}{26} + \frac{6018 \times 10}{1384} + \frac{5479 \times 8.0}{896} \)

= 82.9 + 43.5 + 48.9

= 175.3 < 140.4 \times 1.25 = 175.5 \text{ kips}

Min. pile load, \( Q_a = -82.9 - 43.5 - 48.9 \)

= -9.5 \text{ kips} \quad \text{uplift}

Determine if the piles can resist this amount of uplift.

**ULTIMATE PULL OUT CAPACITY**

The short term uplift capacity of a friction pile in fine-grained material is equal to the skin friction on the pile. The long term uplift capacity is sometimes taken as 0.7
of the skin friction. However, AASHO limits the allowable uplift to 0.4 of the allowable compressive load.

\[ \text{Allowable uplift} = 0.4 \times 140.4 \times 1.25 = 70.2 \text{ kips} > 9.5 \text{ kips actual} \]

The footing dimensions and pile pattern selected are satisfactory.

**DETERMINE IF BATTERED PILES ARE REQUIRED**

The limited data available indicates that the lateral capacity of a pile group is approximately equal to the capacity of a single free-headed pile times the number of piles in the group. Assume a lateral movement of \( \frac{3}{4} \) inch is permissible. From Fig. 27 the allowable lateral load for a single free-headed HP 12\times53 pile in stiff clay is 5 kips per pile for short term loading and 2 kips per pile for long term loading. Since horizontal loads on this pier are secondary forces, use the short term allowable load of 5 kips perpendicular to the pile flange and \( \frac{3}{4} \times 5.0 \) or 3.75 kips per pile parallel to the pile flange.

The allowable lateral load parallel to the \( x \)-axis is:

\[ (26 \times 3.75) \times 1.25 = 122 \text{ kips} > 73.5 \text{ kips} \]

The allowable lateral load perpendicular to the \( y \)-axis is:

\[ (26 \times 5.0) \times 1.25 = 162 \text{ kips} > 92.7 \text{ kips} \]

Therefore, battered piles are not required.

**CAPACITY OF THE PILE GROUP**

The ultimate capacity of the pile group,

\[ Q_u = Q_{uu} + Q_u \]

where \( Q_{uu} = 9c(BL) \)

\[ Q_u = 2(D(B+L)c \]

\[ Q_u = 9c(BL) + 2D(B+L)c \]

where \( B = 20 + 0.982 = 20.982 \text{ ft, say 21.0 ft} \)

\( L = 16 + 1.004 = 17.004 \text{ ft, say 17.0 ft} \)

\( Q_u = 9 \times 3.0(21.0+17.0) + 2 \times 86.0(21.0+17.0)2.0 \)

\( Q_u = 1026 + 6536 = 7556 \text{ kips} \)

Applying a factor of safety of 2.5, the allowable vertical load on the pile group

\[ = 7562 / 2.5 \]

\[ = 3025 \text{ kips} \]

The maximum load on the pile group is 2156 kips. No reduction in pile capacity is required for group action.

**SETTLEMENT**

Settlement of a pile group in a fine-grained material may be approximated by assuming that the imposed loading is applied as a uniform pressure within the perimeter of the group, at a level \( \frac{2}{3} \) down the pile penetration. Determine settlement for \( DL \) and \( LL \).

Settlement is determined for a soil layer beginning a distance \( Z \) below the pile tip and extending upward from the pile tip a distance \( D/3 \). The distance \( Z \) is determined
by assuming that settlement will occur to a depth of twice the largest group width below the pile tip.

\[ Z = 2 \times 21.0 \text{ ft} = 42.0 \text{ ft} \]

\[ \Delta_s = \frac{C_c(D/3+Z)}{1+e_s} \left[ \log \left( \frac{q_e+\Delta_s}{q_e} \right) \right] \]

\[ \gamma = 120 \text{ psf} \]
\[ e_0 = 0.5 \]
\[ C_c = 0.04 \]

where \( q_e = \) existing effective overburden pressure at mid-depth of the soil layer in which settlement is considered.

\( \Delta_s = \) Pressure due to \( Q \) at mid-depth of the soil layer in which settlement is considered.

\( C_c = \) compression index

\( e_s = \) void ratio

From laboratory tests the following values were found.

\[ e = 0.5 \text{ and } C_c = 0.04 \]

**Determine Existing Overburden Pressure**

\[ q_e = \gamma \left[ \frac{2}{3}D + \frac{D}{3} + \frac{Z}{2} + 8.0 \right] \]

\[ = 0.12(58.0 + 35.0 + 8.0) = 0.12[101.0] \]

\[ = 12.12 \text{ ksf} \]

**Determine Change In Pressure Due To Pile Loading**

a. Dead load, \( Q = 2156 \text{ kips} - 206 \text{ kips} = 1950 \text{ kips} \)

\[ \Delta_s = \frac{1950}{61.4 \times 57.4} = 0.553 \text{ ksf} \]
Consolidation Settlement

\[ \Delta_r = \frac{0.04(70.0)}{1+0.5} \left( \log \frac{12.12 + 0.553}{12.12} \right) = 1.866(\log 1.045) \]
\[ = 1.866(0.01912) = 0.036 \text{ ft} \]
\[ = 0.432 \text{ in.} \]

Settlement Due To Elastic Deformation Of The Pile

Average pile load (over length of pile) = \( \frac{1950 \text{ kips}}{26} \times \frac{1}{2} = 37.5 \text{ kips} \)

\[ \Delta = \frac{37.5 \times 84.0 \times 12}{15.6 \times 29 \times 10^2} = 0.083 \text{ in.} \]

Total D.L. settlement = 0.432 + 0.083 = 0.515 in. = \( \frac{1}{2} \) in.

b. Live Load, \( Q = 206 \text{ kips} \)

\[ \Delta_r = \frac{206}{61.4 \times 57.4} = 0.058 \text{ ksf} \]

Consolidation Settlement

\[ \Delta_r = \frac{0.04(70.0)}{1+0.5} \left( \log \frac{12.12 + 0.058}{12.12} \right) = 1.866(1.004) \]
\[ = 1.866(0.00173) = 0.0032 \text{ ft} \]
\[ = 0.039 \text{ in.} \]

Settlement Due To Elastic Deformation Of The Pile

Average pile load = \( \frac{206}{26 \times 2} = 3.96 \)

\[ \Delta = \frac{3.96 \times 84.0 \times 12}{15.6 \times 29 \times 10^2} = 0.008 \text{ in.} \]

Total \( LL \) settlement = 0.039 + 0.008 = 0.047 in. = \( \frac{1}{20} \) in.

Total \( DL + LL \) settlement = \( \frac{1}{8} \) in.

The above magnitude of calculated settlements should be regarded as only a rough approximation. However, it can be used to judge the adequacy of the foundation.

Example 5—Bridge Pier in Water (with Tremie Concrete Seal)

This design is typical for a piled foundation placed in a shallow stream. Steel sheet piling is driven to form a cofferdam, and the area inside the cofferdam is excavated underwater. Bearing piles are driven, and tremie concrete is placed underwater. After the tremie concrete has gained sufficient strength, the cofferdam is dewatered and the pier footing is constructed in the dry.

Tremie seals generally are at least 5 feet thick so as to permit the tremie concrete to flow and cover the full plan area of the cofferdam without excessive movement of the tremie pipe. The tremie seal is made slightly larger than the footing to simplify placing of footing form work and to catch and drain any water seeping into the cofferdam.

Piles are designed as rock bearing in this example. However, the design procedure for friction piles is similar with added steps to determine the depth of penetration.
FORCES AT TOP OF FOOTING
Determine forces at the top of the footing. These forces are used to design the column section and then to estimate the footing size and pile pattern. Loads for AASHO Groups I, II, III, VIII and IX are tabulated.

Group I—Dead Load + Live Load + Impact + Stream Flow

<table>
<thead>
<tr>
<th>Loading</th>
<th>$F_v$ (kips)</th>
<th>Transverse Direction</th>
<th>Longitudinal Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>D.L. of Superstructure</td>
<td>966</td>
<td>$-           $</td>
<td>$-           $</td>
</tr>
<tr>
<td>D.L. of Pier</td>
<td>510</td>
<td>$-           $</td>
<td>$-           $</td>
</tr>
<tr>
<td>L.L.</td>
<td>206</td>
<td>$-           $</td>
<td>680</td>
</tr>
<tr>
<td>Impact</td>
<td>45</td>
<td>$-           $</td>
<td>150</td>
</tr>
<tr>
<td>Stream</td>
<td></td>
<td>$-           $</td>
<td>2.8</td>
</tr>
<tr>
<td>(Normal Pool)</td>
<td></td>
<td></td>
<td>12</td>
</tr>
<tr>
<td>(High Water)</td>
<td></td>
<td></td>
<td>8.8</td>
</tr>
<tr>
<td>Total Group I</td>
<td>1727</td>
<td>2.8</td>
<td>842</td>
</tr>
<tr>
<td>(Normal Pool)</td>
<td></td>
<td></td>
<td>$-           $</td>
</tr>
<tr>
<td>(High Water)</td>
<td></td>
<td></td>
<td>8.8</td>
</tr>
</tbody>
</table>
### Group II—Dead Load + Stream Flow + Wind

<table>
<thead>
<tr>
<th>Loading</th>
<th>$F_v$ (kips)</th>
<th>Transverse Direction</th>
<th>Longitudinal Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$F_v$ (kips)</td>
<td>$H_z$ (kips)</td>
<td>$M_y$ (kip-ft)</td>
</tr>
<tr>
<td>D.L. of Superstructure</td>
<td>966</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>D.L. of Pier</td>
<td>510</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Wind on Superstructure</td>
<td>—</td>
<td>63.0</td>
<td>3930</td>
</tr>
<tr>
<td>Stream (Normal Pool)</td>
<td>—</td>
<td>2.8</td>
<td>12</td>
</tr>
<tr>
<td>Flow (High Water)</td>
<td>—</td>
<td>8.8</td>
<td>88</td>
</tr>
<tr>
<td>Wind (Normal Pool)</td>
<td>—</td>
<td>7.6</td>
<td>220</td>
</tr>
<tr>
<td>on Pier (High Water)</td>
<td>—</td>
<td>5.9</td>
<td>203</td>
</tr>
<tr>
<td>Total (Normal Pool)</td>
<td>1476</td>
<td>73.4</td>
<td>4162</td>
</tr>
<tr>
<td>Group II (High Water)</td>
<td>1476</td>
<td>77.7</td>
<td>4221</td>
</tr>
</tbody>
</table>

### Group III—Dead Load + Live Load + Impact + Stream Flow + 0.3 Wind + Wind on Live Load + Traction + Centrifugal Force + Friction

<table>
<thead>
<tr>
<th>Loading</th>
<th>$F_v$ (kips)</th>
<th>Transverse Direction</th>
<th>Longitudinal Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$F_v$ (kips)</td>
<td>$H_z$ (kips)</td>
<td>$M_y$ (kip-ft)</td>
</tr>
<tr>
<td>D.L. of Superstructure</td>
<td>966</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>D.L. of Pier</td>
<td>510</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>L.L.</td>
<td>206</td>
<td>—</td>
<td>680</td>
</tr>
<tr>
<td>Impact</td>
<td>45</td>
<td>—</td>
<td>150</td>
</tr>
<tr>
<td>Stream (Normal Pool)</td>
<td>—</td>
<td>2.8</td>
<td>12</td>
</tr>
<tr>
<td>Flow (High Water)</td>
<td>—</td>
<td>8.8</td>
<td>88</td>
</tr>
<tr>
<td>0.3 Wind on Superstructure</td>
<td>—</td>
<td>18.9</td>
<td>1180</td>
</tr>
<tr>
<td>0.3 Wind (Normal Pool)</td>
<td>—</td>
<td>2.3</td>
<td>66</td>
</tr>
<tr>
<td>on Pier (High Water)</td>
<td>—</td>
<td>1.8</td>
<td>61</td>
</tr>
<tr>
<td>Wind on Live Load</td>
<td>—</td>
<td>10.0</td>
<td>716</td>
</tr>
<tr>
<td>Centrifugal Force</td>
<td>—</td>
<td>42.0</td>
<td>3000</td>
</tr>
<tr>
<td>Friction</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Traction</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Total (Normal Pool)</td>
<td>1727</td>
<td>76.0</td>
<td>5804</td>
</tr>
<tr>
<td>Group III (High Water)</td>
<td>1727</td>
<td>81.5</td>
<td>5875</td>
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</tbody>
</table>

### Group VIII—Group I + Ice

<table>
<thead>
<tr>
<th>Loading</th>
<th>$F_v$ (kips)</th>
<th>Transverse Direction</th>
<th>Longitudinal Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$F_v$ (kips)</td>
<td>$H_z$ (kips)</td>
<td>$M_y$ (kip-ft)</td>
</tr>
<tr>
<td>Group I at Normal Pool Ice</td>
<td>1727</td>
<td>2.8</td>
<td>842</td>
</tr>
<tr>
<td>Ice</td>
<td>—</td>
<td>115.2</td>
<td>806</td>
</tr>
<tr>
<td>Total Group VIII</td>
<td>1727</td>
<td>118.0</td>
<td>1648</td>
</tr>
</tbody>
</table>
Group IX—Group II + Ice

<table>
<thead>
<tr>
<th>Loading</th>
<th>$F_v$ (kips)</th>
<th>Transverse Direction</th>
<th>Longitudinal Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$H_x$ (kips)</td>
<td>$M_z$ (kip-ft)</td>
<td>$H_y$ (kips)</td>
</tr>
<tr>
<td>Group II at Normal</td>
<td>1476</td>
<td>73.4</td>
<td>4162</td>
</tr>
<tr>
<td>Pool Ice</td>
<td>—</td>
<td>115.2</td>
<td>806</td>
</tr>
<tr>
<td>Total Group IX</td>
<td>1476</td>
<td>188.6</td>
<td>4968</td>
</tr>
</tbody>
</table>

Group III loading with an allowable stress increase of 25% will govern for maximum pile load and for maximum uplift. Group IX loading with an allowable stress increase of 50% will govern for lateral load design and battering of the piles.

PILE SIZE
AASHO Specifications permit pile stresses higher than 9000 psi if substantiated by field load tests. For this design, assume that a load test is performed and the allowable load is based on a pile steel stress of 12,000 psi. Since it is more economical to drive fewer large piles than many small piles, select an HP 14×73 pile section.

Allowable load per pile = 21.5 in.$^2$×12.0 ksi = 258.0 k/pile

Pile Properties

\[ A_s = 21.5 \text{ in.}^2 = 0.149 \text{ ft}^2 \]
\[ d = 13.64 \text{ in.} = 1.13 \text{ ft} \]
\[ b_f = 14.536 \text{ in.} = 1.21 \text{ ft} \]
\[ p = 2(d + b_f) = 4.68 \text{ ft} \]

FOOTING SIZE AND PILE PATTERN
Estimate the required number of piles by assuming that the vertical load will cause one-half of the total pile load.

No. of piles = \( \frac{1727}{2580} \times 2 = 14 \) piles

Assume the footing dimensions and pile pattern shown. The piles are concentrated near the periphery of the footing to resist the overturning moments.
Moment of Inertia of Pile Group

\[ \Sigma x^2 = 2 \times 4(9)^2 + 2 \times 3(3)^2 = 648 + 54 = 702 \text{ pile-ft}^2 \]

\[ \Sigma y^2 = 2 \times 4(7.5)^2 + 2 \times 2(2.5)^2 = 450 + 25 = 475 \text{ pile-ft}^2 \]

**FORCES AT TOP OF PILES**

Transfer the forces and moments acting at the top of the footing for Groups III and IX loading down to the top of the piles. To these forces add the weight of the footing, the weight of the tremie seal, and the saturated weight of the soil and water acting on the footing and tremie seal. Upward buoyant forces for the various river stages are considered to act on the bottom of the tremie seal.

Since the footing and piles are below the ground surface, impact is deducted from the loading.

To determine the above weights assume the tremie seal to be 23\times20\times5 \text{ ft thick.}

**Group III—Dead Load + Live Load + Friction + Stream Flow + 0.3 Wind + Wind on Live Load + Traction + Centrifugal Force + Buoyancy at Normal Pool**

<table>
<thead>
<tr>
<th>Loading</th>
<th>( F_v ) (kips)</th>
<th>Transverse Direction</th>
<th>Longitudinal Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( H_x ) (kips)</td>
<td>( M_x ) (kip-ft)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( H_y ) (kips)</td>
<td>( M_y ) (kip-ft)</td>
</tr>
<tr>
<td>Forces at top of footing— Water at Normal Pool</td>
<td>1727</td>
<td>76.0</td>
<td>5804</td>
</tr>
<tr>
<td>Impact</td>
<td>-45</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Wt. of footing</td>
<td>340</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Wt. of tremie seal</td>
<td>345</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Wt. of sat. soil</td>
<td>154</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Wt. of water</td>
<td>123</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Buoyancy</td>
<td>-515</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>( H_x(5.0) = 76.0 \times 5.0 )</td>
<td>—</td>
<td>—</td>
<td>380</td>
</tr>
<tr>
<td>( H_y(5.0) = 93.1 \times 5.0 )</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Total Group III at Normal Pool</td>
<td>2129</td>
<td>76.0</td>
<td>6034</td>
</tr>
</tbody>
</table>

**Group III—Dead Load + Live Load + Stream Flow + 0.3 Wind + Wind on Live Load + Traction + Centrifugal Force + Friction + Buoyancy at High Water**

<table>
<thead>
<tr>
<th>Loading</th>
<th>( F_v ) (kips)</th>
<th>Transverse Direction</th>
<th>Longitudinal Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( H_x ) (kips)</td>
<td>( M_x ) (kip-ft)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( H_y ) (kips)</td>
<td>( M_y ) (kip-ft)</td>
</tr>
<tr>
<td>Forces at top of footing— at High Water</td>
<td>1727</td>
<td>81.5</td>
<td>5875</td>
</tr>
<tr>
<td>Impact</td>
<td>-45</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Wt. of footing</td>
<td>340</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Wt. of tremie seal</td>
<td>345</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Wt. of water</td>
<td>394</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

(Continued)
(Table Continued)

<table>
<thead>
<tr>
<th>Loading</th>
<th>$F_v$ (kips)</th>
<th>Transverse Direction</th>
<th>Longitudinal Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wt. of sat. soil</td>
<td>154</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Buoyancy</td>
<td>-830</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$H_z(5.0) = 31.5 \times 5.0$</td>
<td>—</td>
<td>—</td>
<td>418</td>
</tr>
<tr>
<td>$H_y(5.0) = 90.8 \times 5.0$</td>
<td>—</td>
<td>—</td>
<td>454</td>
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<tr>
<td>Total Group III</td>
<td>2085</td>
<td>81.5</td>
<td>6143</td>
</tr>
<tr>
<td>at High Water</td>
<td></td>
<td>90.8</td>
<td>5443</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Loading</th>
<th>$F_v$ (kips)</th>
<th>Transverse Direction</th>
<th>Longitudinal Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forces at top of footing—</td>
<td>1476</td>
<td>188.6</td>
<td>4963</td>
</tr>
<tr>
<td>at Normal Pool</td>
<td></td>
<td></td>
<td>41.3</td>
</tr>
<tr>
<td>Wt. of footing</td>
<td>340</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Wt. of tremie seal</td>
<td>345</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Wt. of sat. soil</td>
<td>154</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Wt. of water</td>
<td>123</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Buoyancy</td>
<td>-515</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$H_z(5.0) = 188.6 \times 5.0$</td>
<td>—</td>
<td>—</td>
<td>943</td>
</tr>
<tr>
<td>$H_y(5.0) = 41.3 \times 5.0$</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Total Group IX</td>
<td>1923</td>
<td>188.6</td>
<td>5911</td>
</tr>
<tr>
<td>at Normal Pool</td>
<td></td>
<td>41.3</td>
<td>206</td>
</tr>
</tbody>
</table>

### PILE LOADING

$$Q_n = \frac{F_v}{r} \pm \frac{M_x}{\Sigma x^2} \pm \frac{M_y}{\Sigma y^2}$$  \hspace{1cm} (Eq. B)

#### Group III at Normal Pool

Max. pile load, $Q_n = \frac{2129}{14} + \frac{6034 \times 9.0}{702} + \frac{5461 \times 7.5}{475}$

$$= 152.1 + 77.4 + 86.3$$

$$= 315.8 \text{ kips} < 258 \times 1.25 = 322.5 \text{ kips}$$

*Governs for max. pile load*

Min. pile load, $Q_n = 152.1 - 77.4 - 86.3$

$$= -11.6 \text{ kips}$$
Group III at High Water

Max. pile load, \( Q_m = \frac{2085 \times 6148 \times 9.0}{14 + 702} + \frac{5443 \times 7.5}{475} \)

= 148.9 + 78.8 + 85.9

= 313.6 kips < 259 \times 1.25 = 322.5 kips

Min. pile load, \( Q_m = 148.9 - 78.8 - 85.9 \)

= −15.8 kips

Govern for max. uplift

Group IX

Max. pile load, \( Q_m = \frac{1924 \times 5911 \times 9.0}{14 + 702} + \frac{2067 \times 7.5}{475} \)

= 137.4 + 75.8 + 32.6

= 245.8 kips < 259 \times 1.5 = 387.0 kips

Min. pile load, \( Q_m = 137.4 - 75.9 - 32.6 \)

= + 29.0 kips

ULTIMATE PULL OUT CAPACITY

The ultimate pull out capacity of a pile is equal to the skin friction developed between the pile and soil plus the weight of the pile. The skin friction value is:

\[ Q_u = \frac{1}{2} spK_b \gamma^D \tan \delta \]  

(Eq. 4)

Using Fig. 8 and \( \phi = 30^\circ \), \( K_b = 0.40 \)

\[ Q_u = \frac{1}{2} \times 4.68 \times 0.40 \times 0.063(38)^2 \times \tan 20^\circ \]

= 30.9 kips

Pile weight = 0.073 lb/ft \times 38 \text{ ft} = 2.77 kips

\[ Q_u = 30.9 + 2.77 = 33.67 \text{ kips} \]

Using a safety factor = 2.5

Allowable uplift = \[ \frac{33.67}{2.5} = 13.5 \text{ kips} \]

For Group III Loading at High Water

Uplift force = 15.8 < 13.5 \times 1.25 = 16.8 kips

:. The footing size and pile pattern are satisfactory.

DETERMINE IF BATTERED PILES ARE REQUIRED

The limited data available indicates that the lateral capacity of a pile group is approximately equal to the capacity of a single free-headed pile times the number of piles in the group. Assume a lateral movement of \( \frac{1}{4} \) inch is permissible. From Fig. 26 the allowable lateral load for a single free-headed HP 14 \times 73 pile in saturated sand with \( \phi \) equal to 30° is 7.0 kips per pile perpendicular to the pile flange and \( \frac{3}{8} \times 7.0 \) or 4.67 kips per pile parallel to the pile flange.

The total allowable lateral load parallel to the x-axis is:

\[ (8 \times 7.0) + (6 \times 4.67) = 84.0 \text{ kips} \]

The total allowable lateral load parallel to the y-axis is:

\[ (8 \times 4.67) + (6 \times 7.0) = 79.3 \text{ kips} \]
Group III Loading at High Water

\[ H_z = 81.5 < 84.0 \times 1.25 = 105.0 \text{ kips} \]

\[ H_v = 90.8 < 79.3 \times 1.25 = 99.1 \text{ kips} \]

Group IX Loading

\[ H_z = 188.6 > 84.0 \times 1.5 = 126.0 \text{ kips} \]

\[ H_v = 41.3 < 79.3 \times 1.5 = 119.0 \text{ kips} \]

Therefore it is required to batter the outer row of piles parallel to the x-axis. Large horizontal loads are applied to the pier infrequently. A symmetrical pattern of pile batter is provided so that the horizontal components of load in all battered piles sums up to approximately zero under the usual loading condition of vertical load only on the pier.

Batter four piles upstream and four piles downstream.

\[ \text{Load to be taken by the battered piles is:} \]

\[ \Delta H_r = 188.6 - 126.0 = 62.6 \text{ kips} \]

The average pile load on the downstream piles = 137.4 kips + 75.8 = 213.2 kips

The average pile load on the upstream piles = 137.4 - 75.8 = 61.6 kips

The pile batter can be specified as \( n \) horizontal : 12 vertical. The required pile batter can then be found by the following ratio:

\[ n \frac{12}{12} = \frac{62.6}{4(213.2 - 61.6)} \]

\[ n = \frac{12 \times 62.6}{4 \times 151.6} = 1.238 \]

Batter outer four piles, perpendicular to x-axis, \( 1\frac{1}{2} \) horizontal: 12 vertical. Thus the maximum axial pile load is:

\[ \text{Max. } Q_m = 315.8 \times \frac{\sqrt{(1.5)^2 + (12)^2}}{12} \]

\[ = 318.0 \text{ kips} < 258 \times 1.25 = 322.5 \text{ kips} \]

:. Pile section is adequate
SETTLEMENT AT TOP OF PILES

Since the piles are driven to rock bearing, the only settlement will result from elastic deformation of the piles.

\[
\text{Dead load per pile} = \frac{1476}{14} = 105.4 \text{ kips}
\]

\[
\Delta_{DL} = \frac{QL}{A,E} = \frac{105.4 \times 38 \times 12}{21.5 \times 29 \times 10^3}
\]

\[
= 0.0771 \text{ in.} = \frac{1}{8} \text{ in.}
\]

Generally the elastic deformation is small and need not be calculated.

TREMIE SEAL

The weight of the tremie seal plus the skin friction on the piles must resist the buoyant force on the bottom of the tremie seal. The critical condition for design of the tremie seal and resulting uplift on the piles occurs when the stream level is slightly below the top of the cofferdam.

\[
\text{Buoyant Force} = 23 \times 20 \times 21 \times 0.0624 \text{ k/cf}
\]

\[
= 603 \text{ kips}
\]

Uplift to be resisted by piles = 603 – 345

\[
= 258 \text{ kips}
\]

Uplift/pile = 258/14 = 18.4 kips

The ultimate pull out capacity = 33.67 kips

\[
\therefore \text{Safety factor} = \frac{33.67}{18.4} = 1.83; \text{ adequate for temporary construction.}
\]

Check Bond Between Pile and Tremie Seal

Section 1.5.1 (C)(3) AASHO Specification, specifies an allowable bond between pile and seal of 10 psi.

\[
\text{Perimeter of pile} = (14.586 \times 4) + (13.64 \times 2)
\]

\[
= 85.62 \text{ in.}
\]

Allowable uplift on piles = 85.62 in. \times 5.0 \times 12 \times 0.01

\[
= 51.4 \text{ kips} \geq 18.4 \text{ kips actual}
\]

Thus the 5.0 ft seal thickness is adequate.